Saving Humpty Dumpty: Designing a Bungee Jump for an Egg

Introduction:

In order to design a bungee jump for an egg, we modeled a bungee jump with masses similar of that of an egg by using kinematics equations. A bungee jump is a conservative system – or – a system that maintains the same amount of energy from beginning to end. Initially, the only energy acting in a bungee jump is the potential energy due to gravity. The bungee cord exhibits spring like behavior, so therefore, the final energy can be described using the potential energy of a spring. Because the system satisfies the conditions of the conservation of work and energy - CWE theorem, we chose to base our model on this equation

$$(PE+KE)top=(PE+KE)bottom$$
$$mgh=1/2\ kx^2$$

where m is the mass of the weights hung on the bungee cord, g is the surface gravitational pull of the earth, h is the total length that the weight fell, k is the spring constant of the bungee cord, x is the difference between the maximum displacement the cord incurs from the jump and the original length of the bungee cord.

In order to predict the k value of a long bungee cord that will be used a future bungee jump, we found the k value of a bungee cord at different shorter lengths. We did this by recreating a bungee jump with a variety of different masses, and using the CWE theorem to find k of the shorter strings.

Methods:

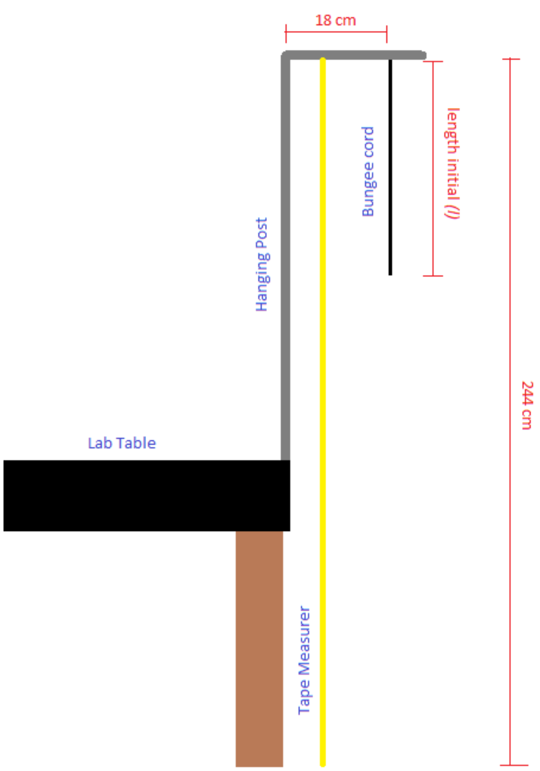
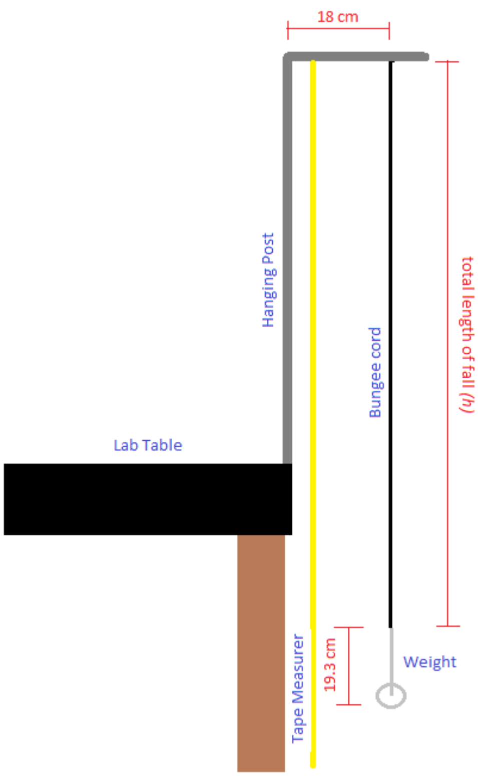


Figure 1:
Depiction of the setup of the Bungee Jump model.

We set up a hanging post on the edge of our lab table, where the length from the top of the post to the floor was 244 cm. From the top of the post, we attached a tape measurer in order to easily document the maximum displacement of the bungee jump easily. The bungee cord was tied to a knob 18 cm away from the vertical post in order to leave room for the bungee weight to bounce without interference. A special loop knot was used to attach the cord in order to not disturb the spring qualities of the rope. In order to test the k value, we chose three different initial lengths of the bungee cord: 20 cm, 40 cm, and 60 cm.

To test how the k value varies with added mass, we attached a 19.3 cm long hanging mass to the bungee cord that weighed either 100, 110, 120, 130, 150, 170 grams. We chose the weights to appropriately match the possible weights of the egg in the future bungee jump, 100-170 grams. To attach the hanging mass, we looped the knot three times around the top of the hanging mass and secured the weights with tape.

For each mass and length, we dropped the hanging mass from the top of the hanging post, lining up the



top of the hanging mass with the hanging post. We would do a test drop in order to approximate the maximum displacement of the bungee cord. At this area, we set up an iPad to record the drop with the CoachMyVideo application. The CoachMyVideo application records video that can later be slowed down and paused in order to further analyze the video. By using this application, we were able to get a more accurate read on the total length of the fall.

For each length and mass, we recorded three bungee jumps. We slowed down the recordings in order to get an accurate reading on the furthest place the weight dropped down. We took the readings from the bottom of the weight, and therefore, had to subtract the height of the weight (19.3 cm) from each reading, in order to isolate the bungee cord. We then took the average total length of fall. In order to calculate the displacement of the cord (x), we subtracted the initial length of the bungee cord from the total length of the fall (h). We then used the averages in the CWE theorem in order to find out the k value. We then took the average of all of the k values in order to find the average k value for the string at each length.

Figure 2:
Depiction of our Bungee Model at the largest displacement of the jump

Results:
In analyzing our data, we were able to show that k of the bungee cord had an inverse relationship to the length of the cord.

Length: 20 cm

mass m	length of total fall *h*	Standard Deviation for *h*	displacement *x*	Standard Deviation for *x*	K value (N/m)
(g, ± 1 g)	(cm, ± 1 cm)		(cm, ± 1 cm)		± 1 (N/m)
100	78	1	39	0	46
110	82	1	43	1	46
120	85	1	46	1	47
130	89	1	50	1	47
150	97	1	58	1	48
170	108	1	69	1	46
					Average k value:47(N/m)

Figure 3: Data Table for the 20 cm Bungee Cord
Data table describing the mass, the total length of the fall h, displacement x, and k value, calculated using the CWE theorem.

The data listed in Figure 3 shows the data for the bungee cord at the length of 20 cm. We calculated the length of the total fall *h* and the displacement *x* by taking the averages from the three trials we had with each mass. We calculated the standard deviation for these values, thus giving us our uncertainty. We used the CWE theorem in order to solve for k. As expected, all of the original k values are similar. We took the average of these k values in order to assign an average k value for the length of the 20 cm string.

Length: 40 cm

mass m	length of total fall *h*	Standard Deviation for *h*	displacement *x*	Standard Deviation for *x*	K value (N/m)
(g, ± 1 g)	(cm, ± 1 cm)		(cm, ± 1 cm)		± 2(N/m)
100	147	1	87	1	25
110	159	2	99	2	24
120	163	3	104	3	25
130	173	1	114	1	25
150	191	2	132	2	25
170	212	1	152	1	24
					Average k value: 25 (N/m)

Figure 4: Data Table for the 40 cm Bungee Cord
Data table describing the mass, the total length of the fall h, displacement x, and k value, calculated using the CWE theorem.

The data listed in Figure 4 shows the data for the bungee cord at the length of 40 cm. Just as in the previous situation, we took the averages of the three trials we did at each mass in order to get the *h* and *x* values. We calculated the k value using the CWE theorem, and again, came up with similar k values for the string. However, these k values were smaller than the values of the first string's (refer to Figure 3).

Length: 60 cm

mass m	length of total fall *h*	Standard Deviation for *h*	displacement *x*	Standard Deviation for *x*	K value
(g, ± 1 g)	(cm, ± 1 cm)		(cm, ± 1 cm)		± 1(N/m)
100	213	3	194	3	18
110	228	2	208	2	17
120	241	2	222	2	17
					Average k value: 17(N/m)

Figure 5: Data Table for the 60 cm Bungee Cord
Data table describing the mass, the total length of the fall h, displacement x, and k value, calculated using the CWE theorem.

The data listed in Figure 5 shows the data for the bungee cord at the length of 60 cm. We did not have any mass measurements for this data table because a jump with heavier masses would hit the floor. We averaged the heights and displacements from our three trials, and calculated the standard deviations and the k values. The k values at this length were the lowest.

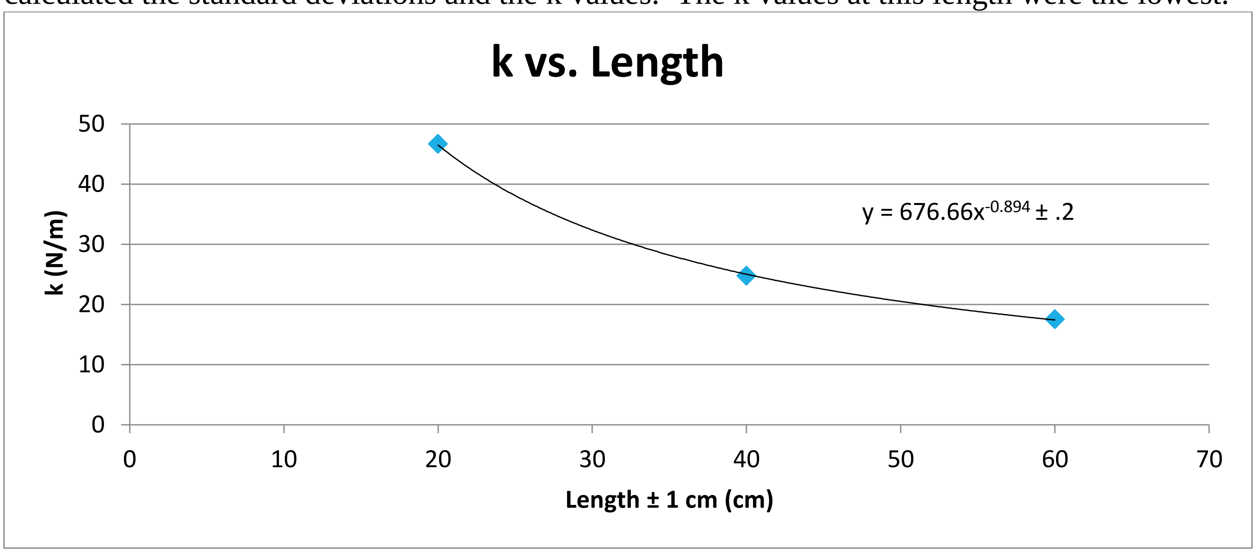


Figure 6: Graph of the k vs. length of the bungee cord
Graph showing the k value on the y-axis and the length of the bungee cord on the x-axis. The power function was fitted to the trendline.

We graphed the average k values for each length and the lengths in order to get a better understanding between them. The graph shows an inverse relationship between the k and the length of the bungee cord. Interestingly, the trendline fit near perfectly to the power function, with a .02% uncertainty of the slope value, found by taking regression analysis of the slope.

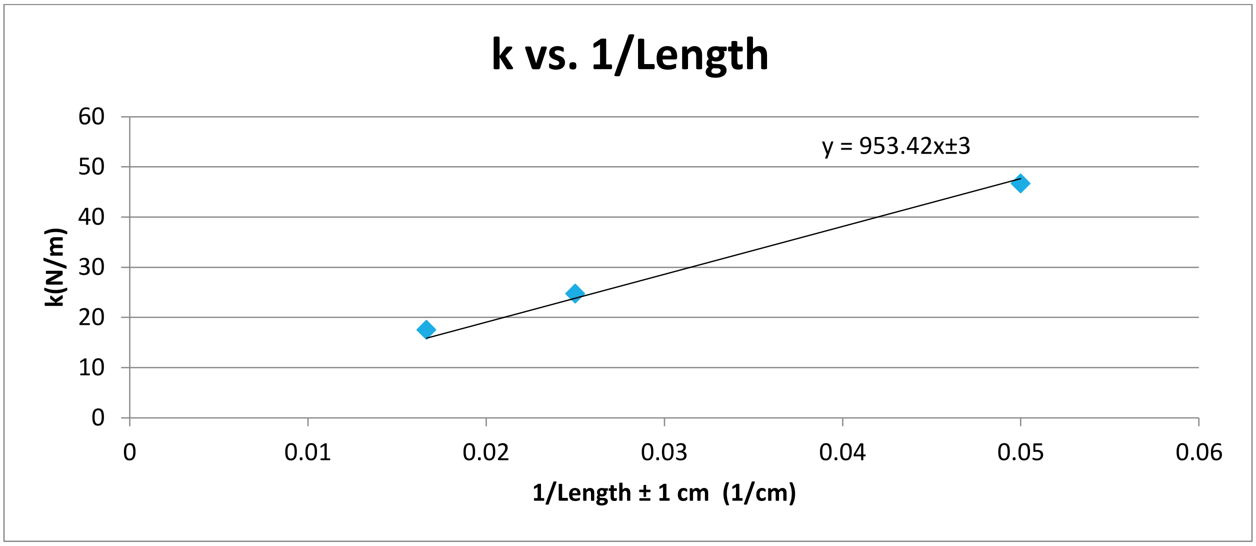


Figure 7: Graph of the k vs. 1/length of the bungee cord
Graph showing the k value on the y-axis and the inverse of the length of the bungee cord on the x-axis. Shows a linear relationship.

In order to give us a better model for when we do our bungee jump, we graphed the k value with the inverse of the length of the bungee cord. This gave us a linear graph. Using this relationship, we can predict the k value of our bungee cord for the bungee experiment by taking the inverse of the length. Using data analysis on Excel, we can find the percent uncertainty of the slope to be 0.3%.

Discussion:

Our results show that there is an inverse relationship between k value of a spring and the length of the bungee cord. Our results also show that the length of the total displacement (h), k value (k) and total displacement (x) all depend on the initial length of the bungee cord, (l). The percent uncertainty of the slope in this relationship can describe our percent uncertainty for the k values. We found this uncertainty using data analysis of the inverse of the length and the k values in Excel. Percent uncertainty of the k value is 0.3%.

We chose to put the k vs 1/length graph (Figure 7) through the origin because the k-value does not have an initial y-intercept that needs to be accounted for.

Through the CWE theorem, we can see that the value of the k constant is proportional to value of h, or the total length that the weight mass will fall

$$mgh=1/2\ kx^2$$

Because of this relationship, the uncertainty for the k value will be at least one of the uncertainties for the h value. Looking forward to our model of the future bungee jump, this is an important factor because we will need to predict our future h value at a larger height. However, our uncertainty for k is very small. If the uncertainty of the k value was the only uncertainty we had to factor in to the total length of displacement (h), our uncertainty for a height of 900 cm would be 2.7 cm.

Our sources of uncertainty can come from inaccurate measuring of the length of the drop. Even though we tried to be accurate about the way we went about measuring the length of the drop, it was sometimes very hard to tell, even on the paused video. Another source of possible uncertainty is in the bungee cord. After several drops, the bungee cord fibers could become looser, thus changing the k value. We made sure to not put weight on the bungee cord in between trials, but the process of experimentation could have stretched out the cord. In the process of dropping the weights, some of the weights would occasionally become loosened. Even though we used tape to secure the weights, sometimes they would slide off. If one of the added weights were to slide off a little, the center of mass of the hanging weight would change, and that might alter the length of the jump. The CWE theorem assumes that the mass starts at rest, and if the person dropping the weight even added a little bit of force, this would change our theorem. This could also contribute to our uncertainty. In order to reduce uncertainty, we would recommend making sure to not stretch out the cord, and making sure the person dropping it is dropping from absolute rest.

Our results are in line with theory. Because of Hooke's law, we know that a spring with a larger spring constant will have a smaller displacement

$$F = -kx$$

Where force is the force pulling the spring, k is the spring constant and x is displacement. Therefore, the bungee cord with the smallest length will have the highest spring constant, even though it has the same properties and force acting on it as the longer cords.

Conclusion:

Our experiment proved that the spring constant and initial length of the bungee cord are inversely proportional. Using this relationship, we can predict the final stretch of a bungee cord by knowing the mass, initial height, and the length of the bungee cord. Therefore, we can use the equation from our graph to predict the displacement for our future bungee experiment. In order to make a more solid model of our experiment, we should get a wider range of initial bungee cord lengths. However, we would have to make a new bungee model in order to account for a larger displacement.