Name: Laura Beth Lavette
Partner: Abby Block Section 113-01
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Bungee II:
Determining an Ideal Cord Length for the Bungee Jump

I. Introduction

In this lab, we aimed to compare our working k from the *Bungee I* experiment, which related the length of our cord to a "spring" constant k, to a newly determined k value, which would be calculated through kinetic (rather than static) experimentation. By comparing two different k values, one measured using Hooke's Law and the other using the conservation of work and energy theorem, we hoped to come up with a working coefficient that accounted for the error and non-ideal characteristics of our bungee cord system. Below is the equation we derived from *Bungee I* using Hooke's Law, which assumes an ideal spring is at work:

$$k = (L-1.0134) / -1.6715 \quad (1)$$

In this equation, the k represents our "working" k and the L represents the varying cord length. As the equation implies, the "spring" constant k increases as the length of the cord decreases. To determine a second k in terms of the conservation of work and energy, we used the following equation comparing the gravitational potential energy to an ideal spring's potential energy:

$$mgh = \frac{1}{2} kx^2 \quad (2)$$

M is mass of the hanging object, g is constant gravitational acceleration (9.8 m/s^2), h is height, k is the unknown "spring" constant, and x is the total displacement. Through this equation and a known mass, height, and kinetic displacement, we can compare both sets of our k values to find a coefficient that accounts for the deviance of our system from an ideal system. Finally, using this coefficient, we constructed a final equation that provided us with an ideal cord length, L, to coincide with our Egg Drop system.

II. Methods

After testing the static effect of mass on our cord's displacement in *Bungee I,* we decided to calculate a second working k based on the kinetic characteristics of the cord. By measuring the total displacement at varying cord lengths (L), we would be able to calculate a second list of k values using equation 2, which we could then compare to k values derived from our original experimental equation 1. We began our experiment by developing the below set up and using a constant hanging mass (m) of 0.05 kg, our bungee cord, a tape measure, and the slo-mo application on our phones:

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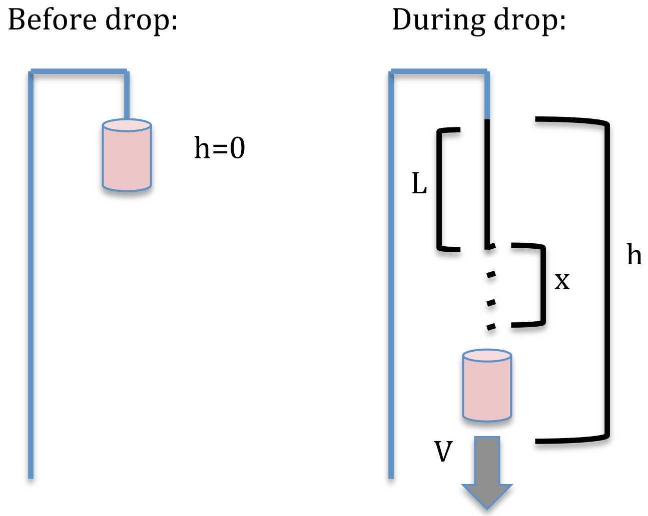

Figure 1. Set-up for dropping mass and measuring vertical displacement. In this set up, we used a constant hanging mass (m), varied the cord length (L) for each trial, and measured the maximum length of the fall, which we deemed as the height (h) of the system. We could then determine displacement (x) by subtracting length of cord from h (x=h-L).

For each of our ten trials, we began by reconstructing the length of the cord and attaching the same hanging mass of 0.05 kg. We used ten different cord lengths as our independent variable: 0.192, 0.317, 0.341, 0.392, 0.513, 0.559, 0.657, 0.717, 0.857, and 0.934 m. We attempted to tie the cord as tightly as possible each time in order to minimize error. Additionally, we attached the tape measure to the top of rod along with our cord so we would not have to hold it up each time and it would be in the same position for each trial. We then measured the length of the cord without any mass attached (which later allowed us to find the total displacement). Next, after attaching the mass, I dropped the object from the same point at the top of the rod while Abby recorded the drop on her phone. Using our recording in slo-mo, we were able to estimate the distance of the fall, which we used to represent the height of the system. Once we had determined ten different heights for our ten different cord lengths, we were able to calculate the maximum displacement for each trial by subtracting the cord length from the height (x=h-L). Finally, using equation 2, we calculated ten different "working" k values to reflect the kinetic characteristics of our cord and ultimately compare to our results from *Bungee I.*

III. Results

Our results provided us with a second list of k values derived from kinetic experimentation; using these k values, we were able to compare static and kinetic "working" k's, determine a coefficient that accounted for non-ideal cord characteristics, and come up with a final equation to find an ideal cord length for the bungee jump. First, after we measured the height for each free-fall trial, we were able to use our known cord lengths to determine the X_{max} by subtracting the length of each cord from the height:

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Table 1. X_{max} calculated through known cord length and height of drop. This method allows us to use total displacement as height and find the vertical displacement (x) using two known values.

Length (± 0.001 m)	Height (± 0.001 m)	X max (± 0.001 m)
0.717	1.320	0.603
0.317	0.605	0.288
0.392	0.735	0.343
0.559	1.065	0.506
0.857	1.560	0.703
0.934	1.680	0.746
0.192	0.375	0.183
0.341	0.635	0.294
0.513	0.945	0.432
0.657	1.220	0.563

Next, using equation 2, a constant mass of 0.05 kg, the gravitational acceleration constant of 9.8 m/s^2, our known heights (m), and our calculated (x) distances from Table 1, we were able to calculate k values for each different cord length:

Table 2. Determining "working" k values based on kinetic experimentation. This data suggests that as the length of the cord increases, the k value decreases, which is consistent with the findings from our static experimentation.

Cord Length (± 0.001 m)	Calculated k (± 0.001 N/m)
0.717	3.558
0.317	7.148
0.392	6.122
0.559	4.076
0.857	3.093
0.934	2.958
0.192	10.973
0.341	7.200
0.513	4.962
0.657	3.772

Using our data from Table 2 along with equation 1 determined from *Bungee I* data, we were able to compose a graph that compared cord length to "working" k values, both from *Bungee I* and *Bungee II.* This way, we had a visual representation of the differences between our static and kinetic results:

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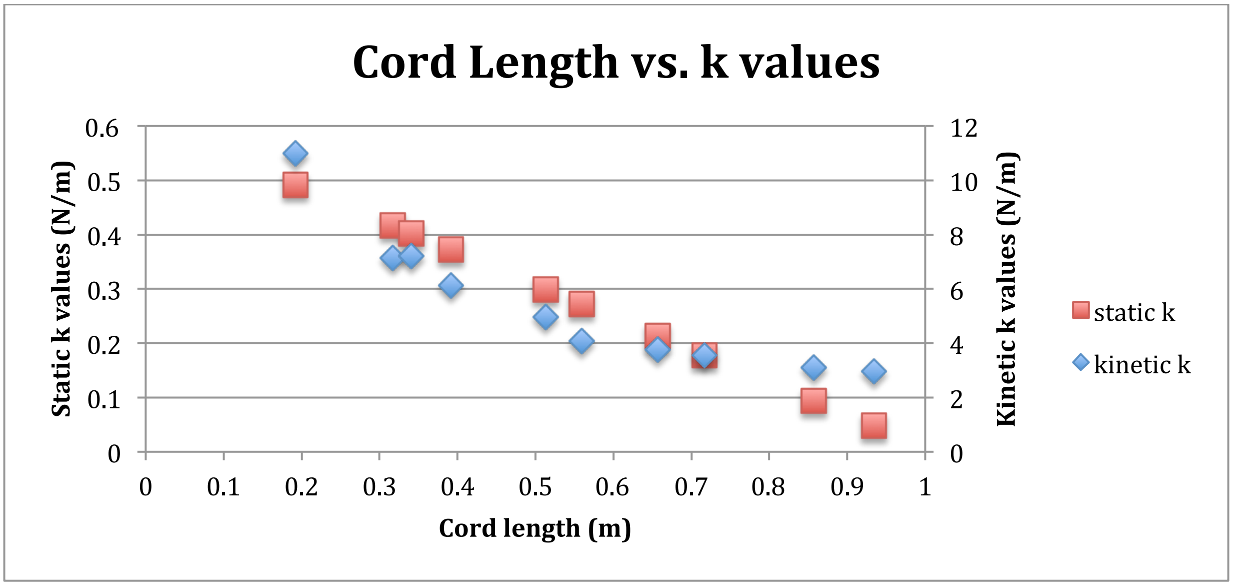

Figure 2. Comparison of static and kinetic k values based on varying cord length. Static k values were determined using equation 1 from our static results in *Bungee I.* Kinetic k values were calculated in Table 2.

Looking to this figure, we can see that the static and kinetic behaviors of our cord follow a nearly identical trend. In other words, as the cord length increases, both sets of k values appear to decrease linearly. However, looking to the different values in the two y-axes, we can see that there is a large disparity between the magnitudes of the k values. Therefore, we decided to find a consistent coefficient that accounts for the differences between the static and kinetic characteristics of our cord. To do this, we began by calculating the percent differences between our static and kinetic k's at each different cord length:

Table 3. Calculated differences between static k values and kinetic k values using $(k_1-k_2)/k_{avg}$. The consistency in difference implies that there is a significant relationship between the static and kinetic characteristics of the bungee cord.

Cord Length (± 0.001 m)	Kinetic k (± 0.001 N/m)	Static k (± 0.001 N/m)	Difference coefficient (± 0.001)
0.717	3.558	0.177	1.810
0.317	7.148	0.417	1.780
0.392	6.122	0.372	1.771
0.559	4.076	0.272	1.750
0.857	3.093	0.0936	1.883
0.934	2.958	0.0475	1.937
0.192	10.974	0.491	1.829
0.341	7.200	0.402	1.788
0.513	4.962	0.299	1.772
0.657	3.772	0.213	1.786

With little disparity between difference coefficients, we went ahead and calculated the average difference between static "spring" constant k and kinetic "spring" constant k to be 1.81. That is, the kinetic k value is consistently about 1.81 times larger than the static k value. Finally, we applied this correction coefficient to our two working equations, one based on the conservation of energy and the other based on Hooke's Law:

$$2mgh / x^2 = 1.81 \times ((L-1.0134)/-1.6715) \quad (3)$$

Which simplifies to:

$$L = (-1.85mgh) / x^2 + 1.0134 \quad (4)$$

Using this final equation, we should be able to determine an ideal length for our bungee cord using known values of mass, height, and total vertical displacement of the free fall.

IV. Discussion

While our final results produced a seemingly effective means of finding an ideal length for our bungee jump cord, testing our results revealed that our data was slightly skewed and the egg will most likely not get close enough to the ground for a full free-fall experience. Because we did not have an accepted value to compare to our final results, we devised a hypothetical free fall, coming up with our own height, mass, and displacement and determining the ideal cord length to use. For this experiment, we replicated the set-up shown in figure 1, using a mass of 0.070 kg, measuring the total height from the floor to the top of the rod to be 2.08 m, and setting our goal displacement to be 2.00 m. Using equation 4, we determined the ideal cord length for our free-fall to be 0.353 m. However, when we conducted the trial run, our hanging mass did not fall close enough to the ground; therefore, our cord length was too short for a successful free fall.

Because equation 4 appears to be an ineffective means of finding an ideal cord length, we must look to possible sources of error and re-evaluate our methods. In our *Bungee II* experiment, we chose to vary cord length in each of our trials, which required untying and retying multiple knots and likely could have accounted for discrepancies in our measurements and results. Furthermore, the phone slo-mo application was not an ideal way to measure the displacement of the object because it was hard to capture the movement on film, the picture was oftentimes blurry on camera, and it was fairly difficult to see the exact point at which the object was at its maximum displacement. Therefore, each of our measurements was an estimate, which could have significantly impacted our final results, especially when working with such small values.

Moving forward, it would be beneficial to conduct more trials in both the static and kinetic experiments, but especially in the kinetic experiment of *Bungee II.* Because it became so difficult to measure the exact distance of the free fall, conducting more than ten trials could increase the consistency and accuracy of our results. More trials would give us more k values to compare to our static equation 1 and could potentially improve our results. However, if we are unable to re-conduct the experiment in *Bungee II*, another way of moving forward is working with our current equation 4, creating more mock trials to test its degree of effectiveness, and finding a consistent discrepancy in the equation. Once we determine a consistent difference between the theoretical displacement of the hanging mass and the actual displacement of the hanging mass (which we could find using a similar set up to figure 1), we could alter our final equation to account for the error. Due to a time limitation, we were unable to follow through with this final step; however, moving forward, this could be an effective means of altering our final results and equation in order to provide the egg with a more enjoyable free-fall experience.

V. Conclusion

We essentially fulfilled the original purpose of our experiment, which was to develop a final equation for an ideal cord length, taking into account the difference between our static and kinetic "working" k values (equation 4). Although we were able to use a method of comparison to construct a final equation, testing our results revealed that our equation is not a completely reliable means of finding an ideal cord length for the bungee jump. In other words, when testing our results, we found that our cord length in a hypothetical bungee jump would be too small, not allowing the egg to fall close enough to the ground. However, if allowed more time before conducting the final experiment, we could use our working equation 4, create more trial runs with hypothetical free-fall conditions, and determine a consistent error in our equation. We could then alter our final equation to account for this error, which would ultimately allow for a more successful bungee jump.