Investigating the Relationship Between Spring Constant and Length in a Bungee Cord

In this experiment, we were interested in deepening our understanding of how a bungee cord's spring constant is affected by the length of the bungee cord when an object is dropped. This value is of interest because of our overall goal of dropping an egg using a bungee cord without breaking the egg. The conceptual basis for this experiment is the equation $\frac{1}{2}k(\Delta y)^2 = mgy$, where k is the spring constant, Δy is the maximum displacement of the mass, m is the mass of the object, and y is the maximum stretched length of the bungee.

For every trial, we used a mass of .135 kg, because that is approximately the mass of an egg. First, we measured the length of the unstretched cord. Then, we hung the mass on then end of the cord, dropped the mass from the top of the cord, and measured using a slow-motion video the maximum length of the stretched cord. The independent variable in this experiment was the initial length of the bungee cord. The dependent variable we measured was the distance the bungee cord stretched when the mass, which was kept constant, was dropped. For every trial, we measured Δy and y to calculate the spring constant, k, for each length of cord.

Our experiment successfully found a mathematical formula relating the the k value to the initial unstretched length of cord in the equation: $k = 16.766\,(y_{0(unstretched)})^2 + 20.604\,(y_{0(unstretched)}) + 8.5922$ +/-0.048. Using this formula, it is possible to calculate the k value for varying upstretched, initial lengths of cord. Furthermore, this k value can be substituted into the equation $\frac{1}{2}k(\Delta y)^2 = mgy$ to calculate the maximum displacement of the mass after being dropped. This relationship between the three values, $y_{0(unstretched)}$, $y_{(stretched)}$, and k, allows us to more accurately predict the length of our initial unstretched cord when dropping our egg from the third floor of the science center without cracking the egg on the floor. The results of this experiment allow us to more accurately predict the initial length of a bungee cord necessary to drop the egg all the way to the ground without breaking it.

The percent uncertainty for our equation was +/-0.048 and our fractional error was 0.093. Our most significant source of uncertainty is the fractional error as the initial length of the cord decreases. Expanding our experimental area beyond the confines of the classroom would alleviate some of this uncertainty.

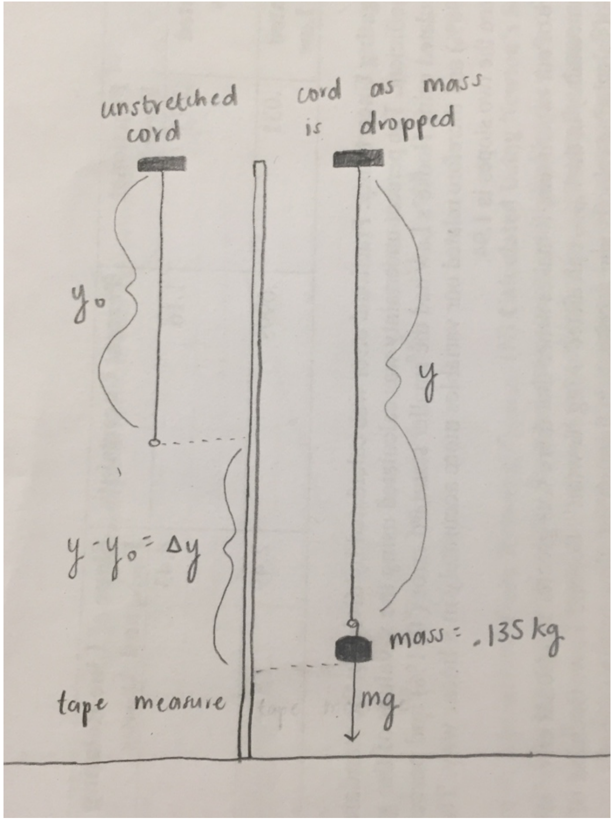

Figure 1: **Diagram of Experimental Variables** Above depicts the two lengths of the bungee cord for one experimental trial. y_0 is the upstretched length of the bungee cord without a mass attached. y is the maximum stretched length of the bungee cord after the mass was dropped from the top of the cord. The Δy value is also evident in the diagram because it is found by subtracting y_o from y. It is important to note that the mass used in every trial was .125 kg during every trial.

$y_{0(unstretched)}$	$Y_{(stretched)}$	Δy (m)	$k_{experimental}$ (N/m)	$k_{poynomial}$ (N/m)	$St.Dev_{norm}k_{polynomial}$ (+/-)
.561	2.11	1.55	2.34	2.31	0.048
.480	1.90	1.42	2.50	2.57	0.048
.440	1.71	1.27	2.81	2.77	0.048
.365	1.49	1.13	3.13	3.30	0.048
.315	1.21	0.893	4.03	3.77	0.048
.250	1.05	0.796	4.39	4.49	0.048

Figure 2: **Experimental and Calculated Values** Above is the raw data collected in addition to the calculated k values. It is significant to note that as the length of the cord increases, the k value decreases. Δy was calculated by subtracting y_o from y. The uncertainty value, the normalized standard deviation of the $k_{polynomial,}$ was calculated by taking the standard deviation of the k values for every trial and then dividing by the average k value.

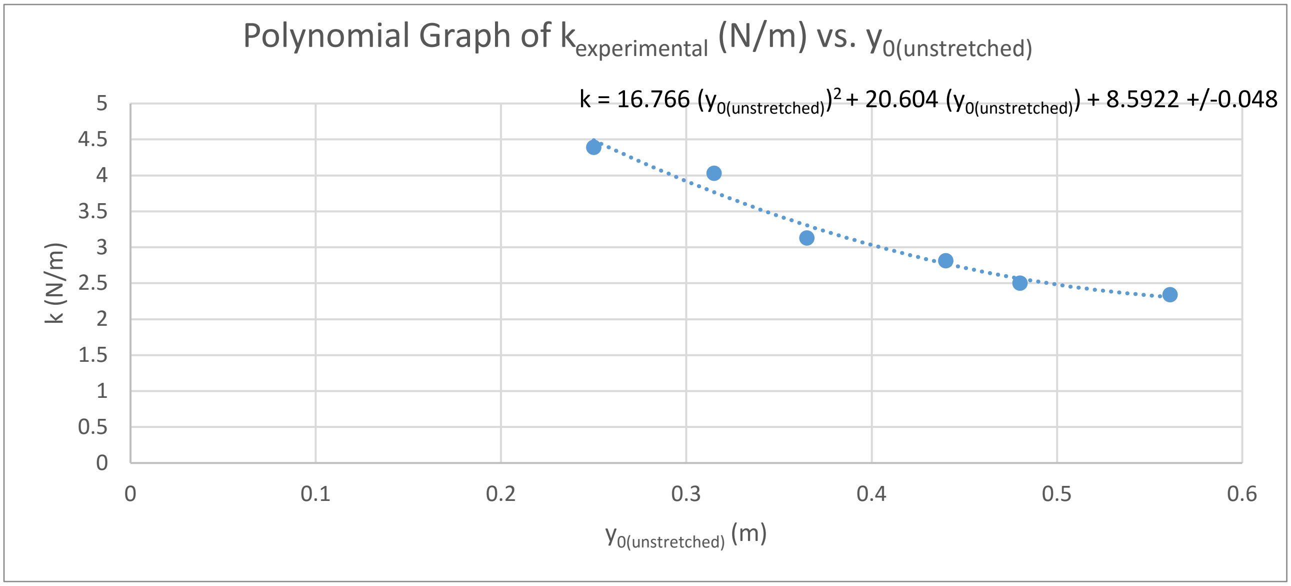

Figure 3: **Polynomial Graph of $k_{experimental}$ (N/m) vs. $y_{0(unstretched)}$ (m)** The values of $k_{experimental}$ were plotted against the values of $y_{o(unstretched)}$. It is significant to note that as the length of the cord increases, the k value decreases. We determined that the polynomial equation was the best fit equation with a normalized standard deviation of .047, as compared to a linear fit, which had a normalized standard deviation of .073. The best fit polynomial trendline has the equation: $k = 16.766\,(y_{0(unstretched)})^2 + 20.604\,(y_{0(unstretched)}) + 8.5922$ +/-0.048.

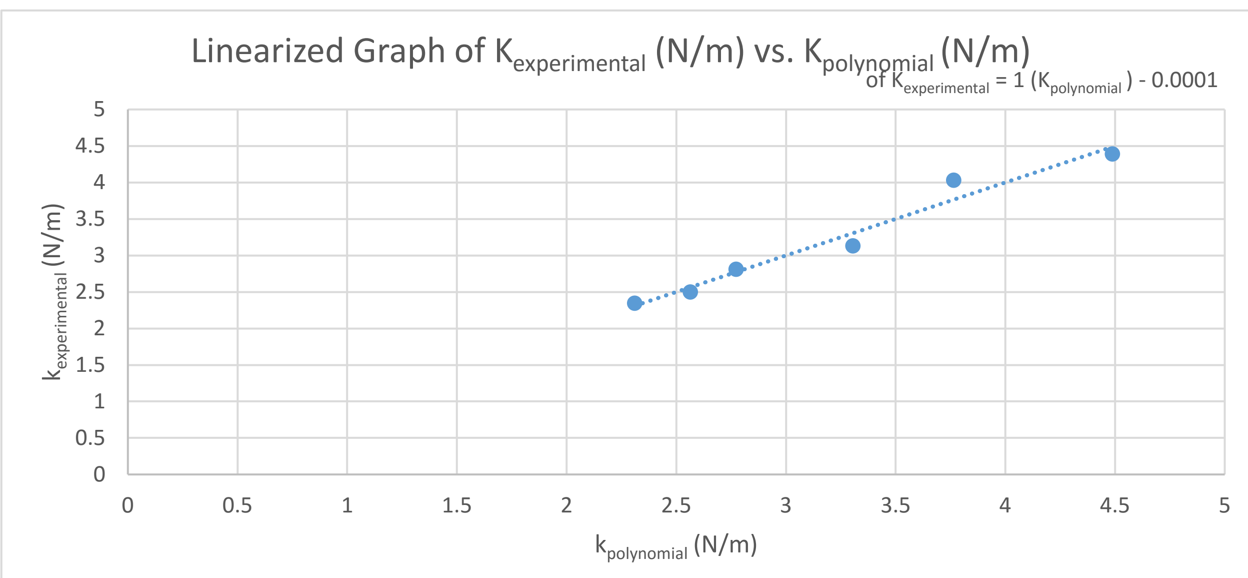

Figure 4: **Quantitative Results: Linearized Graph of $K_{experimental}$ (N/m) vs. $K_{polynomial}$ (N/m)** Pictured above is the linearized graph of $k_{experimental}$ and $k_{polynomial}$. This graph serves to assess how well our equation predicts the experimental values. Because of the line's equation of 1 ($K_{polynomial}$) - .0001, we are confident that our equation $K_{experimental} = 16.7661(K_{polynomial})^2 - 20.6041(K_{polynomial}) + 8.5922$ +/-0.048 can accurately predict the k value based on the upstretched length of the cord.

Percent Uncertainty for $k_{polynomial}$	+/-0.048
Fractional Error for $k_{polynomial}$	0.093

Figure 5: **Propagation of Error:** The percent uncertainty and fractional error for the $k_{polynomial}$ values were calculated by running a regression analysis on the Polynomial Graph of $k_{experimental}$ (N/m) vs. $y_{0(unstretched)}$ (m) data. The difference between the percent uncertainty and fractional error is a value of .045. Although these two values are different, the percent uncertainty and fractional error are still close enough to be in agreement because they are in the same order of magnitude. To test the accuracy of our equation $k = 16.766 (y_{0(unstretched)})^2 + 20.604 (y_{0(unstretched)}) + 8.5922$ +/-0.048 in the future, we could find the $y_{0(unstretched)}$ and $y_{(stretched)}$ lengths and calculate the k experimentally and compare this value to the value found by our equation. Essentially, we would be again comparing $k_{polynomial}$ and $k_{experimental}$ and if the values were within our percent uncertainty of +/-0.048.

Our most significant source of uncertainty is the fractional error as the initial length of the cord decreases. This error can be overcome by increasing all of the overall initial lengths of the bungee cords for each trial. Expanding our experimental area beyond the confines of the classroom would alleviate some of the uncertainty above.

Our experimental value of the greatest interest is the equation that: $k = 16.766(y_{0(unstretched)})^2 + 20.604(y_{0(unstretched)}) + 8.5922$ +/-0.048. Using this formula, it is possible to calculate the k value for varying upstretched, initial lengths of cord. Furthermore, this k value can be substituted into the equation: $\frac{1}{2}k(\Delta y)^2 = mgy$ to calculate the maximum displacement of the mass after being dropped. This relationship between the three values, $y_{0(unstretched)}$, $y_{(stretched)}$, and k, allows us to more accurately predict the length of our initial unstretched cord when dropping our egg from the third floor of the science center without cracking the egg on the floor. The results of this experiment allow us to more accurately predict the initial length of a bungee cord necessary to drop the egg all the way to the ground without breaking it.

Our experiment successfully related the initial length of the bungee cord with the spring constant of the bungee cord, which decreased as demonstrated by the formula $k = 16.766(y_{0(unstretched)})^2 + 20.604(y_{0(unstretched)}) + 8.5922$ +/-0.048. The results of this experiment could be expanded by performing different, additional experiments. For example, keeping the initial length of the cord constant while varying the hanging mass could reveal the relationship between these two values. Our results imply that generally, as one increases the initial length of the bungee cord, the spring constant decreases.