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Bungee Simulation: determining the maximum stretch of an elastic cord from its original length

ABSTRACT:

I present the experimental determination of the maximum extension of the Bungee Challenge elastic cord for any original length with an attached mass of 0.15 kg, a possible mass of an egg dropped in future demonstrations. This experiment aims to determine a constant similar to the k-value spring constant in Hooke's Law for the elastic string, which varies proportionally with string length. An experiment was carried out on a bungee cord of unknown stiffness using a constant mass secured at different lengths. The initial and stretched lengths of the string were measured for a dynamic drop in order to determine the nature of the relationship between these variables with an associated constant value. This constant constant for a mass of 0.15 kg is 3.877 and produces a linear-elastic correlation between string length and maximum elongation. Ultimately, this determination of the constant will allow any desired x_{max} to be reached for a mass of 0.15 kg. This proportionality constant is only valid for a specific mass, however, limiting the applicability of the precise results obtained in future experiments.

INTRODUCTION:

In preparation for a bungee jump simulation of an egg with a mass of 0.10-0.17 kg attached to an elastic bungee cord in order to lower the egg as close to the ground as possible without breaking, a simulation was done using one mass and a variety of string lengths. The purpose of this experiment was to determine the maximum elongation of the elastic cord that will be used in the final simulation when a mass of 0.15 kg attached to the end and dropped from a constant height.

Hooke's Law describes the linear-elastic and proportional approximation of the force, F, needed to extend an elastic material (such as a spring or a bungee cord) with a spring constant, or stiffness, k, a certain distance, x:

(1) $F = -kx$

where F is the force of gravity on the mass;

(2) $F = mass*gravity$

Hooke's Law spring constant of a specific elastic cord at a certain weight by varying the length of the string and measuring the stretch that results from a drop of a constant height. This experiment will produce a relationship similar to a Hooke's Law equation, but since the force was not measured and the stretch is not reported, a k-value cannot be determined from these methods. Instead I produced a value that proportionally related string length and the maximum length of the stretched string.

A phenomenon of elasticity known as hysteresis is known to diminish the "return" of an elastic response to a stress. As determined by Caldwell 2016, it takes one full stretch of an elastic cord to equilibrate the elongation potential of the string used in the Bungee Challenge. Therefore, the results of a bungee drop will only be reproducible after a second drop of a mass or after that specific section of string has been once stretched to the length of the experimental drop.

Considering these theories, I expect that increasing the length of the string will result in a positive linear relationship with the amount of elongation, providing a proportionality constant that will predict the original string length needed to attain a final x_{max}.

METHODS:

A mass is tied to an elastic string of varying lengths secured to a beam and raised to and released from the height of that beam. The experimental variable, the length of maximum elongation, results from the variation of string length, and is recorded and analyzed in order to determine the string constant of the elastic string.

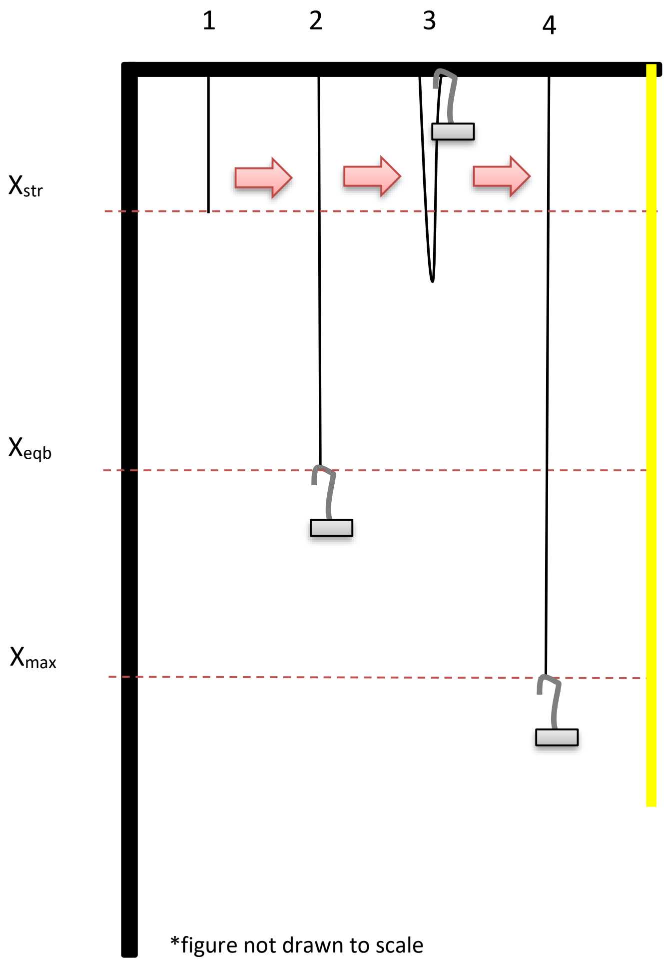

Figure 1. Bungee Drop Simulation Diagram of experimental set up. A measuring tape (in yellow) is hung taunt from a hanging beam. A string of a measured length x_{str} is attached to a hanging beam (1) and a mass is tied to the end, bringing the string to an equilibrium point, x_{eqb}, measured from the hanging point to the knot tying the string to the mass (2). The bottom knot is raised to the level of the top knot (3) and released, stretching the cord to an x of maximum elongation, x_{max} (4).

The experiment is set up and proceeds as follows:

1. Secure a small knot (knot 1) to the end of the string
2. Hang a measuring tape and the string by knot 1 on a horizontal beam over 2 meters from the ground
3. Secure a knot (knot 2) at length x_{str} (approximate)
4. Attach a hanging mass (weighed on a scale) to knot 2
5. Drop the weight from the height of knot 1 (knot 1 to knot 2) without recording an x_{max} in order to account for the effects of hysteresis
6. Drop the weight again from the height of knot 1 while recording the mass on a high-speed camera
7. Review the footage and record the maximum elongation of the cord, x_{max}
8. Remove the mass and <u>measure and record</u> x_{str} post-hysteresis
9. Undo knot 2
10. Repeat steps 3-9, increasing x_{str} for each iteration, keeping the mass constant

RESULTS:

The independent variable of the experiment was the unstretched, post-hysteresis, string length and the dependent variable was the maximum extension of the bungee cord, post-hysteresis as well. These values were correlated to determine a proportionality constant for any initial string length at a mass of 0.15 kg.

x_{str} (m) +/- 0.001	x_{max} (m) +/- 0.002
0.144	0.555
0.19	0.715
0.215	0.806
0.262	1.045
0.296	1.135
0.35	1.359
0.38	1.488
0.408	1.583

Table 1. String length and resulting maximum stretch. The variable x_{str} represents the initial length of the string with no mass attached within an absolute uncertainty of 0.001 m. The variable x_{max} is the maximum elongation of the string within an absolute uncertainty of 0.002 m.

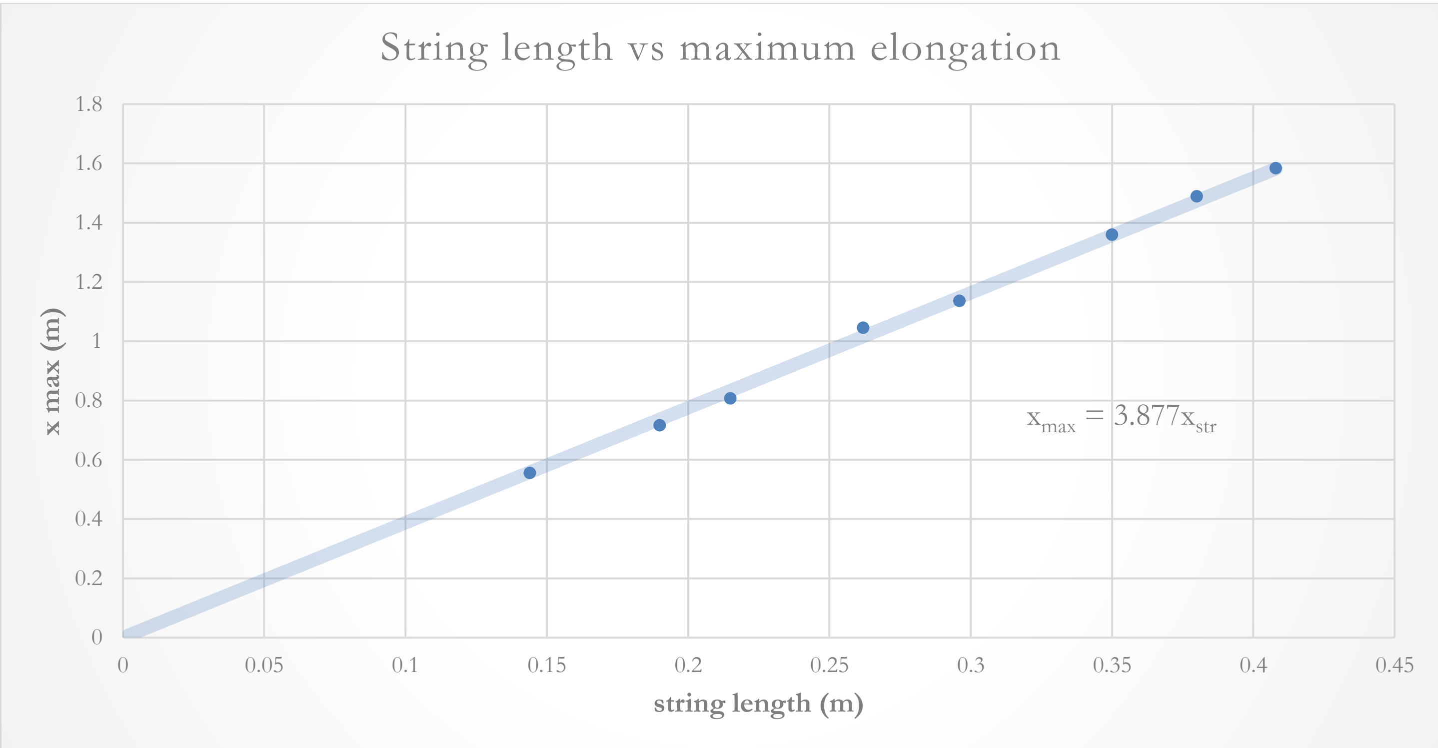

Figure 2. String length vs. maximum elongation. There is a linear relationship between the string length and its maximum elongation, where the coefficient of x (the slope) is a constant value of proportionality of string length and x_{max}. The experimentally determined constant is 3.877.

The experimentally determined equation:

(3) $x_{max} = 3.877x_{str}$ | x_{max} is the maximum elongation of the elastic string, x_{str} is the original string length, and the coefficient of x_{str} (the slope) is the proportionality constant of the elastic string for a mass of 0.14975 kg.

Experimental value of interest:

The proportionality constant = 3.877

uncertainty of experimental value(s) = +/- 0.002 m % uncert= 0.05% | uncertainty by absolute uncertainty

The proportionality constant is 3.877 +/- 0.002 m for a mass of .14975 kg,. Knowing this constant can allow us to determine an x_{str} for any desired x_{max} at this specific mass.

DISCUSSION:

There are no established values to which these results may be compared. However, knowing this proportionality constant, it should be possible measure a length of cord and drop a mass of approximately 0.15 kg from any height, allowing the mass to get as close to the ground as possible without breaking. The Bungee Challenge drop will be from a height of about 9.5 m. If this is the exact height including the height of the attached egg mass, the string should be 2.450 m long. The variability of our result is 0.05%, and x_{max} is therefore 2.450 +/- 0.123 m. This margin of error is fairly small for such a long fall, but should be accounted for in the final experiment by decreasing the string length in order to account for the additional stretch the cord may undergo by about 7 cm, when the uncertainty is less than the resulting remaining distance to the ground.

Sources of uncertainty may have come from many aspects of the experiment. Of utmost concern, it is possible the knots in the string tightened throughout the experiment, contributing an unknown drift to the data. Additionally, the accuracy of the length measurements taken may be questioned. For example, the measuring tape could have hung at an angle to the string, increasing or decreasing the actual measurement of length. Additionally, the high-speed camera may have not captured a frame at the true maximum extension, showing a smaller result than occurred in practice. Likewise, the camera could have been held at an angle to the base of the mass, making the measurement appear to be slightly skewed from the true value.

These results support the hypothesis that with an increasing length of x_{str}, the length of x_{max} increases linearly, and that for a mass of 0.15 kg, a desired maximum elongation can be attained by projecting this trend forwards.

CONCLUSION:

This experiment determined a proportionality constant for a mass of 0.15 kg, explaining the linear proportionality of the variables:

$$x_{max} = 3.877x_{str}$$

For any desired elongation x_{max}, a string length can be tied off to attain that length, as long as the mass is 0.15 kg. Any deviation from this mass, however, requires a new way of determining the proportionality constant. A more determination of the true spring constant would require a relationship between the spring constant and string length be determined by measuring the force on a string of increasing length with a progression of masses on each length.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged:

Jenna Biegel