Quantifying Shifts in Spring Constants for Various Bungee Configurations

ABSTRACT:

This experiment assumes bungees function like oscillating springs, and thus can be approached with Hooke's Law $\vec{f}_{spring} = -kxi$. This experiment quantifies the spring constant k. The spring constant is deduced from a linear regression of force verse displacement plots for two independent test conditions. Force is calculated based on the weight of a known hanging mass. The first test condition is the number of strands. The second test condition is the length of the strands. Force and displacement are measured experimentally by manipulating the mass of the system, and the experimental value k is calculated. This experiment shows that k changes proportionally to the number of strands and inversely proportional to the length of strands. Shifting k is important to understand for the jump experiment so the action of the bungee is properly predicted to insure a safe movement of the egg from equilibrium.

INTRODUCTION:

This experiments considers the bungee like a spring exhibiting simple harmonic oscillation. For the bungee jump, force is known based on the force of weight from the egg. It is crucial to correctly predict the displacement, or vertical distance from equilibrium in order to protect the egg. This distance is affected by the spring constant k**.** The goal of this experiment is to quantify how this value responds to shifts in the two controllable conditions of the bungee, length and number of strands. Hooke's Law allows this calculation.

Hooke's Law for Simple Harmonic Oscillation

$$\vec{f}_{spring} = -kxi$$

Bungees stretch to an equilibrium length, and movement away from that equilibrium is considered oscillation. Displacement is the distance of the oscillation from the equilibrium position. The mass of the system drives both the equilibrium length and the magnitude of oscillation because $\vec{w}_{system} = -\vec{f}_{spring}$. This relationship makes it clear that both the spring constant k also impacts the magnitude of oscillation.

This experiment hypothesizes if the mass of the system and coinciding displacement of the system is known, then the spring constant for the controllable test conditions, length and number of strands, will be proportional to changes in the number of strands and the length of the bungee.

METHODS:

The force acting on the system is controlled by manipulating the mass of an object hanging on the bungee. The displacement is measured at five masses for each test condition of the bungee.

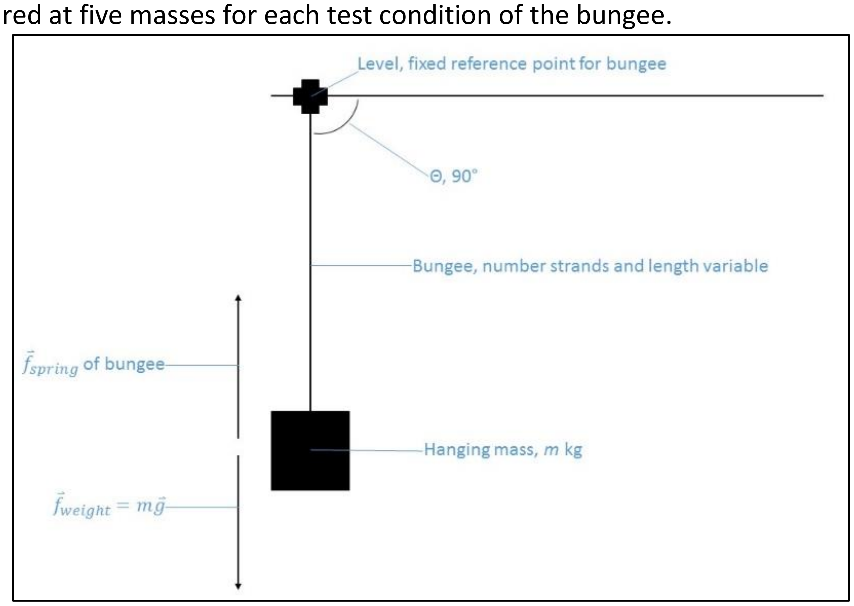

Figure 1: Schematic of the bungee and hanging mass system, include all forces present.

The experiment follows a similar structure for two parts. Part A finds $\boldsymbol{k}$ in relation to a different number or strands of similar length. Part B finds $\boldsymbol{k}$ in relation to different lengths of the same number of strands. These are accomplished by measuring the displacement of the bungee from equilibrium by attaching the bungee to a fixed point with no loop, allowing it to hang vertically, and changing the hanging mass.

Data for categories A and B of test variables are collected, and then analyzed in terms of linear relationships to deduce $\boldsymbol{k}$.

Part A:

1. Fix the length of bungee
2. Measure length of bungee with no mass attached
3. Attach mass with a uni-knot to eliminate a confounding loop in the bungee
4. Measure length of bungee after mass has settled to new equilibrium
5. $Displacement = length_{mass} - length_{no\ mass}$
6. Carry out for 5 masses
7. Repeat steps 1-6 for 1, 2, and 4 strands of the bungee

Part B:

1. Maintain 2 strands.
2. Fix 1 length, measure with no mass
3. Carry out steps 2-6 (above) at first test length, 33.2 cm
4. Repeat for two more lengths, 68.8 cm and 124.6 cm

Analysis:

1. Calculate force ($\vec{\boldsymbol{F}}$ in N) by multiplying hanging mass by $\vec{\boldsymbol{g}}$
2. Plot $\vec{\boldsymbol{F}}$ as a function of displacement
3. Linearize relationship
4. Perform linear regression

RESULTS:

The force applied to the test conditions, 1 strands, 2 strands, 4 strands, 33.2 cm, 68.8 cm, and 124.6 cm, is controlled and known based on the hanging mass. The displacement is measured as the difference of the equilibrium length (hanging mass of 0 kg) from the stretched length. The force (N), hanging mass times $\vec{\boldsymbol{g}}$, is plotted as a function of this measurement. The raw data is linearized for analysis by adjusting force as $\boldsymbol{e^{force}}$, and is still in N.

Figures 2 and 3 show all of the data collected as well as the linearized force values. Figures 4 through 9 show the linear relationships between displacement and force. Figure ten compiles these results and evaluates standard error for each relationship.

The coefficient of $\boldsymbol{x}$ in each equation is the spring constant for that relationship, or the $\boldsymbol{k}$ component of $\vec{\boldsymbol{f}}_{\boldsymbol{spring}} = \boldsymbol{kxi}$. All model equations are summarized in figure 10. This value allows predictions to be made for the behavior of the bungee at the various test conditions. The results for $\boldsymbol{k}$ are shown in figure 11 for each condition.

This experiment shows that $\boldsymbol{k}$ invariably changes in response to conditions applied to the bungee – in length and number of strands. Five of the six studied relationships result in acceptable uncertainties, while the 4 strand treatment is more significantly uncertain. The spring constant shifts roughly proportionally to the number of strands and roughly inversely proportionally to the length of strands. For example, when the number of strands doubles from 1 to 2, the spring constant increases with a proportion on 2.068. When the length of string increases by a proportion of 2.072, the spring decreases by a proportion of .466.

Catalog of Figures:

Part A Results				
1 Strand				
Mass (kg)	**Length (m)**	**Displacement *d* (m)**	**Force *F* (N)**	**e^{Force}**
0.000	0.558	0.000	0.000	1.000
0.015	0.583	0.025	0.147	1.159
0.030	0.615	0.057	0.294	1.342
0.045	0.653	0.095	0.441	1.555
0.060	0.693	0.135	0.589	1.801
0.075	0.742	0.184	0.736	2.087
2 Strands				
Mass (kg)	**Length (m)**	**Displacement *d* (m)**	**Force *F* (N)**	**e^{Force}**
0.000	0.688	0.000	0.000	1.000
0.015	0.701	0.013	0.147	1.159
0.030	0.717	0.029	0.294	1.342
0.045	0.736	0.048	0.441	1.555
0.060	0.756	0.068	0.589	1.801
0.075	0.776	0.088	0.736	2.087
4 Strands				
Mass (kg)	**Length (m)**	**Displacement *d* (m)**	**Force *F* (N)**	**e^{Force}**
0.000	0.607	0.000	0.000	1.000
0.050	0.627	0.020	0.491	1.633
0.100	0.652	0.045	0.981	2.667
0.150	0.683	0.076	1.472	4.356
0.200	0.719	0.112	1.962	7.114
0.250	0.759	0.152	2.453	11.617

Figure 2: The raw results for measured lengths at the masses applied to each test strand condition in Part A. Displacement is calculated by subtracting the equilibrium length from the length of the bungee with the hanging mass. Force is calculated by multiplying the value in the mass column by 9.81. This value is adjusted to provide a linear relationship between displacement and force for analysis.

Part B Results				
33.2 cm				
Mass (kg)	**Length (m)**	**Displacement *d* (m)**	**Force *F* (N)**	**e^{Force}**
0.000	0.332	0.000	0.000	1.000
0.015	0.337	0.005	0.147	1.159
0.030	0.346	0.014	0.294	1.342
0.045	0.353	0.021	0.441	1.555
0.060	0.363	0.031	0.589	1.801
0.075	0.373	0.041	0.736	2.087
68.8 cm				
Mass (kg)	**Length (m)**	**Displacement *d* (m)**	**Force *F* (N)**	**e^{Force}**
0.000	0.688	0.000	0.000	1.000
0.015	0.701	0.013	0.147	1.159
0.030	0.717	0.029	0.294	1.342
0.045	0.736	0.048	0.441	1.555
0.060	0.756	0.068	0.589	1.801
0.075	0.776	0.088	0.736	2.087
124.6 cm				
Mass (kg)	**Length (m)**	**Displacement *d* (m)**	**Force *F* (N)**	**e^{Force}**
0.000	1.246	0.000	0.000	1.000
0.015	1.277	0.031	0.147	1.159
0.030	1.302	0.056	0.294	1.342
0.045	1.334	0.088	0.441	1.555
0.060	1.374	0.128	0.589	1.801
0.075	1.412	0.166	0.736	2.087

Figure 3: The raw results for measured lengths at the masses applied to each test length condition in Part B. Displacement is calculated by subtracting the equilibrium length from the length of the bungee with the hanging mass. Force is calculated by multiplying the value in the mass column by 9.81. This value is adjusted to provide a linear relationship between displacement and force for analysis.

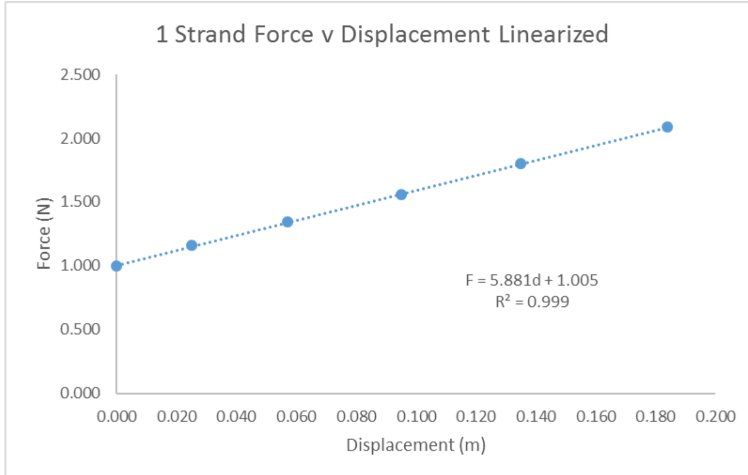

Figure 4: The linearized force as a function of displacement when mass is applied to a single strand. The conditions result in a spring constant of 5.881 and the relationship is functionally linear with an R^2 value of .999.

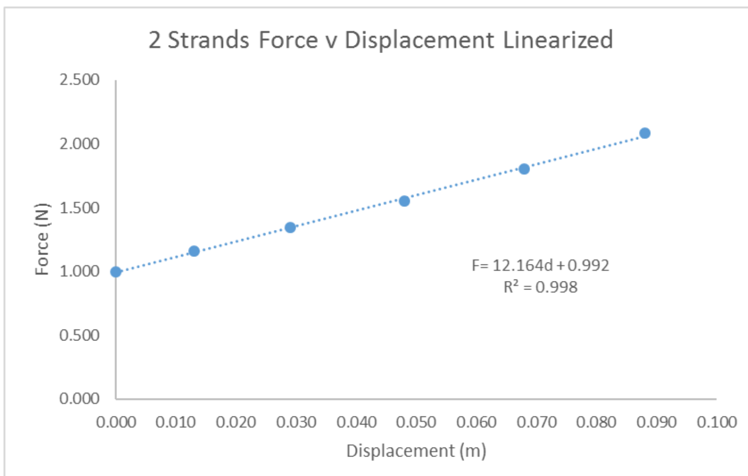

Figure 5: The linearized force as a function of displacement when mass is applied to a double strand. The conditions result in a spring constant of 12.164 and the relationship is functionally linear with an R^2 value of .998.

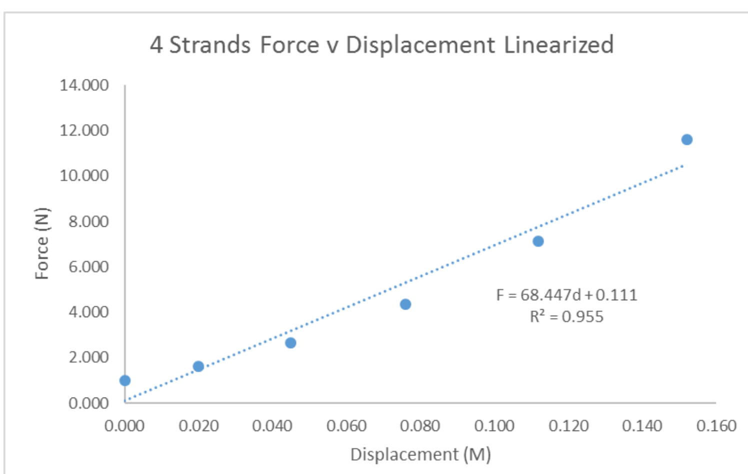

Figure 6: Figure 5: The linearized force as a function of displacement when mass is applied to a quad strand. The conditions result in a spring constant of 68.447 and the relationship does not show a confident level of linearity.

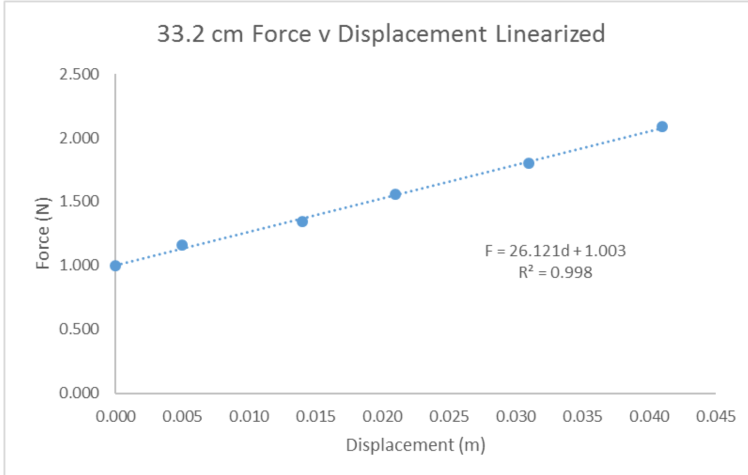

Figure 7: The linearized force as a function of displacement when mass is applied to a double strand with a length of 33.2 cm (.332 m). The conditions result in a spring constant of 26.121 and the relationship is functionally linear with an R^2 value of .998.

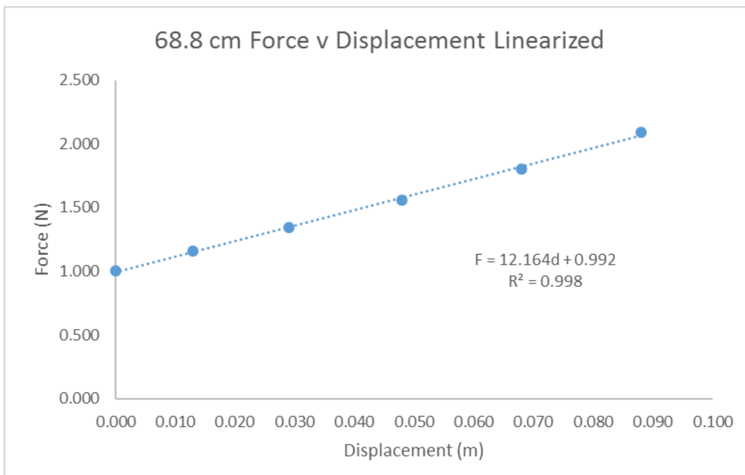

Figure 8: The linearized force as a function of displacement when mass is applied to a double strand with a length of 68.8 cm (.688 m). The conditions result in a spring constant of 12.164 and the relationship is functionally linear with an R^2 value of .998.

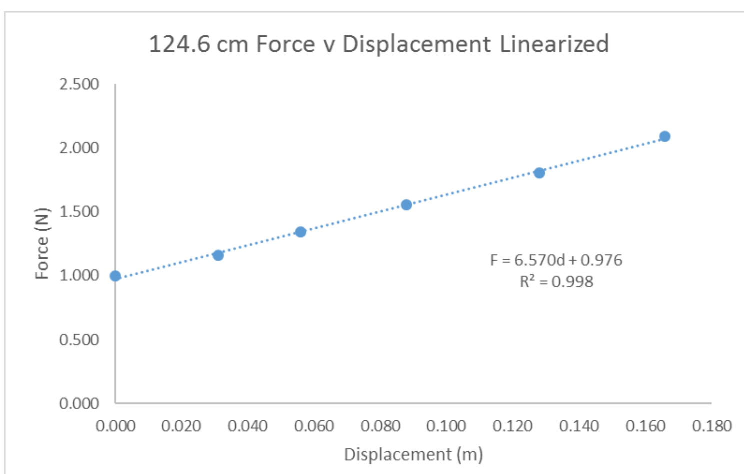

Figure 9: The linearized force as a function of displacement when mass is applied to a double strand with a length of 124.6 cm (1.246 m). The conditions result in a spring constant of 6.570 and the relationship is functionally linear with an R^2 value of .998.

Linear Equation Analysis using Excel Regression Analysis					
Test Condition	**Equation**	**Uncertainty - Slope**	**% Uncertainty**	**Uncertainty - intercept**	**% Uncertainty**
1 Strand	F=5.881x+1.005	0.04	0.7	0.004	0.4
2 Strands	F=12.164x+0.992	0.3	2	0.01	1
4 Strands	F=68.447x+0.111	7	10	0.6	600
33.2 cm	F=26.121x+1.003	0.6	2	0.01	1
68.8 cm	F=12.164x+0.992	0.3	2	0.01	1
124.6 cm	F=6.570x +0.976	0.1	2	0.01	1

Figure 10: Compilation of best fit linear equations for each relationship studied. Uncertainty values and percent uncertainties are produced by running Excel regression analysis on each relationship. Uncertainty in the slope communicates standard error, and uncertainty in the intercept shows the congruence of the results with the theoretical ideal. Percent uncertainty is $\mathbf{100} \times \frac{uncertainty}{value}$, corresponding to slope and intercept.

Relationship Summary		
	Part A	
Number Strands	**Spring Coefficient**	**% Uncertainty**
1	5.881	0.7
2	12.164	2
4	68.447	10
	Part B	
Length Bungee	**Spring Coefficient**	**% Uncertainty**
33.2	26.121	2.227
68.8	12.164	2.126
124.6	6.570	2.249

Figure 11: The spring coefficient for each condition in the left column is the slope of the linear regression line. The percent uncertainty is the standard error of the value in this model. Standard error was calculated using excel regression analysis. Changes in $\boldsymbol{k}$ *are roughly proportional to increases in the number of strands and inversely proportional to the length of the bungee.*

Proportional Summary			
Number Strands	**Proportional Shift**	**Spring Coefficient**	**Proportional Shift**
1	null	5.881	null
2	2	12.164	2.068
4	2	68.447	5.627
Length Bungee	**Proportional Shift**	**Spring Coefficient**	**Proportional Shift**
33.200	null	26.121	null
68.800	2.072	12.164	0.466
124.600	1.811	6.57	0.540

Figure 12: The proportion of change in test condition compared to proportional change of corresponding $\boldsymbol{k}$ *values. Strands appear to be directly proportional while length appears to be inversely proportional.*

DISCUSSION:

Error analysis

Five of the six results exhibit acceptably low uncertainties for both the experimental values and the intercept of the model. The standard error of the experimental values is reasonably low for all test points except the 4 strand condition to draw inferences about how the bungee will behave in response to the egg, based on its condition. This use of the data is further corroborated by the low uncertainty in the intercept of the model. The theoretical intercept should be 1 for all cases due to the linearization process, e^{force}. Only the 4 strand condition has a significantly high uncertainty. An uncertainty greater than 5% is considered significant and invalidates a result. The incongruity between the observed intercept, .111, and the ideal intercept for the 4 strand case indicates that the system is not functioning as a spring-like simple harmonic oscillation. Hence, the spring coefficient for that test condition cannot be used for inference. Other observed but insignificant uncertainties are attributed to the least count of the device used to measure the length of the bungees and provide displacement values. This is considered the primary source of uncertainty in the model Hooke's Law equations, and is largely unavoidable without more elegant measuring devices. Secondary but insignificant sources of uncertainty are inherent to the bungee, such as the possibility of inconsistency across the length or a change in elasticity over trials.

Interpretation of Results

Closer interpretation of the 4 strand test condition offers insight to potential sources of incongruity. A qualitative analysis of the linearized plot reveals a pattern of two distinct linear trends or "steps" within the comprehensive plot. When parsed out, the first "step" of three data points reveals a highly linear relationship $\boldsymbol{F = 1.052x + 0.999,}$ $\boldsymbol{R^2 = 0.999}$, that is congruent with the theoretical ideal. The second step of the next three data points, $\boldsymbol{F = x}, \boldsymbol{R^2 = 1,}$ functions linearly but not ideally. While these portions cannot be used for inference about spring coefficient, they suggest that the bungee has reached a threshold where it no longer functions as a spring at the second "step". Experimental procedure offers further evidence for this interpretation because larger masses, on the order of 200 grams larger, were applied to this system than the other 5 scenarios under the logic that the increased number of strands would require a greater force to provide quantifiable displacements. However, the anomalous results indicate that this is not the case and rather there is a force threshold around 1.5 N (the force at the distinction of the "steps") where the bungee no longer can be considered spring-like.

The four strand system notwithstanding, the majority of the results, on account of low percent uncertainties and a congruence with theoretical ideals, strongly support the broad hypothesis that the spring coefficient of the bungee changes proportionally to changes in the condition of the bungee. In the case of number of strands, the experiment exhibits clear support of the hypothesis. In the case of length of bungee, a discrepancy from the hypothesis is revealed in that the spring coefficient changes inversely proportional to length, but the relationship is strong and still in line with the underlying expectation for relationship implicit in the hypothesis. Both categories of results are accurate considering the nearness of the linearized intercept to the theoretical ideal value of 1. The four strand system suggests that there may be a limiting case to the utility of this model, and further experimentation should be considered to evaluate if a lurking force threshold for simple harmonic oscillation exists in the bungee. This could, dependent on the mass of the egg, have ramifications for the appropriate modeling approach to insure a safe jump.

CONCLUSION:

The experiment concludes there is a relationship between the bungee conditions used and the spring coefficient $\boldsymbol{k}$. The number of strands used and $\boldsymbol{k}$ are proportional. The length of the bungee is inversely proportional to $\boldsymbol{k}$. However, the four strand system is anomalous and may represent a lurking force threshold where the bungee encounters a limiting case and can no longer be modeled by Hooke's Law.

These conclusions are helpful in the design of a bungee apparatus because they allow for a calculation of $\boldsymbol{k}$ in the system being considered. With a known $\boldsymbol{k}$ the designer is able to model both the force of the system and the vertical displacement $\boldsymbol{x}$ of the system from equilibrium. Both of these aspects of the bungee jump are crucial to assure the egg of a safe adventure. Potential next steps include a further investigation into the effects of especially large masses and coinciding large forces on the usability of this model.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

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