Matt Marcus and Juliana Kerper **Section:** 1 **Date:** 10/10/16, 11/8/16

Relationship Between Spring Constant and Length of Bungee Cord

Abstract:

Hooke's Law states that the Force of a Spring is equal to the Spring Constant k multiplied by the displacement value, x. In our experiment, we wanted to see if our bungee cord would behave like an ideal spring and follow Hooke's Law. To do this, we hung a tape measure and our bungee from an apparatus and measured the different lengths when different masses were added to the bottom of the bungee. We tested five different lengths of the bungee cord along with five different masses for each length. We used this data to calculate the spring constant k and then graphed the spring constant versus 1/bungee length to linearize our data. The slope of the graph would then be the force of a spring and follow Hooke's Law. We got the equation k=.97(1/x) which proves there is an inverse relationship between spring constant and the length of the bungee cord. We followed a previously done experiment by Andy Cuthbert and Alex Herr, and compared some of our data to their data.

Introduction:

Purpose:

- This experiment was created to determine the relationship, if one exists, between the spring constant of our bungee cord and the length of the bungee cord. Using this relationship we could derive an equation using Hooke's law to find the spring constant at any bungee length.
- We used Hooke's Law (F=-kx) to determine the static spring constant of the bungee with a hanging mass.
- We assumed that a bungee acts similar to an ideal spring and used F_{spring}= mg, where m is the mass of the hanging object and g is the gravitational force constant 9.81 m/s.
- This experiment was done in order to do the Bungee Jump Challenge, in which we receive an egg and have to use a bungee to get it as close to the ground as possible without it breaking from either too much force on the egg itself or hitting the ground.
- We followed an outline of this experiment made by Andy Cuthbert and Alex Herr. We will compare our data and findings with their data throughout the experiment

Relevant Equations:

- F = ma
- $F_{Spring} = m_h g$
- $F_{Spring} = -kx$
- F = Total Force
- m = mass
- m_h = hanging mass
- g = gravity constant (9.81 m/s^2)
- k = spring constant
- x = Displacement from initial length of bungee to final length ($x_f - x_i$)
- x_i =bungee length with no added mass
- x_f =bungee length with added mass

Theoretical Background:

- The force on a spring is very similar to a bungee cord so we can use Hooke's Law to illustrate how a bungee cord works.
- As the cord lengthens, the spring constant should go down at a decreasing rate

- Since the k value gets smaller as x becomes bigger, we expect the bungee length to vary inversely with spring constant k

Hypothesis:

We hypothesize that the spring constant k will vary with the bungee length inversely. This means that as the bungee length increases, the spring constant k will decrease. We also hypothesize that although the spring constant will decrease, it will decrease at a decreasing rate. As the bungee lengthens, the spring constant will decrease, but it will decrease less and less each time.

Methods:

Overall Method:

To calculate the spring constant k, we needed to measure the displacement of the bungee after each mass is added. We varied the mass connected to the bungee in order to calculate many different spring constants with different lengths of bungee cord.

Diagram:

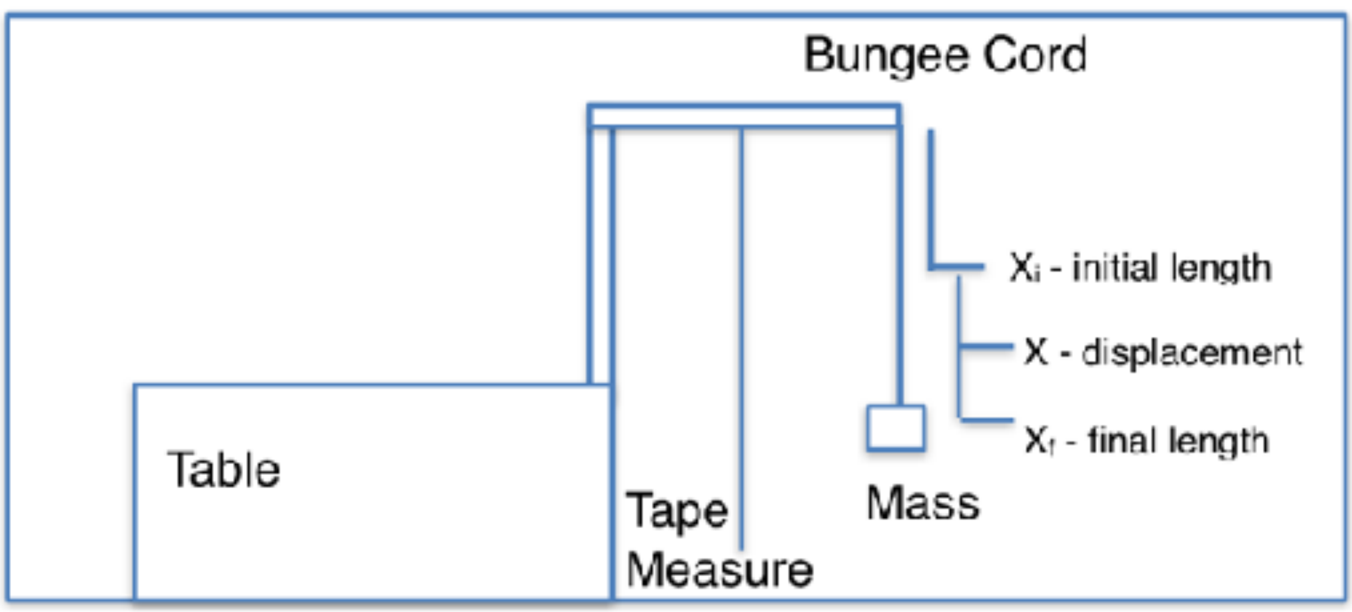


Figure 1: Bungee and Hanging Mass System. A bungee is connected to a hanging apparatus and connected to a mass.

- m = hanging mass
- x = Displacement from initial length of bungee to final length ($x_f - x_i$)
- x_i =bungee length with no added mass
- x_f =bungee length with added mass

Setup:

- Connect a tape measure to an apparatus that will allow the bungee to hang off with and without a mass and not be impeded
- Find an approximate length you would like your bungee cord to be for your first measurement (it will not always be exact because of the knot tying)

- Use a range of masses in order to get a larger distribution of spring constant k values using different lengths and masses
- Vary length of bungee cord but measure each length with the same distribution of masses in order to get a clearer graph
- Stretch Bungee Cord a few times before the experiment to lessen error of stretching as the experiment progresses
- Take off masses after measurements and do not leave any mass on the cord for long amounts of time to stop excessive stretching from occurring

Procedure:

- Tie a knot in bungee cord and hang from apparatus
- Measure the initial distance from the apparatus to the end of bungee cord
- Hang mass from end of bungee cord and record new length
- Measure five different cord lengths and 5 masses for each length
- Multiple measurements of one length with one mass is unnecessary because the cord is not changing when measurements are taken
- Vary lengths between .2 and .7 meters and masses between .025 and

.2 kilograms

Results:

Intro to Results:

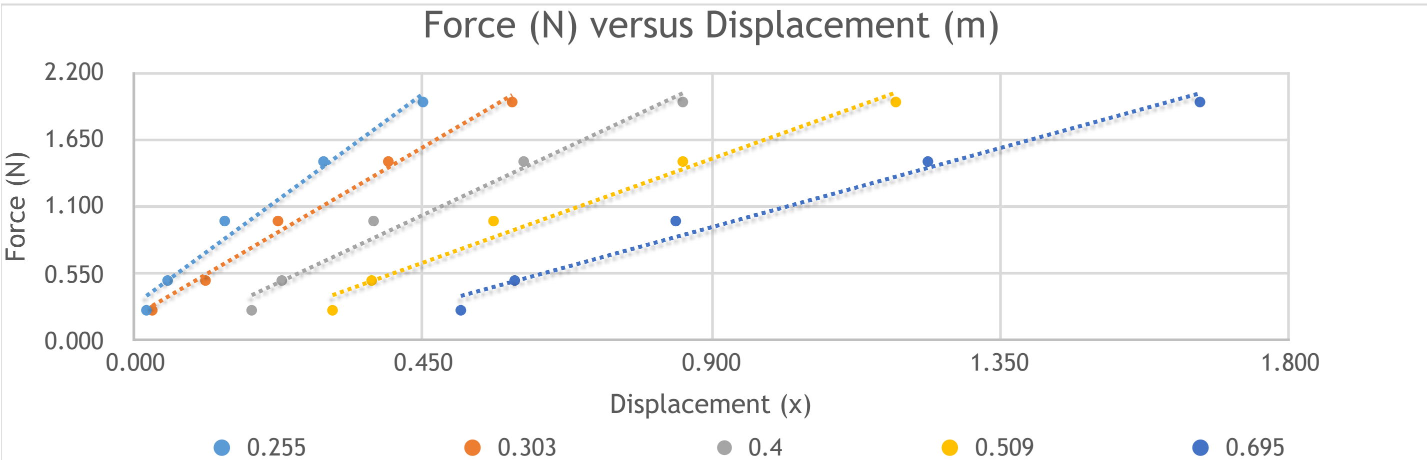
- The mass was recorded by using preset masses and adding the mass of the hanger that holds the mass
- We then used the mass multiplied by g to get the force of gravity
- The displacement was calculated by subtracting the initial value of x from the final value to get the displacement

Table:

Cord length (m)		0.255	0.303	0.4	0.509	0.695
Hanging mass (kg)	Fgrav (N)	Displacement ($x_f - x_i$)				
0.025	0.245	0.019	0.028	0.183	0.309	0.509
0.050	0.491	0.052	0.111	0.230	0.370	0.593
0.100	0.981	0.141	0.224	0.373	0.560	0.844
0.150	1.472	0.295	0.396	0.607	0.855	1.237
0.200	1.962	0.450	0.589	0.855	1.187	1.661

Figure 2: Cord Length and Displacement Data. For each cord length, we measured the initial value of x before adding weight and adding the final value. We calculated the difference of these values to get the displacement

- **Figure 3: Force (N) versus Displacement.** We graphed the data from each cord length all on the same graph to find the differences in displacement when mass is added.



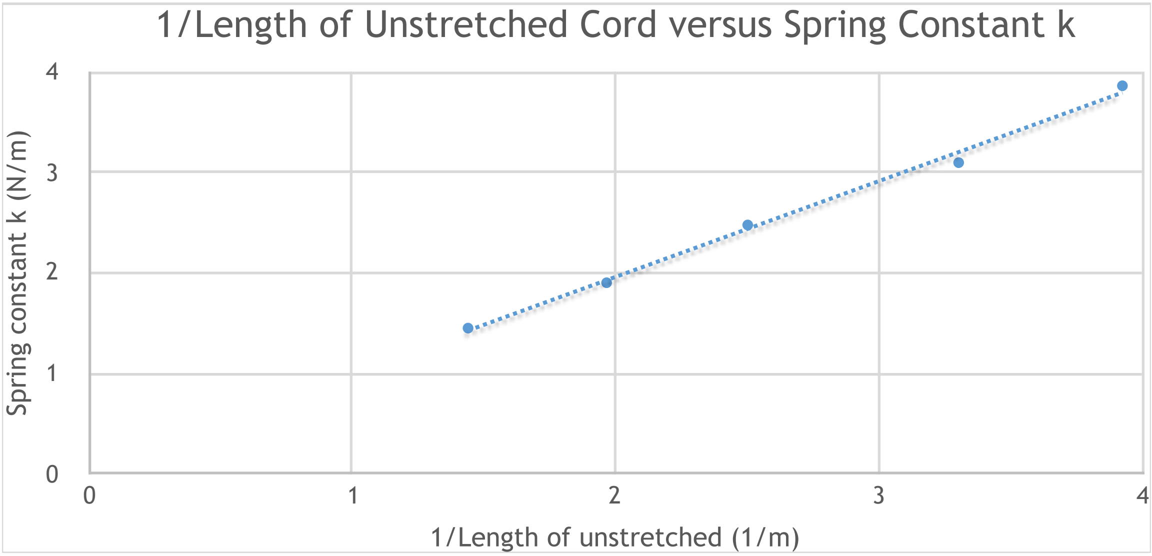
Equations:

- .255 - y = 3.86x + 0.29
- .303 - y = 3.10x + 0.19
- .400 - y = 2.48x -
- .509 - y = 1.90x - 0.21
- .695 - y = 1.45x - 0.37

Table:

- **Figure 4: Linearized Cord Length and Spring Constant.** We took the spring constant from the slope of the above equations and also found 1/Cord Length.

Cord length (m)	Spring constant k	1/Cord length (1/m)
0.255	3.8648	3.92156862745098
0.303	3.099	3.300
0.400	2.478	2.500
0.509	1.903	1.965
0.695	1.450	1.439



Graph:

- **Figure 5: Spring Constant k versus 1/Length of Cord.** We took the spring constant k and the value of 1/Length of Cord to linearize our graph. This is because the slope of this graph is now the Force. $k/(1/x) = kx = F$

Equation:

- $k = 0.97(1/x)$

Excel Regression Analysis and Propagation of Uncertainty Analysis:

- uncertainty for slope = 0.039 N = kgm/s^2
- uncertainty for y-intercept = 0 (our graph went through the origin)
- uncertainty in coefficient(s) = 0.05 m

Identify Experimental Values of Interest:

- Values from the slope of the equation are of interest because they represent the relationship between the Spring Constant and Length of Bungee Cord and show the error
- Uncertainties were obtained from the linear regression, the outputs of standard error in the x-variable and the intercepts

Summarize Results:

The equation $k = 0.97(1/m)$ was achieved by fitting the graph with Spring Constant k versus 1/Length of Bungee Cord. This should give us the force of the system as the slope, because the slope equals kx which is equal to the force of a spring. This proves that there is a relationship between the Spring Constant and the Length of the Cord. Cuthbert and Herr also found a relationship between the Spring Constant and Cord Length.

Discussion:

Acceptability and Extra Testing:

- The uncertainty of our experiment appears to be appropriate and acceptable for our equation and graph
- Our final test of our experiment will be the Bungee Jump Challenge, in which we will have an egg and must connect it to the right length of bungee cord in order to get as close to the ground as possible without breaking.
- Using our results and knowledge from this experiment we can calculate at any length of Bungee Cord the Spring Constant

Sources of Uncertainty:

- We used a standard ruler and line of sight measurements, which could result in small uncertainties in our data
- We only recorded on length for each mass, and so the result may be less accurate than trying many times and recording the data
- As the bungee holds the mass, it slowly stretches, which could skew our data if it becomes less springy as the experiment progresses

Any Further Observations:

- If we had done more data points, by shortening the bungee or lengthening it more and varying or testing more masses, our equation could be more accurate than it is currently
- Cuthbert and Herr used a very small length of .1 m for the bungee and had an extremely high spring constant. This caused their graph to have a slightly higher slope.
- We tested their graph without this one point, and our slopes became very similar

Results Support Hypothesis:

Our results agreed with our hypothesis. The Spring Constant k when graphed versus Cord Length supported our view that the k value would decrease at a decreasing rate as the bungee length increased.

Conclusion:

Experimental Outcomes:

- This experiment proved that there is a relationship between Spring Constant and Cord Length
- Using this relationship, we can calculate the Spring Constant at many different lengths to defeat the Bungee Jump Challenge

Implications of these Conclusions:

- We now know that the Spring Constant will decrease as Cord Length increases, so we will have to use this to find the Spring Constant at different lengths for the Bungee Jump Challenge.
- This experiment proved that most bungees will behave like ideal springs and obey Hooke's Law, which allows for easier analysis

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Matthew Marcus