Section: 06 **Date: Nov. 28, 2016**

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TITLE: To model our bungee cord's k value with the CWE theorem by changing bungee length.

ABSTRACT:

In this experiment we want to model the behavior of a dynamic bungee as it drops from some height with an object attached. If done successfully it will give us vital information about our bungee cord that can help us make sure that anyone (or any egg) that uses it for a free fall experience will have a pleasant one. To do this we measured the behavior of the bungee by attaching a variety of massed objects to different lengths of our bungee cord and observed the 'stretch' of the bungee as the objects were dropped from some height. The results were able to show a theme that is common with many springs and bungees, that as the length of the bungee decreases, its spring constant, k, or 'strength of spring', also increased. Though as my results show, it was unclear whether the relationship between length and k was linear or not due to large amounts of error. Throughout the entire experiment many sources of uncertainty arose, the largest being that we did not keep the height from which we dropped objects constant, which did not allow us to have any definitive results.

INTRODUCTION:

To allow a bungee jumper to use our bungee cord and have a successful experience without stopping too early or fast, and obviously not hitting the ground, we need to know the behavior of that bungee. In this experiment we are learning more about the effect of the connection of the bungee length and k value when the cord is being used dynamically. In this experiment we utilize the CWE theorem, which states that in a conservative system the amount of total energy at the top of the jump, will equal the amount of energy at the bottom. Total energy in this case being, PE (potential energy) + KE (kinetic energy). This equation is:

$$(PE + KE)_{top} = (PE + KE)_{bottom}$$

When the bungee jumper is at the top, ready the jump, they are relatively still, meaning that there is only potential energy, or the energy that exists purely form the object's placement within the system. In contrast the object at the very bottom of its decent, will have only kinetic energy, which is the energy from its motion. By removing KE for the top, and PE for the bottom, and stating the definitions of the sources of energy left, this equation becomes:

$m * g * h = (1/2) * k * \Delta x^2$

Where m is mass, g is gravity due to acceleration, h is height of where the jump begins, k is the spring constant, and **Δ**x is the total stretch that the elastic bungee does.

When speaking about the spring constant, k, I am referring to the constant that comes form Hooke's law of an ideal spring. This equation states:

$F = f * x$

Where, F is the total amount of force, and x is the displacement. This shows that for larger spring constant, f, you need a proportionally larger force to make the object move the same amount of distance, x.

I hypothesized that my bungee cord would follow the same logic behind Hooke's law in this dynamic system where I used the CWE theorem, being that the larger k values of the bungee, the larger amount of force would be needed to cause the same elastic stretch.

METHODS:

Our basic procedure was to drop different massed objects while attached from our bungee cord and to measure the change in stretch. We measured this stretch by knowing the length of cord when it was attached to the object and allowed to hang in equilibrium, and then we measured the 'bottom' point of the drop by using a high-speed camera. A diagram of set up is below.

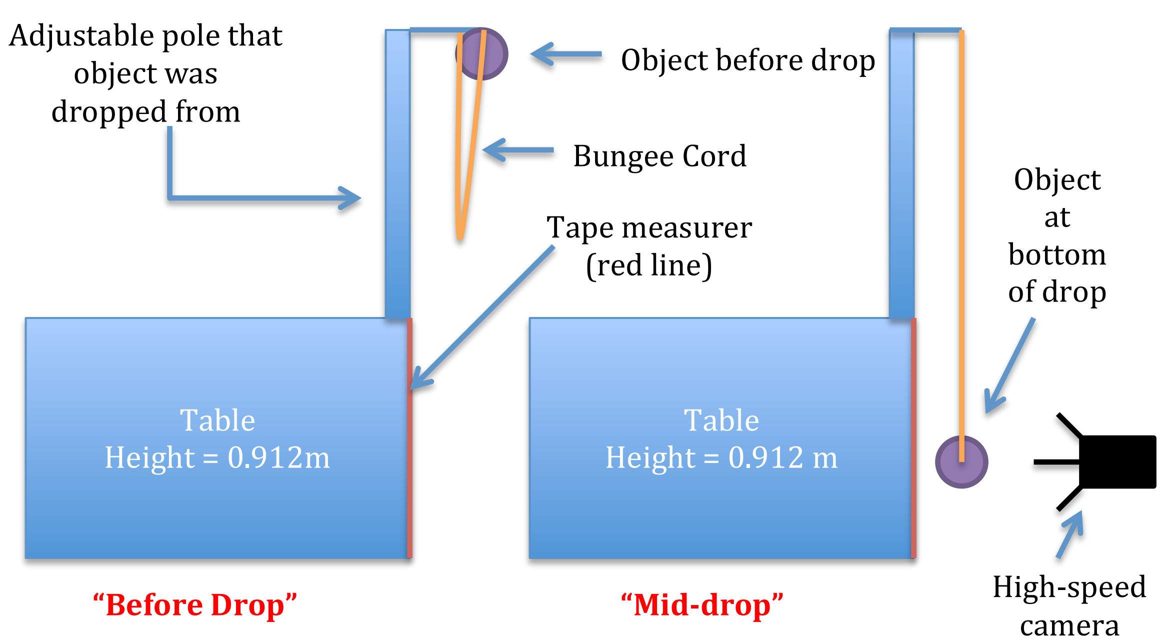


Diagram 1: Setup of experiment

As Diagram 1 shows, our set up was very simple. Attached to a stationary table was a pole that had a bar at the top, from which we were able to securely, attach our bungee cord to. We attached a tape measurer that ran the length of the table to the ground that would be visible along with the falling object in the view of the high-speed camera.

Procedure:

- Once the experiment was set up, we choose a length of bungee cord and tied an approval slipknot that we could attach the object to (one that will remain that length before and after drops).
- Before each drop we would make sure that we would measure the height from the top of the table to the height of the bar we were dropping the object from. The height of the table is known, (0.912m in this experiment), so the combination of these two heights would give you the total height of the drop.
- We would choose an object that we wanted to test and attach it to the bungee cord and allow it to hang from the cord at rest. The new length of the bungee cord is referred to as, x_o, or x-equilibrium.
- With x_o now known we took our object and positioned it at the same height as the bar, ready to drop (a chair might be needed for some).
- Meanwhile, the other partner needs to be positioned with a high-speed camera ready to film the object during its 'bottom' point.
- With both partners ready, the object is dropped and the high-speed camera should show the object as it drops and eventually is pulled back upward, with the tape measurer in clear view as well.
- The length of the bungee cord while the object is at its lowest point is x_{max}. This is found by adding the height from the top of the table to the bar the object was dropped from, and the measurement that is obtained from the high-speed camera.
- The stretch of the bungee cord, Δx is (x_{max}-x_o).
- We tested 5 lengths of bungee cord and did four different masses for each.

RESULTS:

In order to use the CWE theorem, we needed all the different components talked about earlier in the introduction so we can better model our spring constant, k. To do this we need to measure the mass and height of drop of the experiment for the left side of the CWE equation (gravity is constant at 9.81 m/s^2). The other variable we need is the **Δ**x value, or change in x, which we will get through our experiment procedure. Heights changed throughout the experiment so we could have a larger variety of massed objects, although we later regretted that choice since it produced another non-constant in our data. In our graphs for our raw data we graphed Δx^2 vs. (m*g*h), since those are the values in the CWE theorem. This left our slope to be, (1/2)*k.

Table 1-5 - Raw Data from Experiment

***All measurements of length, m, have an uncertainty of: +/- 0.001m**

Bungee Length**: 0.26m**

Mass (kg) (+/- 0.01 kg)	Height of Drop (m)	x_0 (m)	x_{max} (m)	Stretch (Δx) (m)
.05	1.432	.32	0.56	.24
.07	1.432	.355	0.66	.305
.1	1.432	.395	0.765	.37
.12	1.432	.465	0.91	.445

Bungee Length: **0.305m**

Mass (kg) (+/- 0.01 kg)	Height of Drop (m)	x_0 (m)	x_{max} (m)	Stretch (Δx) (m)
.05	1.432	0.37	0.65	.28
.07	1.432	0.41	0.76	.35
.09	1.432	0.455	0.885	.43
.12	1.432	0.54	1.06	.52

Bungee Length: **0.39m**

Mass (kg) (+/- 0.01 kg)	Height of Drop	x_0 (m)	x_{max} (m)	Stretch (Δx) (m)
.05	1.692	0.48	.825	.345
.09	1.882	0.58	1.135	.555
.12	1.882	0.7	1.415	.715
.14	1.882	0.785	1.545	.76

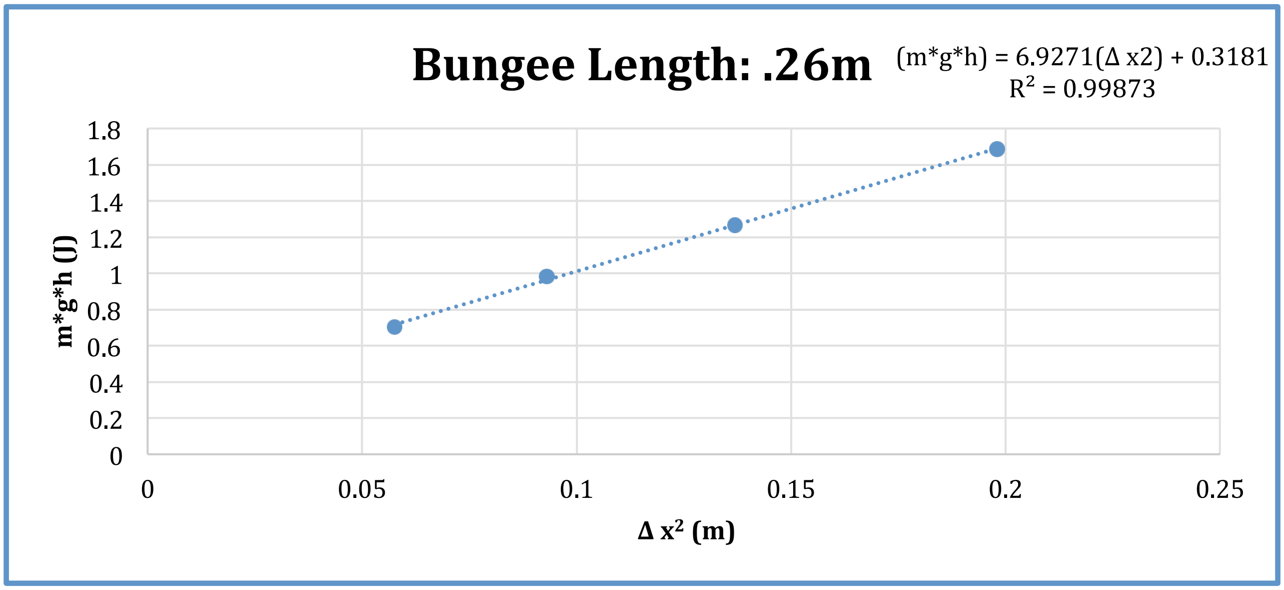
Bungee Length: **0.475m**

Mass (kg) (+/- 0.01 kg)	Height of Drop	x_0 (m)	x_{max} (m)	Stretch (Δx) (m)
.05	1.692	0.582	1.02	.438
.07	1.882	0.677	1.18	.503
.1	1.882	0.75	1.45	.7
.12	1.882	0.84	1.605	.765

Bungee Length: **0.55m**

Mass (kg) (+/- 0.01 kg)	Height of Drop	x_0 (m)	x_{max} (m)	Stretch (Δx) (m)
.05	1.882	0.655	1.135	.48
.07	1.882	0.715	1.325	.61
.09	2.027	0.85	1.65	.8
.12	2.027	0.953	1.785	.832

Graph 1: Graph showing the results from Box 1, when the bungee cord was at length: .26m.



*Graphs for the rest of the bungee lengths were done in a similar format and the results are listed below. *

Table 6: Regression Analysis from all tested lengths of bungee

Length	Equation	Uncertainty for Slope	% Uncertainty of slope	Uncertainty of intercept	% Uncertainty of intercept
.26m	$(m*g*h) = 6.9271(\Delta x^2)+0.3181$	.1367	**1.97%**	.02037	6.4%
.305m	$m*g*h) = 5.0281(\Delta x^2)+0.3341$	.1191	**2.36%**	.024	7.18%
.39m	$(m*g*h) = 3.6496(\Delta x^2)+0.4399$	.2484	**6.8%**	.1119	25.4%
.475m	$(m*g*h) = 3.8424(\Delta x^2)-0.7639$	.3539	**9.2%**	.23	30%
.55m	$(m*g*h) = 2.7632(\Delta x^2)+0.2613$	1.3	**47%**	.766	293%

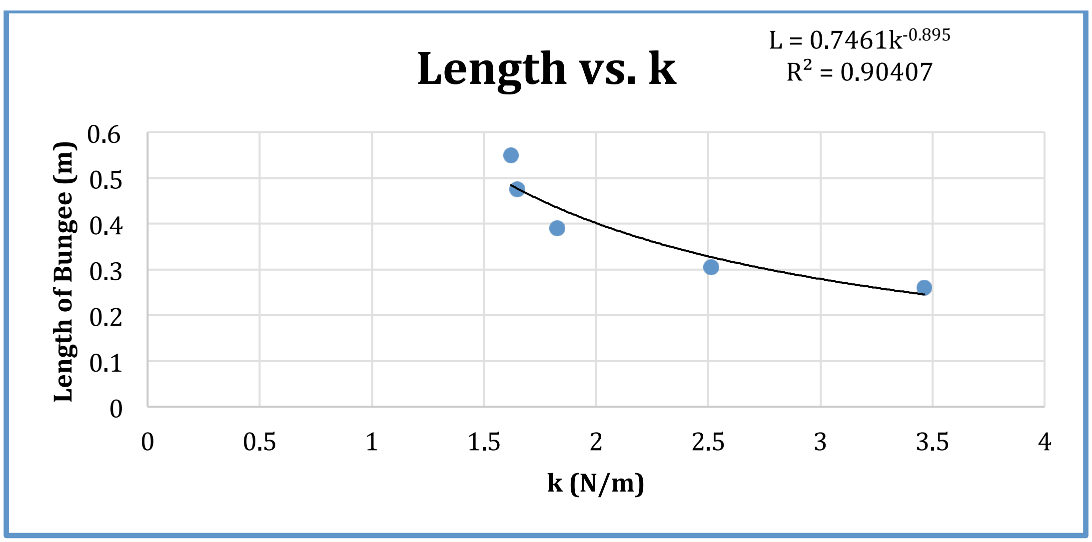
Experimental Values of Interest:

Looking at our original CWE theorem, $m*g*h=(1/2)k*\Delta x^2$, the slope of each of these graphs are, (1/2)*k. So the k constant for each length is listed in Table 7:

Table 7: Data points for Graph 2

Bungee Length (m)	k value (N/m)
.26	3.46355
.309	2.51405
.39	1.8248
.475	1.64835
.55	1.6197

Graph 2: This graph shows the relationship between all the different lengths of bungee cords' k values in this dynamic experiment. I applied a power curve to model its downward decay, as the bungee cord gets shorter. I choose against other curves because they fit the data either about the same or worse, nor did they go through more points. A polynomial regression gave me about the same level of fit, but a dramatically worse linearized graph.

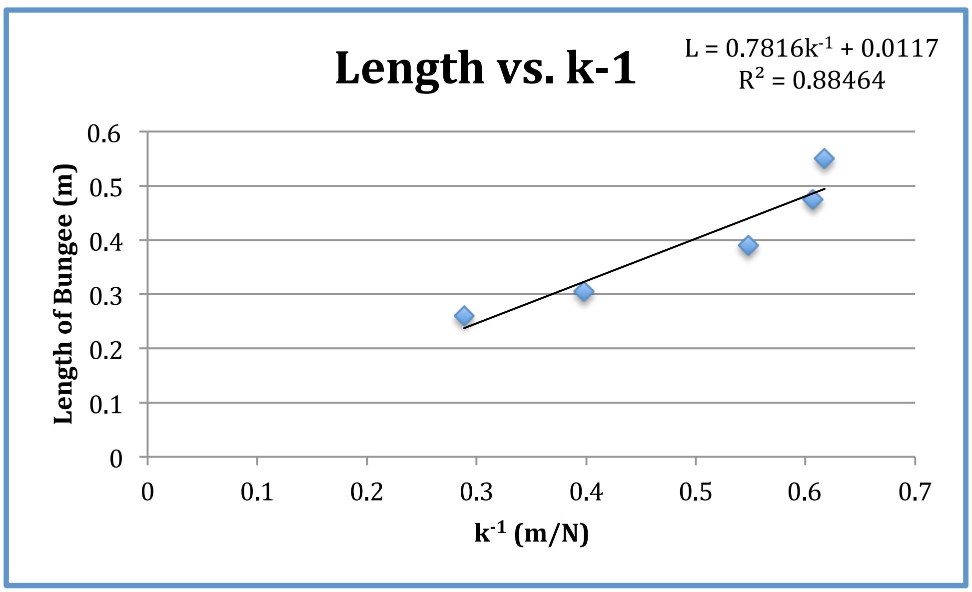


After plotting Length of bungee vs. k (from Table 7) we received a power regression of, $L = 0.7461k^{-0.895}$. In an ideal situation with more precise data the exponent would be closer to, -1, and we would be able to linearize this graph by plotting, Length vs. k^{-1}. As shown below, when that was done my data was not a very good linear fit since my exponent is so far from, -1.

Table 8: Data points for Graph 3

Bungee Length (m)	k^{-1} (N/m)
.26	0.28872111
.309	0.397764563
.39	0.548005261
.475	0.606667273
.55	0.617398284

Graph 3: Since I used a power curve to model my data to create a linearized curve I plotting, length vs. k^{-1}.



Regression Analysis results:

Uncertainty of Slope: .2805 % Uncertainty: 35.89%

Uncertainty of Intercept: .15413 % Uncertainty: 1317%

The results of our data are largely inconclusive due to the large amount of uncertainty that exists in our data, in some more than others. Looking at our linearized graph it clearly shows that the linear fit is not a good one, as the percent uncertainty of the slope is over 35%. Although it is hard to say whether that is from a non-linear relationship existing between the different length's k's, or whether my large uncertainties are to blame for the non-linear fit, especially at length 0.55m.

DISCUSSION:

We used a preexisting formula, the CWE theorem, to model our bungee cord's dynamic k value according to its length. Though since each cord has a unique physical composition, there is no preexisting data to quantitatively compare it to. Perhaps with more time and more strings, I could repeat this experiment, with all of my sources of error reduced, and I could test a variety of similar strings and create an average k value that can act as my 'ideal string's k' for this brand of bungee cord.

Though I do not have any data to compare it to currently, looking at each of the different length's regression analyses, it can be seen that some could be labeled as more 'acceptable' comparatively to others due to their smaller percent error. Yet others, such as the length at 0.55m, have an extremely large percent error, calling into question the entire experiment. This means that our data is outside of the 'acceptable' amount of error.

I only applied a power curve to Graph 2 due to the fact that the curve of the data resembled a power curve and was just as good of fit as other options but eventually gave a better linearized graph as well. Although in reality this was not a very good fit either, so it can be assumed this is not the correct way to model my bungee. Looking at my linearized graph of k values, it is confusing whether it shows if a non-linear relationship exists, or if the data points for lengths, 0.475m and 0.55m, with their large uncertainties skewed the linearized graph. Although what it does show you, as does the combination of the data and their accompanying graphs, is that there is a relationship between the length of the bungee and the k value, where as the length goes up, the k value goes down, resulting in a larger 'stretch' in the bungee. Regardless of either option, it can be seen that the linearized model of our k value is not acceptable since it has a percent uncertainty of slope of over 35%.

There were many sources of uncertainty in our experiment that might of help create the inconclusive data that resulted. The bungee cord itself could of offered some minor sources of uncertainty, such as the slipknot changing lengths in between trials or wearing out over time. There is also the possibility that while we were video taping with the slow motion camera that we were not completely parallel with the object, meaning that our point of view would skew how we read the tape measurer behind the object in the video.

Although the largest source of uncertainty, and where I think our data became inconclusive, was that we did not keep our height constant throughout the experiment. While running the experiment we wanted to make sure that it had a variety of masses for each length we tested, and the way we set up our experiment with the tape measure starting at the table, and then measuring from the table to the height of object separately meant that the object had to at least reach the table during its drop. Because of this we had to move the pole down or up according to the mass of our object. Although what we did not realize at the time was that the equation we were using to model our cord's behavior, the CWE theorem, is reliant on the potential energy, PE, of an object, which involves its height. Meaning that we did not keep all factors constant in our experiment that we needed to, which is an obvious source of large amounts of error. There was also confusion on the way in which height was recorded in the lab journals, leading to more possibilities of error.

Although there is much error in our experiment, we were able to see the obvious trend that existed in our data. Our hypothesis was partially supported, that as the length of bungee increases, the k value decreases.

CONCLUSION:

The fact that we did not keep height constant through our whole experiment means that it greatly decreases this experiment's value as a sufficient way to model our cord's behavior. Although we were not able to successfully receive a model from this experiment, the process we went through was logical and the results we received followed the reasoning that we hypothesized beforehand. Looking at our 5 trials of different lengths, there is an obvious outlier that exists in length .55m, which means that removing it from the experiment may greatly decrease our uncertainty, creating a better model relatively. The most effective thing regarding this experiment would be to do redo it completely now that we now all the sources of uncertainty. To fix our mistakes we would keep height constant throughout the experiment. We would also change our strategy of measurement so that the tape measurer went from height of the drop, allowing us to keep the adjustable bar high so we could still accommodate a variety of massed object. Another way to greatly increase the effectiveness of this lab is to find a larger apparatus to drop objects from so your data would be more similar to the final bungee challenge where you drop an egg with a bungee cord off a much larger height than the table set up we had available. With my experiment improved in these various ways, hopefully I could find a length that that would give the correct k value that will support of the weight of the falling egg enough to allow it an enjoyable bungee experience.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Patrick Murphy