Modelling an Elastic Cord
The Relationship between Acceleration (*a*) and Cord Length (*l*)

Abstract:

The ideal bungee jump is one in which the "jumper" is attached to the longest length of cord possible but does not hit the ground and the deceleration of the jumper upon recoil of the elastic cord does not exceed 3g (g = gravity, 9.81 m/s^2). This experiment aims to find a relationship between the length of the elastic cord and the greatest magnitude of acceleration experienced by this cord, in order to determine the greatest length that the cord can be without the deceleration upon recoil of the object exceeding 3g. Our hypothesis was that the maximum magnitude of acceleration, which occurs at the recoil of the cord, would increase as the length of the cord increased. We approached this question by taking an elastic cord and placing two knots in the cord to adjust the length. Five lengths were used and five masses were hung from one end of the cord, one at a time. Each mass was hung from each length of cord and maximal force of the bungee jump was recorded for each jump. This data was used to determine the acceleration of each object at each cord length. Once complete, we were able to plot the acceleration of each cord length against the length of cord to find a relationship between the cord length and the maximal acceleration. We found that our hypothesis was correct and that the greatest magnitude of acceleration of the hanging masses decreases as the length of the cord increases. This relationship can be used to determine the minimum length, 1.046 meters, that we can make the bungee cord without having the jumper experience a deceleration of magnitude greater than 3g, which is hazardous.

Introduction:

This bungee experiment was performed in order to determine the relationship between *a*, the acceleration of an object of a certain mass *m* hung from an elastic cord modelled as an ideal spring, and the un-stretched length *l* of the elastic cord used. This will be used to understand the inherent characteristics of the cord, which can be used to predict how the cord will react to an attached, falling mass during the Bungee Challenge.

The relevant equation specific to this experimental purpose or setup are:

$$|\mathrm{F_{total}}| = m_{\mathrm{hanging}}a$$

Where $|\boldsymbol{F}_{\mathrm{total}}|$ is the magnitude of the total forces acting on the elastic cord when the mass is released.
Note that $m_{hanging}$ is the mass of the hanging mass; in kilograms and *a* is the acceleration of the hanging mass; in m/s^2

Background:

$$|\mathbf{F_{total}}| = \boldsymbol{m_{\mathrm{hanging}}a}$$

This is the equation that is derived from Newton's Second Law. We know that the sum of the forces will be equivalent to the acceleration of the object at any given time multiplied by the mass of the object hanging from the cord. By the previous Bungee Lab, we know that for an optimal bungee experience, our object cannot have a deceleration of magnitude greater than 3g (g = 9.81 m/s^2). Thus we will use

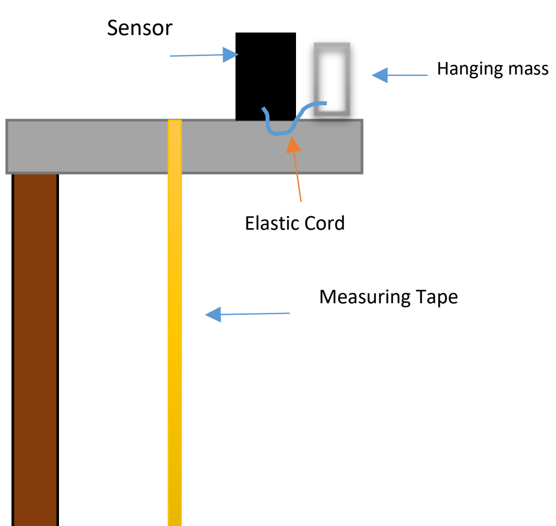
Newton's second law equation to determine the deceleration of the object after the initial recoil of the jump. Taking these measurements for different masses *m* and different lengths of cord *l* will give us a relationship between acceleration *a* and *l* that we can use to pick an optimal cord length for the bungee jump.

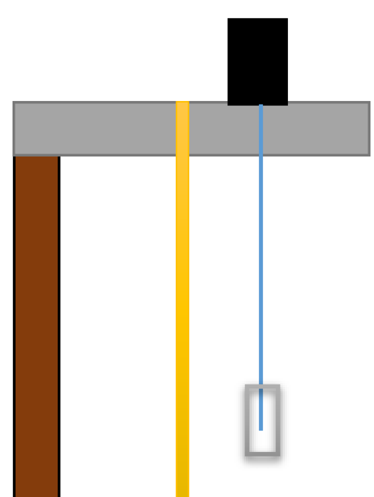
Hypothesis: A cord of longer length has a reduced k constant compared to a cord of lesser length. By Hooke's law, $F_{spring} = kx$, so a cord of longer length has a reduced restoring force. And because F = ma, the deceleration of the mass would be reduced. So the longer the cord, the less the magnitude of deceleration of the jumper.

METHODS:

Various masses were hung from a pre-tied length of cord. From this, force on the cord $|\boldsymbol{F}_{\text{total}}| = m_{hanging}a$ was plotted against each mass *m*. This yielded a relationship modeling Newton's Second Law in which the slope of the line gave us acceleration. This procedure was proceeded with five different masses of .01 kg, .05 kg, .07 kg, .09 kg, and .110 kg. Each mass was attached to a cord that was tied to a sensor at the top of the beam where the object would be released. The sensor was connected to Capstone which would collect the values of force vs time. The largest magnitude of force that was detected by the Capstone program was the value that we used for our experiment as this would be the point of initial deceleration and would also be the greatest magnitude of deceleration that the object would experience. The purpose of the experiment is to ensure that the deceleration of the object does not exceed 3g so the largest magnitude of acceleration must be the value used. After each mass was released, we plotted the total force of the bungee jump against the mass of each object to get the acceleration of the bungee jump for that particular cord length. Then the experiment was repeated for a total of five different cord lengths; .235 m, .27 m, .378 m, .415 m, and .535 m. The relationship between Force and mass was plotted for each cord length to get an acceleration *a* and length *l*. We then plotted the relationship between *a* and *l* to determine the effect of cord length on the maximum acceleration experienced by the object.

Setup:
A metal rod is extended vertically with a horizontal beam attached at the top as illustrated below. An electronic sensor is placed at the top of the beam with the elastic cord attached to a hook on the sensor. The other end of the elastic cord is tied into a loop that the object is hung from. Measuring tape is also hung from the metal beam and is used to measure the un-stretched length of the elastic cord before the mass was attached.





Results:

We began by collecting data to plot Force vs mass for each different cord length. Collected data is shown in the tables below.

Length = .225 meters

Mass (kilograms)	Force (Newtons)
.01	.73
.05	1.73
.07	2.15
.09	2.43
.110	3.1

Length = .270 meters

Mass (kilograms)	Force (Newtons)
.01	.4
.05	1.43
.07	1.77
.09	2.15
.110	2.67

Length = .378 meters

Mass (kilograms)	Force (Newtons)
.01	.9
.05	1.81
.07	2.15
.09	2.58
.110	3.05

Length = .415 meters

Mass (kilograms)	Force (Newtons)
.01	.7
.05	1.63
.07	1.89
.09	2.36
.110	2.86

Length = .535 meters

Mass (kilograms)	Force (Newtons)
.01	.5
.05	1.5
.07	1.86
.09	2.17
.110	2.68

Using these values for maximal force of the hanging mass after being dropped, we were able to plot the values in the figure below.

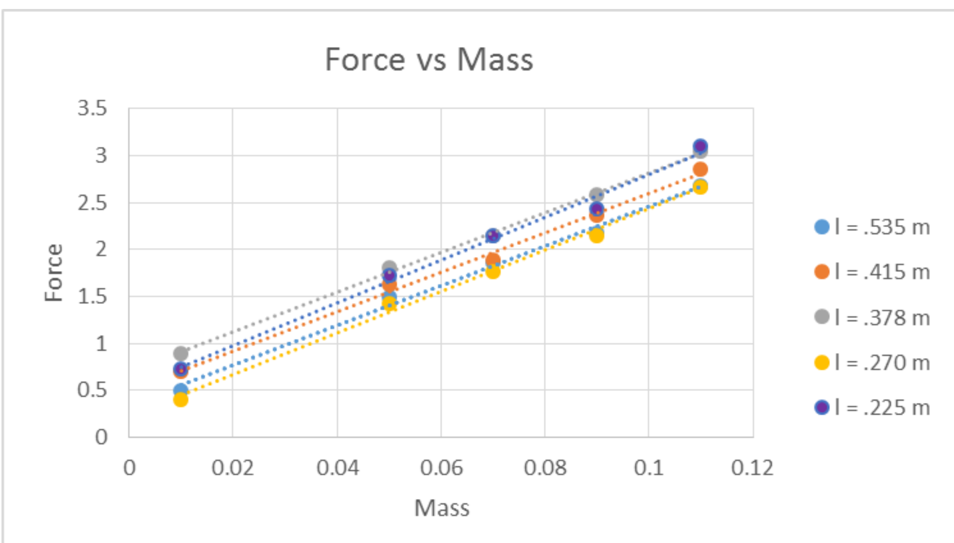


Figure 1: Plot of total force acting on bungee vs the mass of the hanging object at un-stretched lengths of .535 m, .415 m, .378 m, .270 m, and .225 m

After plotting Force vs mass of the varying lengths of cord, we were able to find the linear equations of each line, given in the following table.

Length (meters)	Linear Equation (F =ma)
.225	F = 22.764m + .5256
.270	F = 22.108m + .2249
.378	F = 21.176m + .7004
.415	F = 21.074m + .4971
.535	F = 21.189m + .3435

Table 1: Linear equation for each un-stretched length provided by the Force vs Mass plotted data.

Where the slope of each linear equation represents the acceleration experienced by the hanging mass for that length of cord.

Un-stretched Length *l*	Acceleration *a*
.235	22.764
.270	22.108
.378	21.176
.415	21.074
.535	21.189

Table 2: Un-stretched lengths of cord and the calculated acceleration at each cord length.

This data was then used to plot Length *l* vs Acceleration *a*.

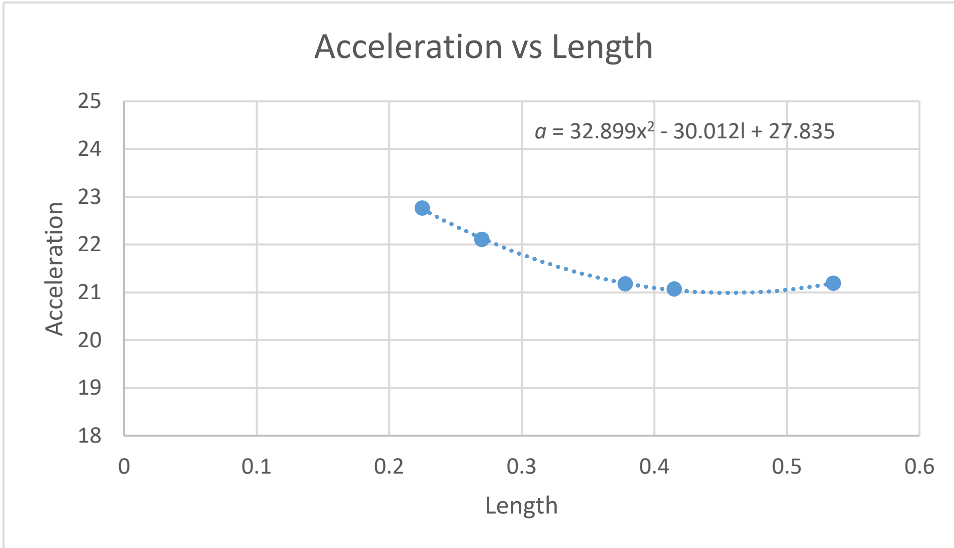


Figure 6: Plot of un-stretched length vs acceleration of the bungee. A polynomial relationship is observed.

After observing the polynomial relationship between Un-stretched length and acceleration, we linearized the data and achieved the below results.

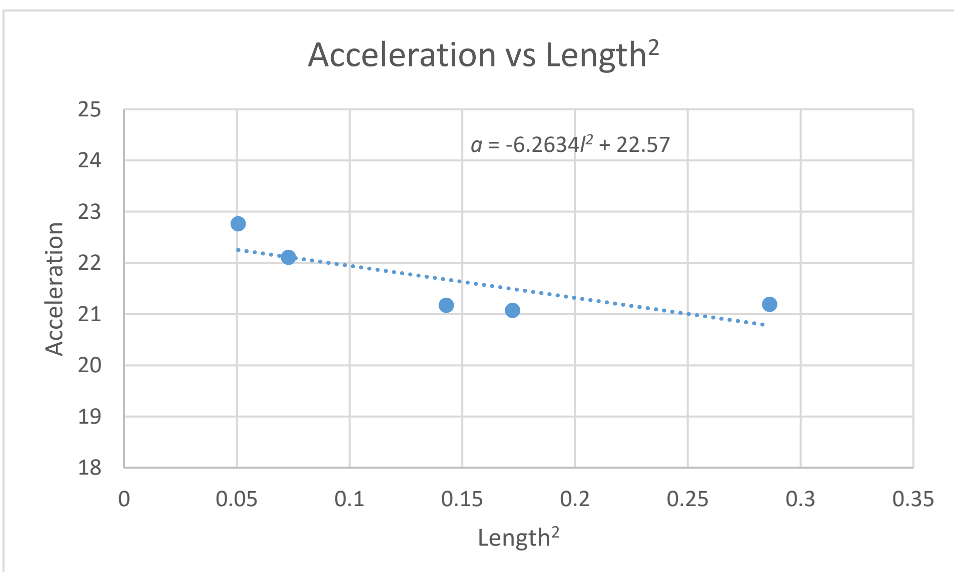


Figure 7: Plot of un-stretched length^2 vs acceleration. A linear relationship is observed.

We performed an Excel regression analysis on Figure 7 to determine the uncertainties of our experiment. The uncertainties for mass and length are given by the least count of the scale (.001 grams) and the measuring tape (.001 meters).

Uncertainty of Mass: 1x 10^{-6} kg

Percent Uncertainty of .01 kg: .01%

Percent Uncertainty of .05 kg: .002%

Percent Uncertainty of .07 kg: .0014%

Percent Uncertainty of .09 kg: .0011%

Percent Uncertainty of .11 kg: 9.09 x 10^{-4}%

Uncertainty of Length: .001 meters

Percent Uncertainty of .225m : .44%

Percent Uncertainty of .270m : .37 %

Percent Uncertainty of .378m : .26%

Percent Uncertainty of .415m : .24%

Percent Uncertainty of .535m : .19%

Uncertainty of Slope: 2.86

Percent Uncertainty: 45.7%

Uncertainty of Intercept: 0.478

Percent Uncertainty: 2.12%

Where percent uncertainty is calculated using the formula **Percent Uncertainty =** $\boldsymbol{\frac{Uncertainty}{Mean} x\ 100.}$

Experimental Value of Interest:

The experimental value of interest is the slope of the line obtained in the acceleration vs length graph. This is because this is the value that relates the length of the cord that we will use to the magnitude of the initial point of deceleration after the object recoils. This relationship will be used to determine the longest length that the cord can be in meters without exceeding a maximal deceleration of 3g.

<u>Discussion:</u>

Thus if we do not want the acceleration to exceed 3g (-29.43 m/s2), then we can determine the length using -29.43 = -6.2634l2 + 22.57. So we get that *l* = 1.046 meters. We observed an inverse relationship between acceleration and length of cord, thus confirming our hypothesis, as *l* increases, *a* should decreases. Thus the length of our cord should not be less than 1.046 meters. Using this length, we can use our knowledge about the relationship between the length of the cord and the stretch of the cord to determine the proper length so that the jumper will not touch the ground.

Sources of Uncertainty:

The main source of uncertainty in this model is that the relationship between length of cord and deceleration during the recoil of the cord is unknown. Thus we do not have this measure for an ideal cord to compare our findings to. Thus we are unable to use the relationship that we found to determine if our cord is behaving like an ideal spring.

Another source of uncertainty for our model would be that different individuals were measuring the length of the cord. We decided that the cord would be measured from the knot on one end of the cord, where the cord is looped to the hook of the sensor, to the knot on the other end of the cord, where the mass is being hung. There is a possibility of inconsistency in this measurement because length cannot be taken at the exact same point of the knot each time and each individual may take the measurement from a different point on the knot each time they are measuring the length.

We predicted that as the length of the cord increased, the magnitude of acceleration of the cord would increase. We found the relationship between a and l to be $a = -6.26l^2 + 22.57$. Thus as the length of the cord increased, the acceleration increases. By setting a = 3g, we can determine the length that cannot be exceeded in order to keep the deceleration of the object below 3g.

Conclusion:

This experiment was developed to model the maximum deceleration of an object during the recoil of a bungee jump as determined by the length of the elastic cord of the "jumping" object. We investigated the relationship between the acceleration of the object and the length of the cord in order to determine what the maximum length of the cord can be without having the acceleration reach a magnitude greater than 3g. A longer length of cord will ensure a more satisfying bungee jump as long as cord does not stretch so far that it reaches the floor and so long as the object does not experience a recoil deceleration greater than 3g in magnitude. This experiment will allow us to address one of these issues when developing our bungee for the Bungee Challenge.

This experiment was limited in that we could not use a cord that exceeded in .535 meters because the objects would hit the ground and we would not be able to detect their maximum magnitude of acceleration. Because we could not raise the beam higher than 2.1 meters, we were forced to only measure cords of shorter length. The accuracy of our results could be improved by performing the experiment from a taller height so that we could use a larger variety of cord lengths and masses.

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