David Williams and Stevan Kriss **Section:** 113-01 (Wed afternoon) **Date:** 11/15/16

TITLE: The Relationship Between Bungee Cord Length and K Value

ABSTRACT:

In this experiment, we attempt to define the relationship between a bungee cord's unstretched length "X_L" and the k value of the cord in order to determine the maximum stretch the cord experiences when an object of known mass is attached to the bungee cord and dropped from rest. The bungee cord is assumed to behave as an ideal Hooke's law system, and the behavior of the cord is analyzed using the CWE theorem. A constant mass attached to the bungee cord was dropped from rest, and a slow-motion video determined the maximum distance the mass fell. This was repeated with 8 different unstretched lengths of bungee cord. From these recorded values, we were able to calculate a k value for each different length of bungee cord. We then graphed the k value vs. the unstretched length of the cord to determine a relationship between the unstretched length of cord and the corresponding k value. This graph was linearized by graphing 1/k vs. the unstretched length, giving the equation 1/k = 0.53(X_L) + 0.01. The uncertainty in this equation was sufficiently small, and we were able to say that our equation was accurate. Using this equation, we can predict the k value for a given length of bungee cord with a small amount of uncertainty. Then, using the equation mgh = ½ kx^2, we can predict the maximum distance "h" a given mass will fall when attached to a given length of bungee cord, which will enable us to design a bungee jump in which the jumper does not hit the ground.

INTRODUCTION:

The purpose of this experiment was to determine how the unstretched length of a bungee cord affects the k value of the cord, and thereby affects the maximum length the cord will stretch.

Relevant Equations:

1) $F_{spring} = -kx$
2) $(PE + KE)_{top} = (PE + KE)_{bottom}$
3) $mgh = \frac{1}{2}kx^2$

The bungee cord is assumed to behave as an ideal Hooke's law system, where k is a spring proportionality constant that is characteristic of our bungee cord. Our experiment was designed to determine how this k value changed with the length of the bungee cord.

Our bungee system is assumed to be a conservative system, meaning that it neither gains nor loses energy from external sources. The CWE theorem states that in a conservative system, the total sum of the potential and kinetic energy of the system remains constant. Since we assume ours is a conservative system, the total sum of the potential and kinetic energy of the system should be the same when the mass is at rest at the top before being dropped and when the mass has fallen to the bottom of the drop and the bungee has stretched its maximum distance. This relationship is written as $(PE + KE)_{top} = (PE + KE)_{bottom}$.

At the top of the jump, all the energy of the system is gravitational potential energy corresponding to the position of the mass relative to the bottom of the jump. This gravitational potential energy is given by the equation mgh where m is the mass of the jumper, and h is the height of the jump. At the bottom of the jump, all the energy of the system is spring potential energy given by the equation $\frac{1}{2}kx^2$ where k is the spring constant of the bungee and x is the displacement, or amount of elongation of the bungee, calculated as (x_{max} – X_L).

Hypothesis:

We predict that if the original unstretched length of the bungee increases, the k value will decrease proportionally.

METHODS:

We attached a constant mass to the bungee cord and dropped the mass from rest. This was repeated for 8 different lengths of bungee cord while keeping the mass constant. For each drop, we measured the maximum position of the mass on the bungee cord at the bottom of the fall.

Figure 1: Bungee Cord System

- X_L is the unstretched length of the bungee cord with no mass attached
- X_{max} is the length of the stretched cord at the bottom of the drop
- x is the displacement of the mass, calculated as ($X_{max} - X_L$)
- h is the height of the jump, equal to X_{max}, or the position of the mass at the bottom of the drop

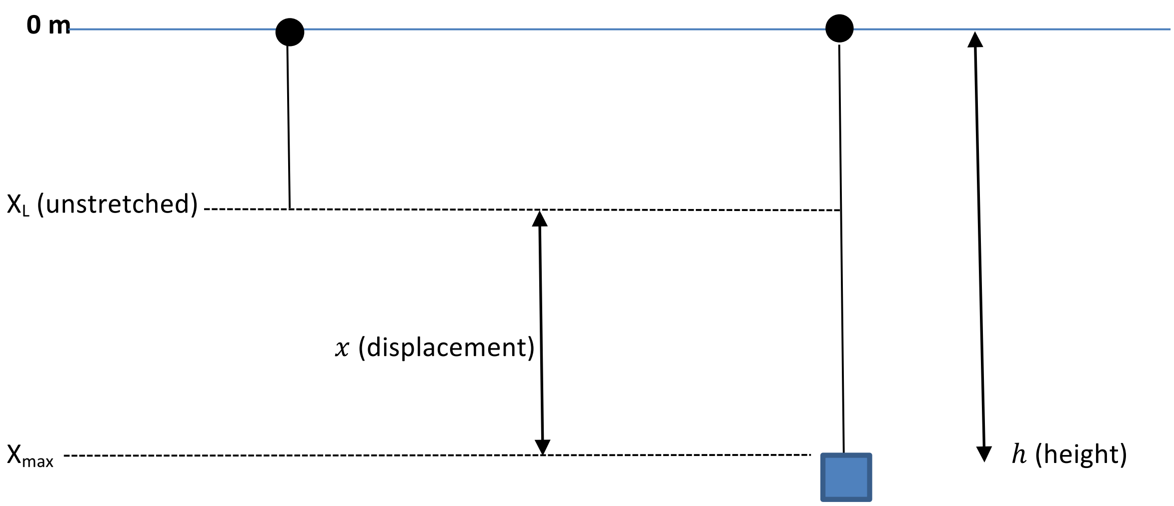
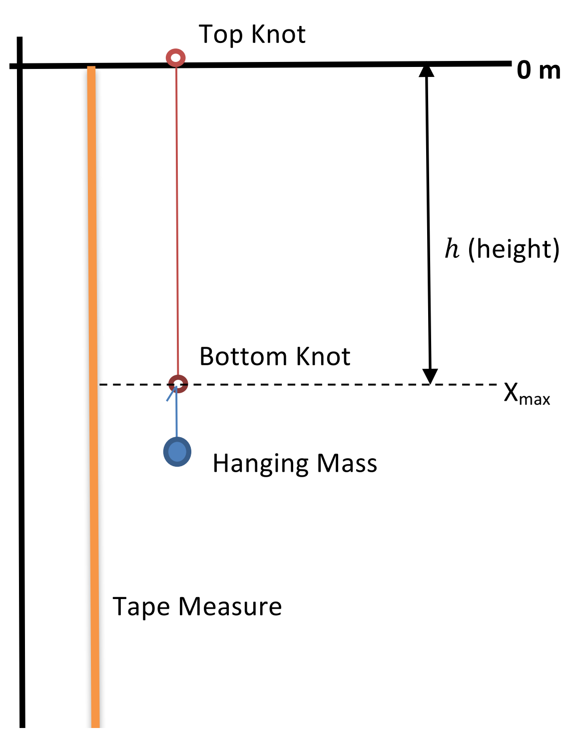


Figure 2: A diagram of our exact set-up. Measurements were made from knot to knot. The mass was dropped from rest at the position 0 m.



The set-up consisted of a bungee cord suspended from a metal pole. A knot was tied at the bottom of the cord, and a hanging mass was suspended from this knot. Measurements were made with a tape measure that was suspended alongside the bungee cord.

Procedure:

1. First the unstretched length of the bungee cord was measured as it hung alongside the tape measure. This length was measured from the top knot to the bottom knot.
2. A hanging mass of 0.05 kg was attached to the bungee cord through a loop tied in the bottom
3. The hanging mass was held at rest at the top of the "jump" in a position such that the bottom knot was lined up even with the top knot, both as a position of 0 meters. The mass was dropped from rest from this position.
4. A slow-motion video of the fall was taken using an iPhone camera. This slow-motion video was analyzed to determine the position of the mass at the bottom of the drop (X_{max}).
5. Steps 1-4 were repeated for 8 different lengths of bungee cord while keeping the hanging mass constant.

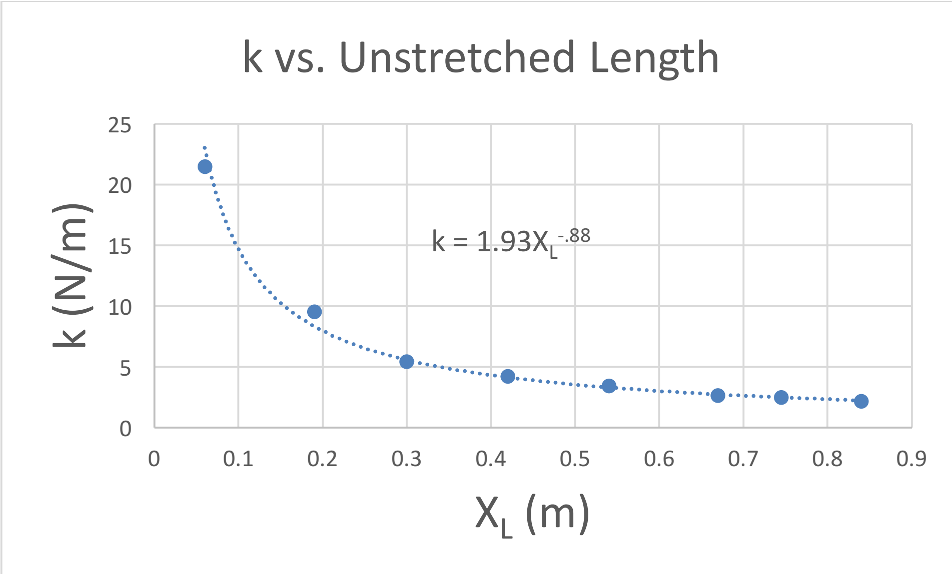
RESULTS:

The unstretched length X_L and the position at the bottom of the drop X_{max} were measured for 8 different lengths of bungee cord. From these values, the displacement x was calculated, and a k value was calculated for each bungee cord using equation 3: $mgh = \frac{1}{2}kx^2$ and solving for k.

Table 1: This table shows unstretched length, X_{max}, displacement, and k values for each of the eight different lengths of bungee cord used. The mass attached to the bungee was 0.05 kg each time. Uncertainties in X_L and X_{max} come from raw uncertainty in making a measurement with the tape measure. Uncertainty in displacement is calculated using a quadratic sum.

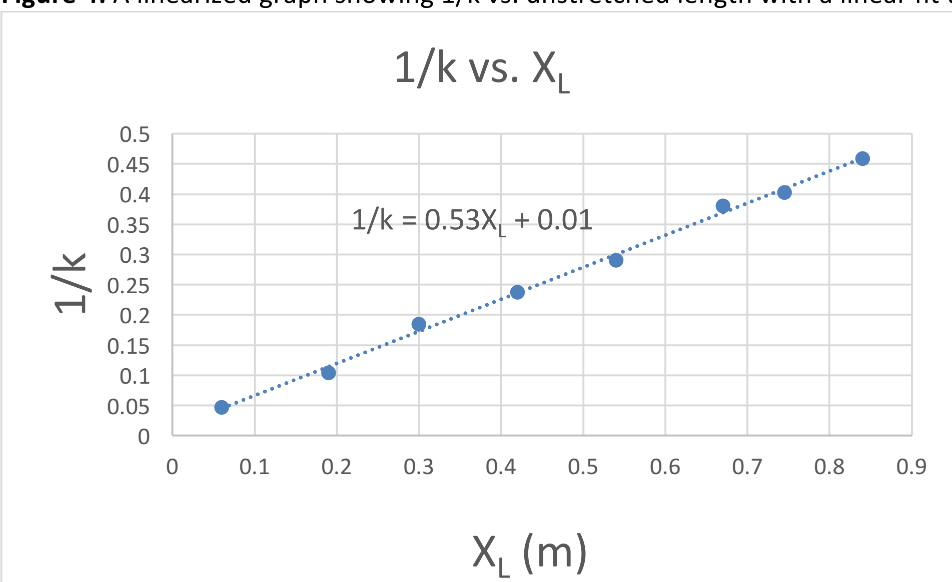
Unstretched Length $\mathbf{X_L}$ (m +- .01 m)	$\mathbf{X_{max}}$ (m +- .02 m)	Displacement "x" ($X_{max} – X_L$) (m +- .02 m)	**k** value (N/m)
0.06	0.14	0.08	21.46
0.19	0.39	0.2	9.56
0.3	0.64	0.34	5.43
0.42	0.87	0.45	4.21
0.54	1.1	0.56	3.44
0.67	1.39	0.72	2.63
0.745	1.52	0.775	2.48
0.84	1.72	0.88	2.18

Figure 3: A graph showing k value vs. unstretched length with a power fit trendline.



The graph of k vs. unstretched length gave us the equation $k = 1.93X_L^{-.88}$. This graph was linearized by graphing 1/k vs. unstretched length.

Figure 4: A linearized graph showing 1/k vs. unstretched length with a linear fit trendline.



The graph of 1/k vs. unstretched length gave the equation $1/k = 0.53X_L + 0.01$.

Uncertainty was calculated using Excel regression analysis for the linearized graph:

uncertainty for slope= 0.01	% uncert= 2%
uncertainty for y-intercept= 0.007	% uncert= 70%

Experimental values of interest:

The experimental values of interest are the slope and the y-intercept of the graph of 1/k vs. X_L.

Slope: 0.53 +- 0.01 y-intercept: 0.01 +- 0.007

Based on these values, we can calculate a k-value for a given length of our bungee cord with acceptable uncertainty by using the equation 1/k = $0.53X_L$ +0.01. We can then plug these values in to the equation $mgh = \frac{1}{2}kx^2$. Because all variables are known except h, we can then solve for h to find how far a given mass will fall when attached to a given length of bungee cord.

DISCUSSION:

The graph of 1/k vs. X_L was not based on a previously known equation. There was no previously known relationship between k-value and unstretched length for our bungee cord, and there were no known values to compare our experimental values of interest to. However, based on the linear regression uncertainty for the slope of 2% and the small y-intercept compared to the slope, we can be reasonably confident in our equation's ability to predict the k value of a bungee cord of known length.

To further determine the acceptability of our results, we performed a test of our equation. This was done by taking a bungee cord of known unstretched length (.33 m) and attaching a known mass (.15 kg). These values were chosen in an attempt to mimic the conditions of the actual bungee jump, but we were limited in height by the laboratory set-up. Based on these known values, we predicted that the h, or position of the jumper at the bottom of the jump, would be 1.11 m. We then allowed the mass to drop, and measured an actual h of 1.08 m. Because there was only a 3% difference in our predicted value and the actual value, we determined that our equation is sufficiently accurate to predict how far the jumper will fall.

There were several sources of error in our experiment. One source of error is the fact that X_{max} values were measured while the mass was moving. Because of this, we chose a greater raw uncertainty for these measurements than for X_L values, which were measured while the mass was stationary. Uncertainty in the displacement was propagated and calculated using a quadratic sum. Uncertainty in the values in our linear equation come from a linear regression analysis.

We hypothesized that if the original unstretched length of the bungee increased, the k value would decrease proportionally. Based on our graph of k vs. unstretched length, we were correct that the k value would decrease proportionally, but it was not a linear relationship.

CONCLUSION:

This experiment allowed us to determine an equation to describe the relationship between the k value and the unstretched length of our bungee cord. If we are given a length of bungee cord and a mass, we can now determine how far the mass will fall when dropped from rest at the top. This is useful for the bungee challenge because it will allow us to determine the correct length of bungee cord to use so that our egg comes close to but does not make contact with the floor.

Our experiment does not tell us how much acceleration the egg will experience during its fall. Going forward, we will need to find a way to ensure our egg does not experience too great of an acceleration as it slows down so that it does not break.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: David Williams