Lab Report Outline

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TITLE: Measuring the Effect of Varying Ratios of Static and Elastic Cord in a Composite Bungee

ABSTRACT:

To create a model that can predict the effect of using a composite cord in a bungee jump scenario on acceleration, we dropped a mass from a constant height while it was suspended by a cord composed of both static and elastic material. The ratio of static to elastic cord was varied with each trial, the maximum force experienced by the mass was recorded, and the acceleration was calculated using Newton's Second Law, f=ma. Because the mass is essentially in free fall while falling with the static cord, one would expect that the force necessary to stop the mass would increase as the static cord becomes longer in comparison to the elastic cord. We found that this expectation was correct. However, the first model we derived, r = 1.8a - 5.4 (where r is the ratio of cords and a is the maximum acceleration), is inaccurate due to the fact that the elastic cord no longer exhibits ideal spring behavior at greater deformations. The second model, r = 1.29a - 3.7, is accurate; however, it is limited to lower ratios where the cord still exhibits behavior that can be modelled as a Hooke's Law approximation.

INTRODUCTION:

Purpose:

- The purpose of this experiment was to determine the maximum acceleration experienced by a hanging mass dropped from a constant height while suspended by a composite bungee
- By finding the maximum acceleration for various ratios of static to elastic cord in the composite bungee, we can create a model for the effect of using a composite bungee in a bungee jump scenario

Relevant Background and Theory:

- As the hanging mass, m, falls with the static cord, the force of gravity, f_g, acts on it in the downward direction, causing it to increase in velocity, v
  - Under Newton's Second Law, f=ma, v increases at a rate equal to the force divided the mass, f_g/m (or the acceleration due to gravity)
- As the elastic cord begins to stretch, a restorative force, f_s, begins to act upon the mass in the upward direction
  - f_s is equal to, through a Hooke's Law approximation, f_s=kx, where k is the spring constant of the cord and x is the stretch of the cord from its equilibrium point
  - The acceleration of the mass is then equal to $(f_g - f_s)/m$
- Therefore the more time the mass can accelerate due to gravity without the spring force acting in the opposite direction, the greater the velocity of the mass will be
- As the velocity increases, the impulse, p, which is equal to $m \times v$, increases as well
  - Impulse is also defined as the integral of the total force $\int f\, dt$
  - As impulse increases, the force necessary to stop the motion of the hanging mass must also increase
  - As the maximum force increases, the acceleration experienced by the mass must increase as well

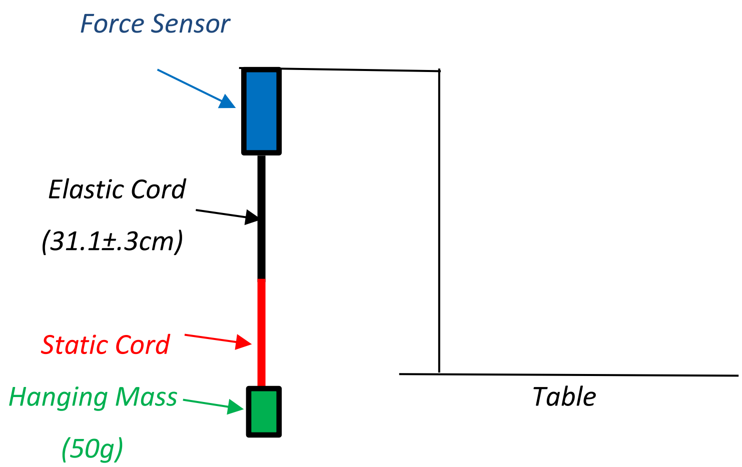
Hypothesis/Expectation:

- Our expectation for this experiment is therefore that as the ratio of static to elastic cord in our composite bungee increases, the maximum acceleration experienced by the mass will increase as well

METHODS:

We gathered data by dropping the mass from a constant height, recording the force exerted on it, and varying the ratio of static to elastic cord

Figure 1 Diagram of Experimental Procedures. The force sensor, cords, and mass were set up as shown below



1. Rigging was affixed to the table at a right angle as pictured above
2. To begin a force sensor which records forces using Capstone Data Analysis software was attached to the rigging
3. An elastic cord of length 31.1±.3cm was tied to the force sensor
4. A static cord (braided fishing line with a 30lb test strength) was tied to the elastic cord
5. A mass of 50g was then bound to the static cord
6. The mass was then raised to a height that the knot on the hanging mass was level with the knot tying the force sensor to the elastic cord
7. The mass was dropped, and the force sensor was used to record the maximum force exerted on the mass
8. To reduce error, the mass was dropped a second time and the average force for both drops was calculated
9. To vary the ratio of static to elastic cord, the static cord was then cut to a shorter length and then retied to the mass
10. Steps 4-9 were then repeated eight more times with varying lengths of static cord

RESULTS:

The maximum net force exerted upon the mass as it was stopped by the composite bungee was recorded using Capstone data analysis software.

The error in the experimental data comes from the least count in the Capstone Program and measuring tape.

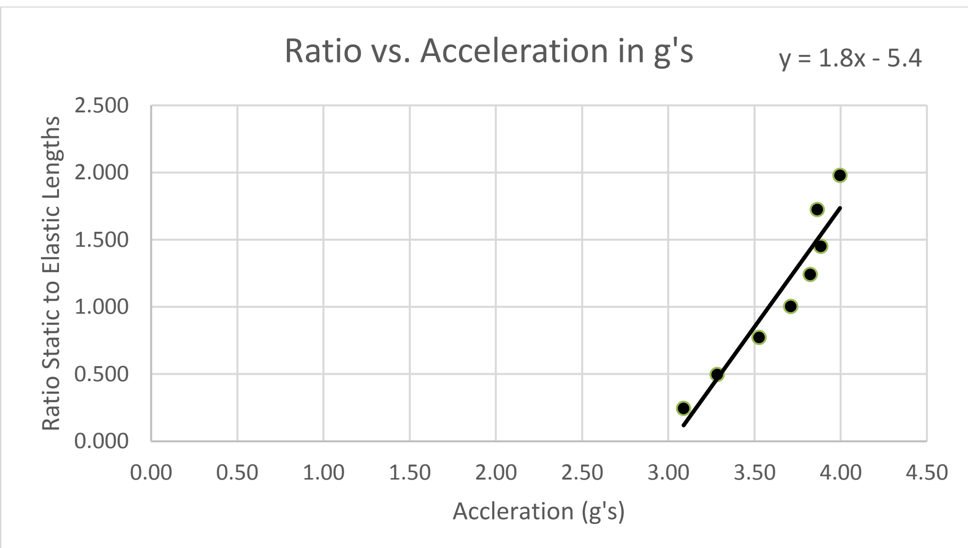
Figure *2* Maximum Force on a 50g Mass for Varying Ratios of Static to Elastic Lengths. The average maximum force was recorded for two drops per ratio while the ratio was changed by approximately ¼ for each trial

Trial	Static Length (m) $\pm$.002m	Ratio of Static to Elastic Lengths $\pm$1%	Max Force 1 (N)$\pm.01$N	Max Force 2 (N) $\pm.01$N	Avg. Max Force (N) $\pm.01$N
1	0.615	1.977	1.97	1.95	1.96
2	0.536	1.723	1.89	1.90	1.89
3	0.451	1.450	1.87	1.94	1.91
4	0.386	1.241	1.88	1.87	1.88
5	0.312	1.003	1.84	1.80	1.82
6	0.24	0.772	1.74	1.72	1.73
7	0.154	0.495	1.61	1.61	1.61
8	0.076	0.244	1.52	1.51	1.52
9	0	0.000	1.41	1.43	1.42

Figure *3* Acceleration in g's for Varying Ratios of Static to Elastic Lengths. Using the average maximum force the acceleration in g's (g's being the acceleration divided by the value of the acceleration due to gravity)

Trial	Ratio of Static to Elastic Lengths $\pm 1\%$	Avg. Max Force (N) $\pm .01\text{N}$	F/mg (g's) $\pm .01\text{g}$
1	1.977	1.96	4.00
2	1.723	1.895	3.86
3	1.450	1.905	3.88
4	1.241	1.875	3.82
5	1.003	1.82	3.71
6	0.772	1.73	3.53
7	0.495	1.61	3.28
8	0.244	1.515	3.09
9	0.000	1.42	2.90

Figure *4* Maximum Acceleration vs. Ratio of Static to Elastic Lengths. The ratio of lengths was found to have a linear relationship



	Coefficients	*Standard Error*
Intercept	-5.4	0.6
Slope	1.8	0.2

Values of Interest:

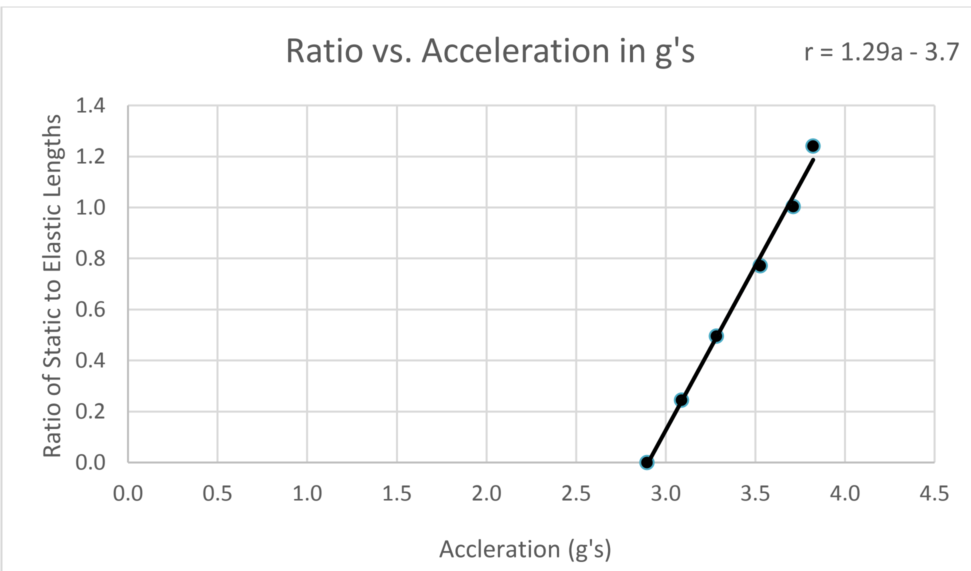
- The linear equation for the relationship between the ratio of static to elastic cord and maximum acceleration, r = 1.8a - 5.4, represents a model of the effect of a composite bungee on the force exerted on the mass
  - The standard error for this model's x variable and intercept are ± 0.6 and ± 0.2 respectively; these values were obtained through linear regression using Excel data analysis software

As the ratio increases to values greater than one and a half, the data points become significantly less linear. As such, a more linear model can be found by removing trials one through three. This can be justified as the cord stops exhibiting ideal spring behavior at greater deformations.

Figure *5* Acceleration in g's for Varying Ratios of Static to Elastic Lengths. In order to create a more linear model for the maximum force exerted for lower ratios, the first three trials have been removed

Trials	Ratio of Static to Elastic Lengths $\pm 1\%$	Avg. Max Force (N) $\pm .01\text{N}$	F/mg (g's) $\pm .01\text{g}$
1	1.241	1.88	3.82
2	1.003	1.82	3.71
3	0.772	1.73	3.53
4	0.495	1.61	3.28
5	0.244	1.52	3.09
6	0.000	1.42	2.90

Figure *6* Maximum Acceleration vs. Ratio of Static to Elastic Lengths. In order to create a more linear model for the maximum force exerted for lower ratios, the first three trials have been removed



	Coefficient	*Standard Error*
Intercept	-3.7	0.2
Slope	1.29	0.05

Values of Interest:

- The linear equation for this restricted data set, r = 1.29a - 3.7, represents a more accurate model for the effect of a composite bungee on the force exerted on the mass
  - However, this model is only accurate to ratios of static to elastic lengths less than 1.24
  - The standard error for this model's x variable and intercept are $\pm$0.05 and $\pm$0.2 respectively; these values were obtained through linear regression using Excel data analysis software

DISCUSSION:

Testing the Accuracy of Results:

- The experimental values of interest here are both the linear models' slopes and intercepts, which can be used to predict the maximum force exerted on a mass dropped while suspended by a composite bungee
- To test the accuracy of the first model, we chose an arbitrary ratio and calculated the maximum acceleration the mass should experience using the model
    - The arbitrary ratio was 0.70, in which case the acceleration should be equal to 3.94g's
    - The actual acceleration experienced by the mass was 3.45g's
    - The percent error between these values is 14.3% which is greater than the percent uncertainty for the model, 6.3%, therefore the first model is inaccurate
- To test the accuracy of the second model we used it to calculate the maximum acceleration the mass should experience for the same ratio
    - For a ratio of 0.70, the maximum acceleration should be equal to 3.41g's
    - As the actual acceleration was 3.45g's, the percent error between these values is 1.2%, which is less than the percent uncertainty for the model, 5.4%, therefore the second model is accurate

Interpretation of Data:

- As the first linear model has a greater slope, we can conclude that the change in the ratio of static to elastic cord yields a significantly smaller average change in the maximum acceleration
    - We can therefore assume that some factor was causing it to reach an upper limit of acceleration
    - If acceleration is reaching an upper limit, the force must be as well
    - Because the cord is modeled to a Hooke's Law approximation, f_s=kx, we can assume that the k constant is changing and the cord is no longer acting like an ideal spring as it stretches to greater lengths
- Because the k constant is significantly less predictable at these larger ratios, the second linear model obtained (which discounts ratios greater than 1.24) is more accurate for ratios less than 1.24

Sources of uncertainty:

- The random uncertainty in this experiment comes from the Capstone data analysis software; this uncertainty was minimized by running each trial twice and taking the average of the two values
- The elastic cord may have experienced deformation after stretching out over the course of the experiment; the elastic cord was measured before and after the experiment to ensure the length had not changed
- The height the mass was dropped from may have varied, resulting in inaccuracies in the maximum force value

Hypothesis/Expectation Reexamined:

- As the maximum acceleration experienced by the mass increases with the ratio of static to elastic cord, our expectation is correct

CONCLUSION:

- As the percent error, 14.3%, was greater than the percent uncertainty, 6.3%, for the first model, the first model is inaccurate and should not be used to calculate the maximum acceleration experienced by a mass falling while suspended by a composite bungee cord
- As the percent error, 1.2%, was less than the percent uncertainty, 5.4%, for the second model, the second model is accurate and can be used to calculate the maximum acceleration experienced by a mass falling while suspended by a composite bungee cord
    - However, due to the apparent change in the elastic cord's k constant at greater stretches, this model cannot be used to predict accelerations for ratios of static to elastic cord greater than 1.24
- As this cord does not function like an ideal spring, a valuable continuation of this experiment would be to replicate this experiment with various masses and record the change in the maximum acceleration for different ratios of static to elastic cord

On my honor, I have neither given nor received any unacknowledged aid on this assignment.
Pledged: Daniel Scarbrough Clark