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Testing the validity of a CWE Theorem model on an elastic bungee jump system

ABSTRACT:

For a successful and thrilling bungee jump, we must determine how the initial conditions of the jump affect the forces that act on the jumper over the course of the jump. One method is to apply the CWE Theorem to our bungee system, because it relates the initial energy to the final energy, thus allowing us to see how the initial conditions (which determine the initial energies) affect the outcome of the jump. This gives us the equation

$$mgh = \frac{1}{2}kx^2$$

which relates the mass of the object (m), the height of the drop (h), the spring constant of the string (k), and the total stretch of the string (x). Our experiment tested the accuracy of the CWE Theorem model by comparing it to 'real world' data, obtained by measuring the total stretch of the string for various masses dropped from the same height (while keeping the un-stretched length constant), which were then compared to the theoretical stretch obtained for the same mass. Our results suggest that, for early drops, the CWE Theorem can accurately predict the stretch of our system. However, as the trials continued, the model's accuracy quickly dropped, most likely due to irreversible deformation for the string. Thus the CWE Theorem fails for long-term accuracy (accuracy across several trials). However, because our bungee jump will only be performed once and the model is accurate for early trials, we can still apply the CWE Theorem model to our system to maximize both safety and thrill of our bungee jump.

INTRODUCTION:

Applying the CWE Theorem to our bungee system gives the following model:

$$mgh = \frac{1}{2}kx^2$$

where m is mass (kg), g is the gravitational acceleration constant (9.81 m/s^2), h is the total height of the drop (m), k is the spring constant (N/m), and x is the total stretch of the string (m), which can be derived by the following.

From the CWE Theorem, assuming no outside forces, (KE_i + PE_i) = (KE_f + PE_f)
We know that $KE_i = 0$ because we start at rest, so $v_i = 0$
$PE_i = mgh$
$PE_f = 0$ because origin is set to be the final height, so $h_f = 0$
$KE_f = (1/2)kx^2$

In order to see whether our system followed the CWE Theorem model, we varied the mass of the falling object and measured the resulting stretch of the cord. These values were then compared to the values obtained from the model.

If our system seems to follow the CWE Theorem model, then we will have a concise equation that relates the initial conditions of the drop to the final conditions, thus giving us a very powerful tool in designing a thrilling and successful bungee jump.

We expect the system to be consistent by the CWE Theorem model

METHODS:

We can experimentally control m, h, and k while x is determined by the equation. Our value of k was found using equations derived by Mah and Pamlatary (unpublished data)

So we varied m while keeping the un-stretched length of the cord unchanged and measured its effects on x.

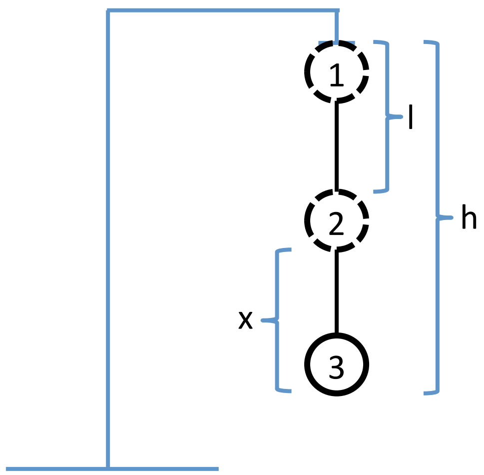

Figure 1. Experimental Setup
1. represents mass in initial state before the drop
2. represents mass when string is a full un-stretched length
3. represents mass at final state when string has maximum stretch

l is un-stretched length of string (known)
h is total height travelled by mass (measured)
x is total length stretched of string (derived by x = h-l)

Mass was tied to elastic bungee cord (the same cord used in previous bungee experiment) using a standard knot. The bungee cord was then attached to a horizontal metal bar with a metal screw, rather than by a knot, to reduce measurement issues with knots (e.g., how do we factor into account the size of the knot, what if the knot loosens as the experiment proceeds, etc.). The length of the cord was kept constant throughout the experiment.
An iPad with a slow-motion camera was placed at the bottom of the drop, along with a tape measure, in order to determine the total distance of the fall (h).

Procedures:
1. Positioned mass by horizontal bar (Figure 1, position 1.)
2. Released mass from position 1.
3. Measured total distance travelled during the drop (Figure 1, position 3.) using slow-motion camera on iPad
4. Steps 1-3 were repeated for 4 separate masses (0.05 kg, 0.07 kg, 0.10 kg, 0.12 kg) on the same length of cord

RESULTS:

We varied the mass of the object, and measured how that affected the stretch of the elastic cord. The results were then compared to the theoretical stretch obtained by the CWE Theorem equation above for the same conditions by finding the percent difference.

m (kg)	h (m)	k (N/m)	x (m)
0.12	1.49	4.21	0.86
0.10	1.47	4.21	0.84
0.07	1.23	4.21	0.60
0.05	1.03	4.21	0.40

Table 1. Data obtained from experiment.
- m is the mass of the object
- h is total height of the drop. Found using slow-motion camera
- k is spring constant. Found using the equation $k = 2.05l^{-1.05}$ derived from previous experiment (Mah and Palmatary, unpublished data; l = length of cord)
- x is total stretch of the string. Found by $x = h - l$

Each value of x was obtained by subtracting l from h. So to find the uncertainty of each x, we must propagate uncertainty for h and l. So we have

$$\begin{aligned} \Delta x &= \sqrt{\Delta h^2 + \Delta l^2} \\ &= \sqrt{(0.5)^2 + (0.1)^2} \\ &= 0.51 \end{aligned}$$

Where the uncertainty of h is 0.5 (measured from video, so we expect a higher uncertainty) and uncertainty of l is 0.1 (least measure on tape measure)

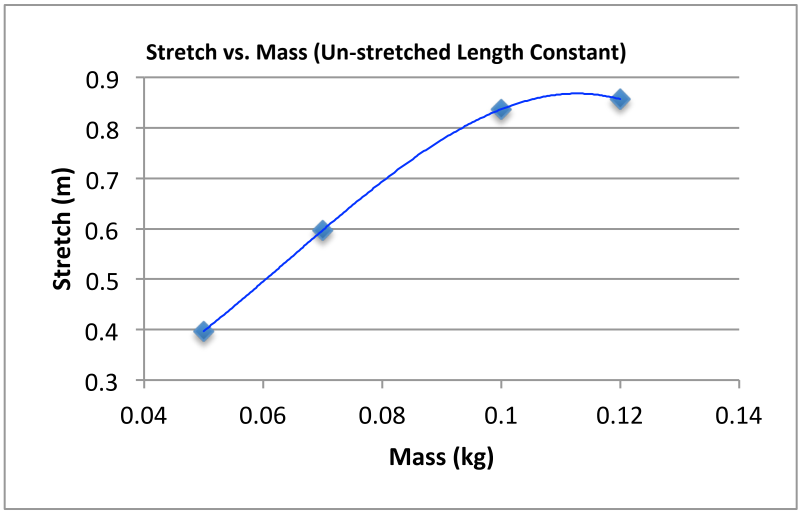

Figure 2. Graph of stretch as a function of varying mass.

Formula of trendline:

$$x = -1428.6m^3 + 274.29m^2 - 7.34m + 0.26$$

It should also be noted that because the data almost perfectly follows a cubic trendline (R^2 = 1), the data cannot be linearized. Furthermore linearization of the data does not serve any real purpose in interpreting the results. The slope of the trendline does not have any important physical interpretation.

Because we cannot linearize the trendline, we must use more sophisticated methods to calculate its uncertainty. Rather than interpret the uncertainty in the coefficients (because again, the trendline has no real significance in the context of our problem; it is just helpful for visualizing the pattern), we will interpret the uncertainty of the function as a whole. The uncertainty of a function at a point x is given by the formula

$$\Delta f(x) = f'(x)\Delta x$$

So for a given mass, m, the uncertainty of the trendline is

$$\begin{aligned}\Delta x &= (-4285.8m^2 + 548.58m - 7.34)\Delta m \\ &= (-4285.8m^2 + 548.58m - 7.34)0.01 \\ &= -42.86m^2 + 5.49m - 7.34 \bullet 10^{-2}\end{aligned}$$

Using this formula, we can find the uncertainty for each point of the trendline (Table 3).

x_{exp}+/-0.51 m	$\Delta\ x_{exp}$	%Uncert
0.86	0.03	3.71%
0.84	0.05	5.62%
0.60	0.10	16.90%
0.40	0.09	23.66%

Table 2. Uncertainty of x_{exp}

Uncertainties found for x_{exp} found using above equation for uncertainty, where m is

DISCUSSION:

To begin, when comparing our experimental data with our theoretical predictions, we have:

m (kg)	x_{exp}+/-0.51 m	x_{theo}	% difference
0.12	0.86	0.91	6.12%
0.10	0.84	0.83	1.12%
0.07	0.60	0.63	5.75%
0.05	0.40	0.49	18.36%

Table 3. Comparison of observed and theoretical stretch

- x_{exp} = x obtained from experiment (Table 1)
- x_{theo} = x obtained from CWE Theorem Equation

$$x = \sqrt{\frac{2mgh}{k}}$$

where m, h, k are as they appear in Table 1.

In Figure 2., we would expect a trendline to be represented by a square root function, rather than a cubic function. However, when we remove the data point for m=0.05 (which had 18% error from theoretical prediction), the graph had trendline $x = 4.02m^{0.71}$, which is closer to what we would expect. But because we do not have solid evidence (e.g., a known experimental error) to remove the data point, we will continue to use the cubic trendline in further analyses.

The main source of uncertainty in our experiment is the irreversible deformation of the string because of stretching. In our experiment, we started with the heaviest weight and progressively removed weights. These methods most likely maximized the deformation of the string, and could potentially explain the 18% error on the final mass (Table 3). This could explain why our uncertainties consistently rose as the experiment progressed (Table 2).

Furthermore, we did not perform multiple drops for each weight, because we were afraid that multiple drops would further deform the string and skew the data more so than if we only used one data point. Further testing, with more replicates, perhaps with new strings each time, will be needed to validate the results of this (admittedly) smaller scale experiment.

While early theoretical predictions of the CWE Theorem model were in agreement with our experimental data, later predictions did not match with experimental data. We can conclude that the CWE Theorem model lacks *long-term* accuracy, most likely due to irreversible deformation of the string. However, in early trials before the string was deformed, the model is able to accurately predict the outcomes. So because the CWE Theorem does not account for deformation of the string, it cannot accurately model our system indefinitely. But until the string deforms, our system can be accurately modeled by the CWE Theorem equations.

Finally, even though our model lacks long-term accuracy, because our egg drop will only be performed once on brand-new string (i.e., non-deformed string), we can still apply the CWE Theorem model to our system.

CONCLUSION:

The CWE Theorem model, while theoretically sound, does not take into account the deformation of the bungee cord. While this is not an issue in early trials, during which deformation has not occurred, with successive trials, the accuracy of the model appreciably drops. However, by the nature of the actual bungee jump, we will be performing only one drop, so this loss of accuracy should not affect the overall results. So we can apply the CWE Theorem model in order to more easily relate the initial conditions of the jump to the final conditions so that we can maximize the thrill of our jump while still ensuring the safety of the egg.

Works Cited

I acknowledge pervious lab work done by Andrew Mah and Hannah Palmatary.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Andrew Mah