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Physics 113-02

Bungee II: Designing a Bungee Jump

ABSTRACT:

This report presents the relationship between the mass added to the bungee cord and the resulting stretch of the bungee (Δx) when dropped from the height (h) above the ground. A work and energy equation was derived considering the position of the mass at the top and bottom of the drop. After testing 5 different masses ranging from 0.05kg to 0.07 kg and measuring the lowest point of the drop using slow-motion technology, the initial potential energy was calculated. Due to concerns about parallax, uncertainty for the measurement was relatively high at +/-0.01m. Other sources of uncertainty include dropping the mass with an unintentional initial velocity and in a direction that was not completely vertical. Because the bungee was modeled as a spring system, all energy at the bottom of the drop was in the form of elastic potential energy. Therefore, using the classical work energy theorem and including the derived equation for the "spring" constant (k) of the bungee cord from the previous bungee lab, an experimental equation was derived as a function of Δx and L. Since the theoretical height of the drop is the change in the length of the bungee added to the initial length of the cord, a second equation was derived for Δx and L of the system. Solving this system of equations presented the ideal initial length (L) for any given mass and height. Following the experiment, the equations and the system was tested to model the final bungee drop in the coming weeks with a calculated 5.11% error. The mass did come up short of hitting the ground, which, in the case of the egg drop, is a preferred result, but with a relatively great quantity of percent error, a certain level of caution in terms of measuring the initial length of the bungee must be taken.

INTRODUCTION:

Purpose: To determine the mathematical relationship between the mass added to the bungee cord and the length of the resulting stretch of the bungee cord when the mass was dropped from a set height to use to establish an equation for final bungee jump.

Relevant equations:

$$mgh = (½)k\Delta x^2 \quad [1]$$

$$k = 1.2908L^{-1.02} \quad [2]$$

$$h = L + \Delta x \quad [3]$$

$$mgh = 1.3576\Delta x^{1.5636} \quad [4]$$

Variables:

m = mass (kg)

g = Force due to gravity (m/s^2) = 9.8 m/s^2

h = height of the drop measured from the top of the bungee to the ground (m)

k = spring constant (N/m)

Δx = stretch of the bungee caused by the weight force of an added mass (m)

L = initial (un-stretched) length of the bungee (m)

Theoretical Background:

According to the work energy theorem, the work done by non-conservative forces is equal to the change in kinetic energy plus the change in potential energy ($W_{non-conservative} = \Delta KE + \Delta PE$). Because the work done by non-conservative forces on the bungee system is zero, the initial kinetic and potential energies equal the final kinetic and potential energies (initial meaning the top of the drop and final meaning the bottom of the drop) ($KE_{initial} + PE_{initial} = KE_{final} + PE_{final}$). Since the mass attached to the bungee is not moving at the top and bottom of the drop, the initial and final kinetic energies is zero, leaving all energy in the form of potential energy. At the top all energy is gravitational potential energy and at the bottom all energy is spring potential energy caused by the restoring force of the bungee. Through this reasoning, Equation [1] is formed. Equation [2] is the experimental equation found in the previous bungee lab in which the relationship between the spring constant of the bungee and the initial length of the bungee was found. Equation [3] represents the ideal situation for the bungee jump where the initial length of the bungee and the stretch of the bungee after adding a mass is exactly equal to the height of the drop from the ground to the top of the bungee.

Hypothesis:

After analyzing the results from the previous experiment which showed as the initial length of the bungee increased, the k value, otherwise known as the spring constant) decreased, it was predicted that the bungee cord would not act exactly as a spring. Because the bungee does create a restoring force on the mass, which does experience oscillatory motion to a certain extent, it was hypothesized that the final potential energy of the bungee would somewhat model that of the power function of the spring system, only with a different exponent representing the differences between the two.

METHODS:

In order to find the ideal initial length of the bungee, another equation in terms of Δx and L was needed. To do so, L was kept constant throughout the experiment, the mass added to the bungee was varied, thus, serving as the independent variable, and Δx served as the dependent variable as it was varied as a result of the differing masses. Three trials of five different masses ranging from 0.05 kg to 0.07 kg were performed. Once the relationship between Δx and L was found graphing initial potential energy versus Δx, the equation of the graph was combined with the equation from the previous bungee lab to produce one multivariable equation with Δx and L as the variables. The system of equations of this equation and Equation [3] was solved for L to find the ideal initial length for any given mass and height.

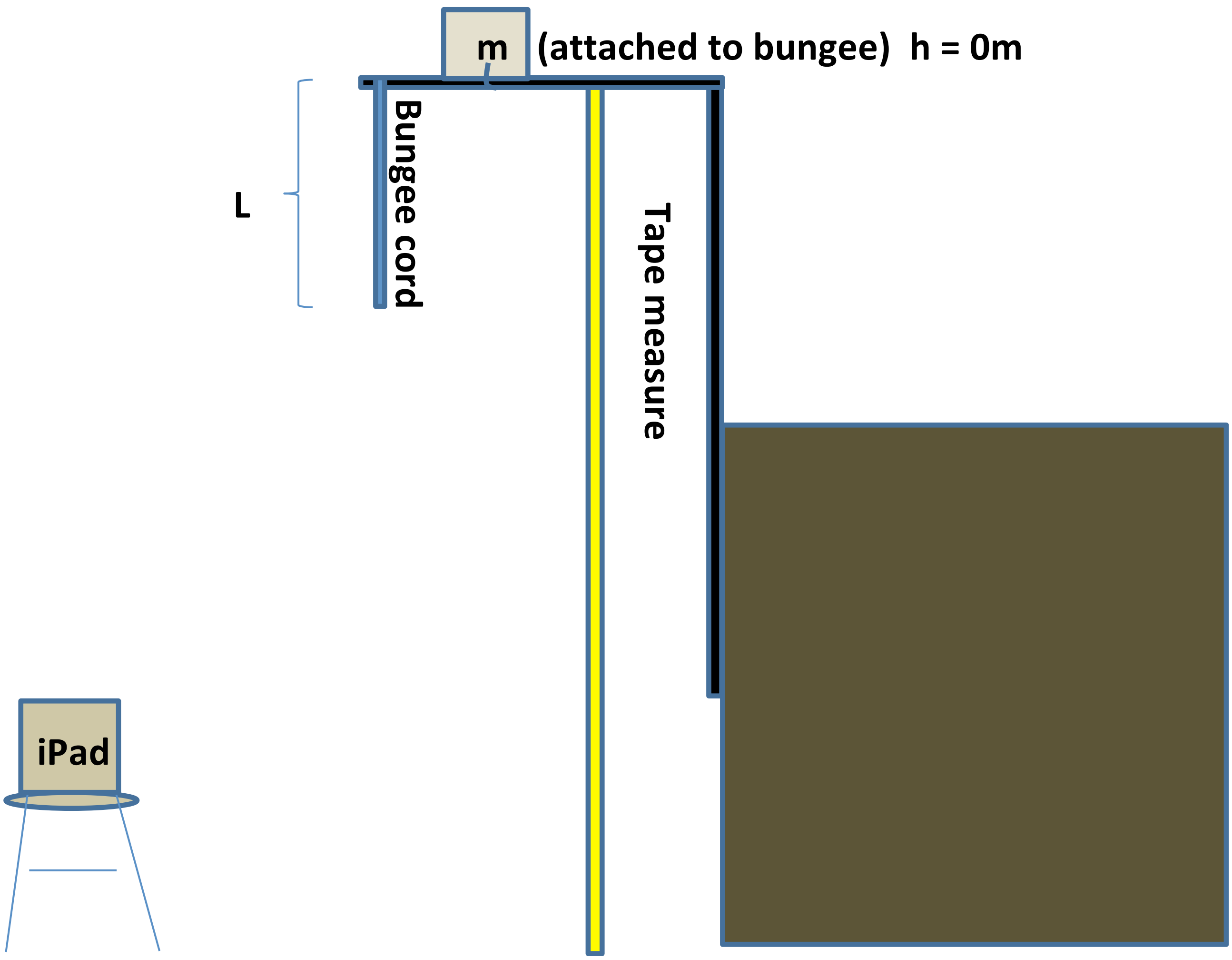

Figure 1: Diagram of the set up and the top of the drop.

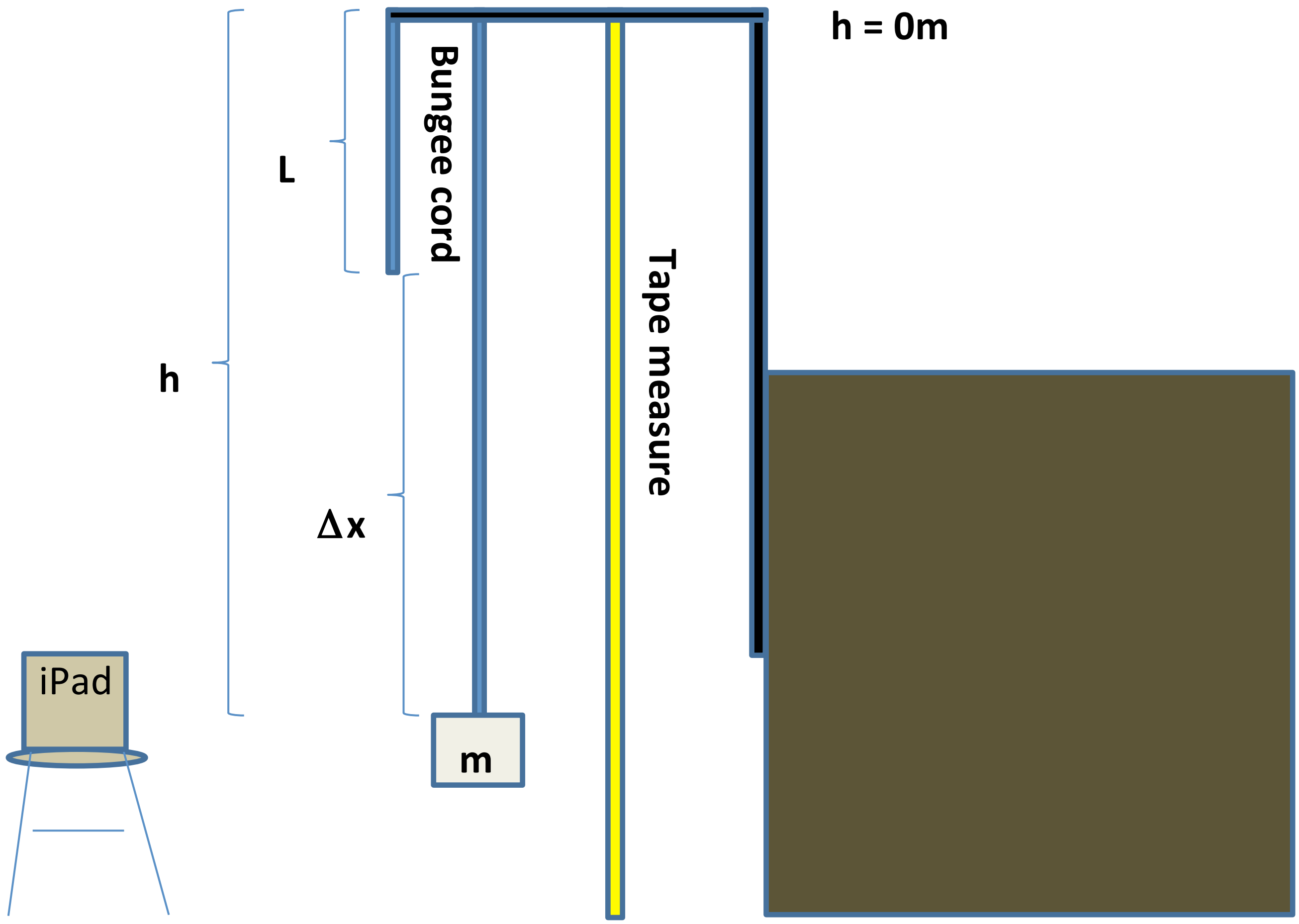

Figure 2: Diagram of the bottom of the drop.

Setup:

1. The bungee cord was tied to the overhang.
2. The tape measure was hung from the overhang and carefully positioned so that it was perfectly vertical.
3. A knot was tied at the end of the bungee cord so that the bungee length could be easily measured and so that the mass could be hung from the bungee.
4. The initial length (L) of the bungee cord was measured.
5. The iPad containing the slow motion camera was positioned on a stool and test runs were performed so that the camera on the iPad appeared parallel with the knot at the bottom of the drop

Procedure:

1. The mass was attached to the bungee by hooking it through the knot made in the bungee.

2. The mass was prepared to be dropped by hovering it at h = 0m and keeping it as still as possible
3. The slow motion camera was prepared and began recording
4. The mass was dropped vertically and stopped after it reached its lowest point.
5. The slow motion camera stopped recording.
6. The lowest point of the knot was measured using the slow motion video and recorded as the height (h). This distance minus the previously recorded and constant initial length of the bungee (L) was the Δx of the trial.
7. Steps 1-6 were repeated for two more trials with the same mass.
8. Steps 1-7 were repeated with 4 different masses.
9. The height of the overhang was adjusted with different masses so that the mass would not hit the floor when dropped
10. Different stools were used to adjust the height of the iPad so the slow motion camera remained parallel with the bottom of the drop
11. The initial potential energy (mgh) was graphed versus the stretch of the bungee (Δx) and a power function was fitted to the data points on the graph. The Equation [4] is the equation of this function
12. The coefficient on x in Equation [4] was known to be a constant times k. Equation [2] was substituted for the k value in the coefficient on x. Solving for the constant resulted in a function of Δx and L.
13. Solving the system of equations of this equation and Equation [3] presented the optimum value for L for any given height (h) and mass (m).

RESULTS:

Five different masses were attached to the bungee and dropped three times each. The initial length (L) was held constant throughout the experiment. The slow motion camera on a iPad was used to calculated the lowest point of the drop, which was recorded as the height (h) of the drop. The average of the three measured heights was calculated. The stretch of the bungee (Δx) was calculated by subtracting the constant initial length (L) from the average height for each mass. The average height for each mass was multiplied by the mass and the acceleration due to gravity to produce the initial potential energy, which was graphed versus the stretch of the bungee.

L = Length (m) +/- 0.002m	m = Mass (kg)	h = Height (m) +/- 0.01m	Average Height (m)	Initial Potential Energy (J) (mgh)	Δx = h-L (m)
0.6	0.05	1.17	1.174666667	0.575586667	0.574666667
		1.174			
		1.18			
0.6	0.055	1.235	1.237666667	0.667102333	0.637666667
		1.24			
		1.238			
0.6	0.06	1.29	1.293666667	0.760676	0.693666667
		1.295			
		1.296			
0.6	0.065	1.346	1.345333333	0.856977333	0.745333333
		1.348			
		1.342			
0.6	0.07	1.398	1.396666667	0.958113333	0.796666667
		1.39			
		1.402			

Table 1: Data from each trial for each mass used

Average height (m)	Statistical Uncertainty
1.174666667	0.005033223
1.237666667	0.002516611
1.293666667	0.00321455
1.345333333	0.00305505
1.396666667	0.006110101

Table 2: Statistical uncertainty (standard deviation) for each average height value

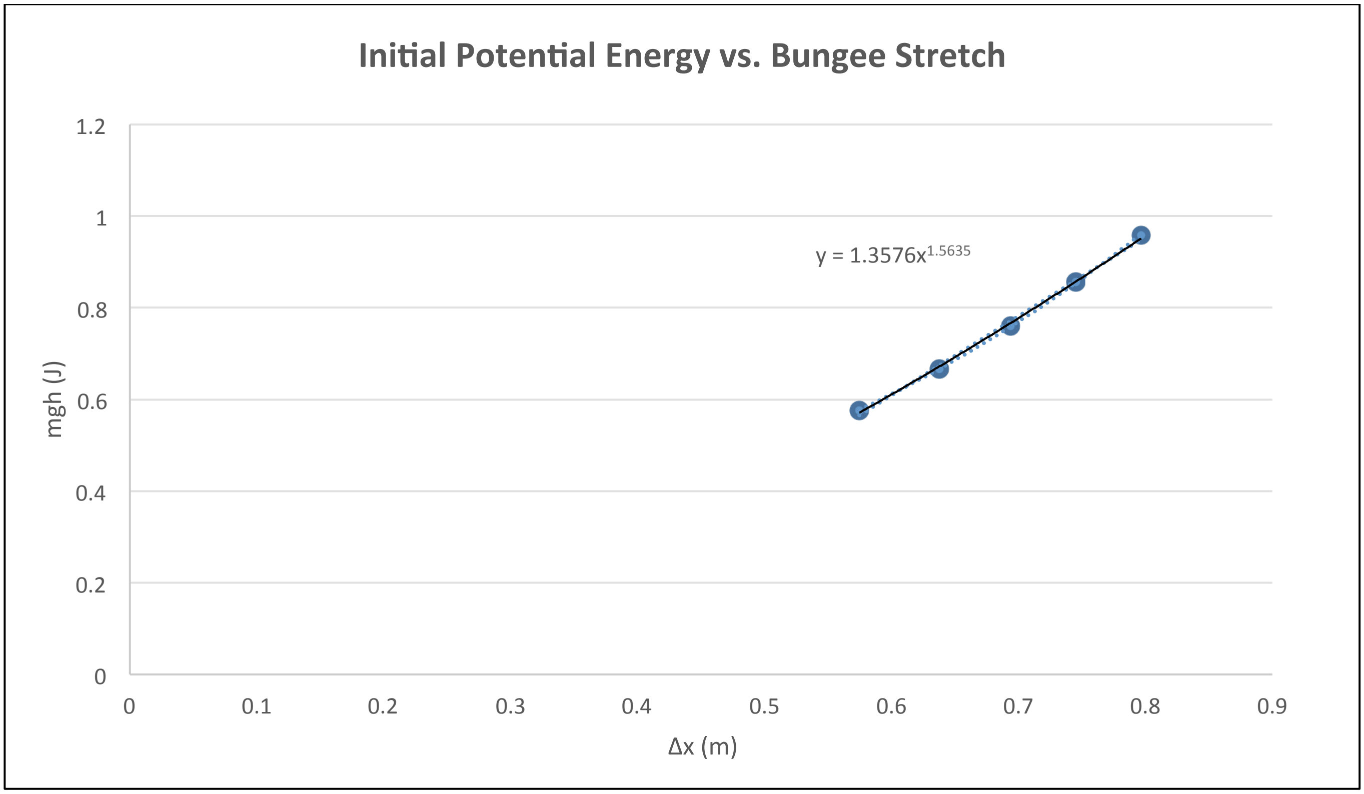

Figure 3: Graph showing the initial potential energy (mgh) versus the stretch of the bungee (Δx) as a power function.

Equation of the power function from Figure 3: $y = 1.3576x^{1.5635}$

Values of interest

In comparing the equation representing the power curve of best fit from the graph in Figure 3 and Equation [1] the following results are obtained:

$y = mgh$

uncertainty in mgh: +/- 0.01m

$x = \Delta x$

$1.3576 = ck$ (c = constant)

- Recall Equation [2]: $k = 1.2908L^{-1.02}$
- Substituting k: $1.3576 = c(1.2908L^{-1.02})$
- Substituting L = 0.6 (held constant throughout experiment): c = 0.6246
- $mgh = 0.6246k\Delta x^{1.5635}$
- Substituting k: $mgh = 0.6246(1.2908L^{-1.02})\Delta x^{1.5635}$
- Solving the system of equations with this equation and Equation [] enables the ideal initial length to be calculated.

Uncertainty

Uncertainty in initial length (L): +/- 0.002m

Uncertainty in height (h): +/- 0.01m

Uncertainty in the stretch of the bungee (Δx): +/- 0.0102m

obtained using quadratic sum technique

DISCUSSION

To test the results of the experiment, a sample bungee jump was performed. A mass of 0.1 kg and a height of 1.955 m were given as they will be for the final bungee jump. The system of equations was solved to yield an ideal initial bungee length of 0.622 m. The bungee cord stretched approximately 1.233 m (data was measured using the same methods as the experiment), or about 0.1 m short of the actual height given. Upon considering the application to the upcoming bungee jump, a stretch of the bungee that is shorter than the given height is the preferred outcome since a stretch of the bungee that is longer than the given height would result in a crushed egg and a failed challenge. The test produced 5.11% error in the calculated equation. This error as well as the uncertainty in the experiment can be attributed to parallax between the iPad camera, the knot in the bungee cord, and the tape measure, unintentional initial velocities given to the mass when dropped, not dropping the mass completely vertically, and the sources of error and uncertainty that range back to the previous bungee lab. In that lab, a procedural flaw was discovered in the bungee cord that carried over into this bungee lab: as the bungee was stretched more and more, the bungee permanently expanded because of the constant stress of the weight force caused by the mass attached to the bungee. Thus, the uncertainty calculated in Equation [2] in the previous bungee lab carried over as uncertainty in this lab. Despite this amount of error and uncertainty, the results of the lab did, however, support the hypothesis that the equation for the bungee would model that of a spring and could be used to predict the optimum initial length of the bungee (L) for a given height and mass.

CONCLUSION

This experiment established the mathematical relationship between the initial length of the bungee cord and the resulting stretch of the bungee. Adding these two values together produces the ideal height or the distance the mass attached to the bungee will fall before the restoring force of the bungee pulls the mass back upwards into oscillatory motion. In terms of the bungee jump challenge in a few weeks when the height of the drop and the mass of the egg are given, the equation will be used to calculate the initial length of the bungee. A certain amount should be subtracted from the height that is substituted into the equation, however, because the height given is the height to the ground, and in this experiment the height was measured at the lowest point of the not. In other words, the height of the mass attached to the bungee was not calculated into the equation, so the height of the egg is another factor to consider when contemplating the height inputted in the equation. Also, because of the 5.11% error in the test run, a certain amount of caution should be taken due to all of the discussed error and uncertainty.