Your name and your lab partner(s): Theodore Bowie Andrew Blair **Section:** 6 **Date:** 11/15/16

TITLE: Relating a Bungee Cord to the CWE Theorem

ABSTRACT:

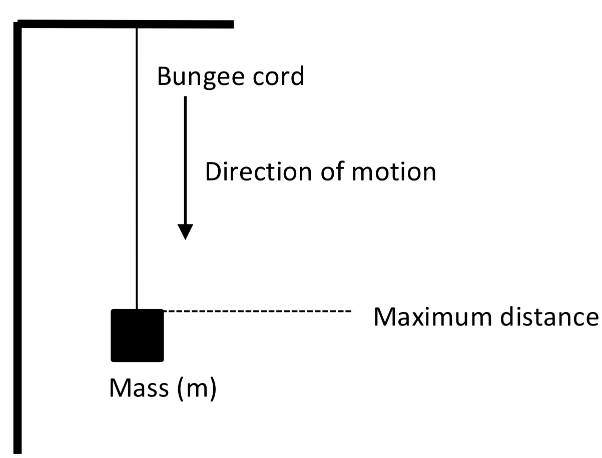
In order to further understand the properties of our bungee, we assessed the relevance of the CWE theorem to a mass being dropped, and attached to the bungee. In a previous lab, we analyzed the relationship between the length of our bungee and the static displacement of the bungee under the force of a known mass. However, to accurately design a bungee jump, we studied the effects of the free fall on the displacement of the bungee. Therefore, we dropped various masses with different bungee lengths to determine how different masses and bungee lengths affect the displacement of the bungee. More specifically, we dropped a known mass from the top of the bungee cord, then using a slow motion camera, we measured the maximum distance the mass travels from where it was dropped. Then, we can find the displacement created from the force of the falling mass by subtracting the static equilibrium from the maximum distance traveled. After repeating this process for various bungee lengths and masses, we can then analyze their relationship. Hopefully, our bungee cord will act similar to a spring, and the CWE theorem will provide an accurate estimation for our bungee. After plotting mass against displacement for our measurements, we observed a parabolic relationship between mass and the displacement. We were able to confirm that the CWE theorem does in fact provide an estimate within our uncertainty for our data, thus confirming that our bungee does indeed act somewhat like a spring. Therefore, when attempting to predict what length bungee we will need for designing a bungee jump, the CWE theorem will apply to provide a helpful estimate.

INTRODUCTION:

For this experiment, we will attempt to find a relationship between the mass of an object and the displacement of a bungee cord from the force of that mass being dropped. This will aid us in designing a safe, yet “exciting” bungee jump for an egg, since we will then be able to estimate the displacement caused by our egg on our bungee cord. From the CWE theorem, note that if our bungee was an ideal Hooke’s Law system, we would have the theoretical relationship where $mgh = .5kx^2$, where m is the mass of jumper, h is the height of the jump, k is the spring constant, and x is the amount of elongation that the elastic undergoes. Clearly, we do not have an ideal spring since we are using a bungee cord, however, from our previous bungee experiment, we confirmed that our bungee cord does act somewhat like a spring. Therefore, we will test the validity of the CWE theorem for our bungee, as if our bungee behaved like a spring.

METHODS: *How are you getting at the purpose or question?*

Essentially, we will drop various masses attached to a bungee cord, and then measure the maximum distance the mass travels from the top of the bungee.



Now, let's more accurately describe the setup and procedure. First, attach a bungee cord to a stable object. Also, attach a tape measure to the top of the stand to measure the appropriate distances. Then, measure and record the length of the bungee cord that is hanging down from the stand. Now securely tie a known mass to the bottom of the bungee. Measure the static equilibrium of this mass hanging from the bungee. After you have the static equilibrium, we can then measure the displacement caused by the mass falling. To do so, drop the mass from the top of the bungee cord. Using a slow motion camera, record the motion of the mass at its maximum distance from the top of the bungee, and measure this distance on the tape measure. It is important to note that you should measure from where the mass is attached to the bungee, and not from the bottom of the mass. Now that you have your maximum distance, you can subtract the equilibrium distance to find the maximum displacement caused by the falling mass. Repeat this procedure for various masses at various lengths.

RESULTS:

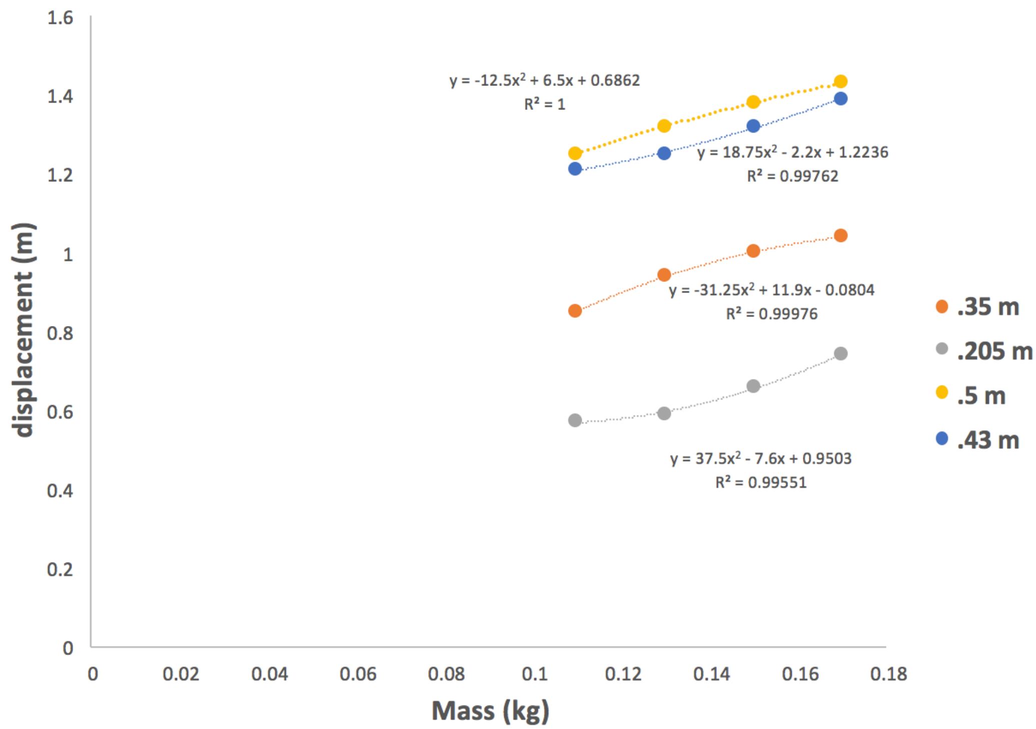
We measured the displacement of 4 masses at 4 different bungee lengths. These masses are .11kg, .13kg, .15kg, and .17kg. The bungees lengths we dropped these masses from are .205m, .35m, .43m, and .5m. First we measured the static equilibrium distance from the top of the bungee for every mass and bungee length. We then measured the maximum distance the mass travels from the top of the bungee. Subtracting those two quantities yields displacement, our main focus. The following tables and graphs contain the appropriate measurements:

unstretched bungee (m)	**0.205**			
mass (kg)	total distance (m)	Equilibrium (m)	Displacement (m)	Displacment square rooted
0.150	0.890	0.230	0.660	0.812
0.170	0.990	0.250	0.740	0.860
0.130	0.770	0.180	0.590	0.768
0.110	0.700	0.130	0.570	0.755

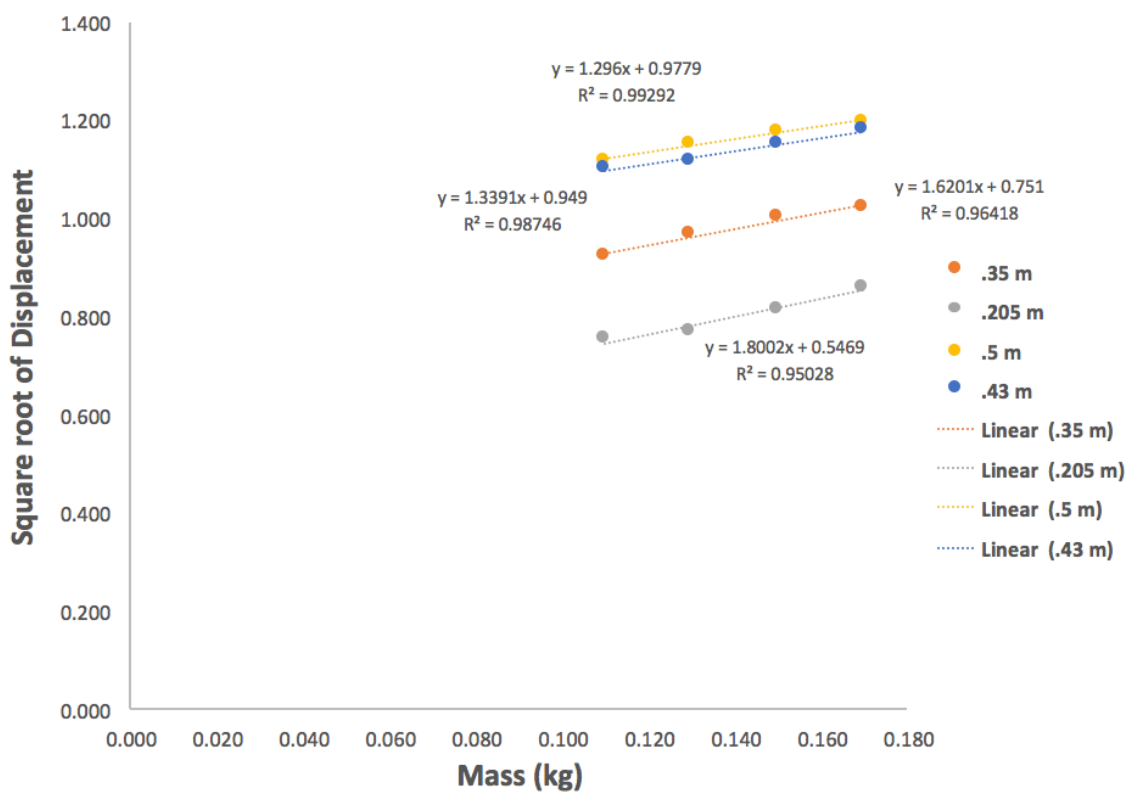
unstretched bungee (m)	**0.35**			
mass (kg)	total distance (m)	Equilibrium (m)	Displacement (m)	Displacment square rooted
0.150	1.400	0.400	1.000	1.000
0.170	1.530	0.490	1.040	1.020
0.130	1.250	0.310	0.940	0.970
0.110	1.090	0.240	0.850	0.922

unstretched bungee (m)	**0.5**			
mass (kg)	total distance (m)	Equilibrium (m)	Displacement (m)	Displacment square rooted
0.150	1.980	0.600	1.380	1.175
0.170	2.170	0.740	1.430	1.196
0.130	1.780	0.460	1.320	1.149
0.110	1.590	0.340	1.250	1.118

unstretched bungee (m)	**0.43**			
mass (kg)	total distance (m)	Equilibrium (m)	Displacement (m)	Displacment square rooted
0.150	1.860	0.540	1.320	1.149
0.170	2.050	0.660	1.390	1.179
0.130	1.690	0.440	1.250	1.118
0.110	1.550	0.340	1.210	1.100



Clearly, we can see above that the equations are parabolic, and fit our data quite nicely. However, we still need to linearize the graph to perform the proper regression analysis. Below, we can see our new linearized data.



So, we have the following linear equations, and let *d* be the displacement with the following uncertainties from the regression analysis:

- at .205 meters: $d^{1/2} = (1.8 \pm 0.3)m + (0.5 \pm 0.04)$
  - % uncertainty of slope: 16.2%   % uncertainty of y-intercept: 7.5%
- at .35 meters: $d^{1/2} = (1.6 \pm .2)m + (0.75 \pm .03)$
  - % uncertainty of slope: 13.6%   % uncertainty of y-intercept: 4.2%
- at .43 meters: $d^{1/2} = (1.33 \pm .1)m + (0.99 \pm 0.02)$
  - % uncertainty of slope: 8%   % uncertainty of y-intercept: 1.6%
- at .5 meters: $d^{1/2} = (1.29 \pm 0.08)m + (.99 \pm 0.01)$
  - % uncertainty of slope: 6%   % uncertainty of y-intercept: 1.1%

Interestingly, our uncertainties get smaller as the length of the bungee increases, implying that as the length of bungee increases, there is less error. From the previous lab, we found an equation which yields a "spring" constant value of $k = 1.2x^{-.954}$. Therefore, we can solve the CWE theorem equation from earlier to get

$$d^{1/2} = (2mgh/k)^{1/4}.$$

We can then calculate estimated *k* values for each length and get the expected values at each bungee length:

unstretched bungee (m)	**0.43**
CWE Expected Displacement	Percent Error in Displacement
1.429171036	8%
1.597288981	15%
1.268227403	1%
1.117234392	8%
Average:	8%

unstretched bungee (m)	**0.35**
CWE Expected Displacement	Percent Error in Displacement
1.122497216	12%
1.249239769	20%
0.987420883	5%
0.84817451	0%
Average:	9%

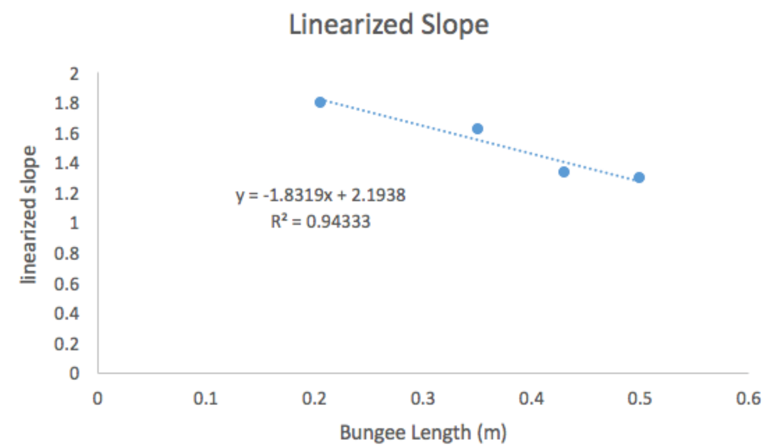
unstretched bungee (m)	**0.205**
CWE Expected Displacement	Percent Error in Displacemen
0.693890089	5%
0.779098036	5%
0.600851418	2%
0.526981555	8%
Average:	5%

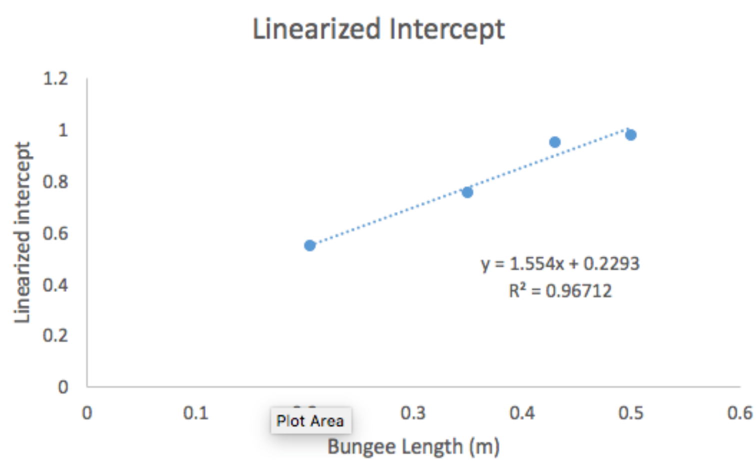
unstretched bungee (m)	**0.5**
CWE Expected Displacement	Percent Error in Displacemen
1.584833832	15%
1.76628116	24%
1.398901663	6%
1.216187159	3%
Average:	12%

Note that for our bungee length of 0.5 meters, the average of the percent errors is greater than the percent uncertainty. Also, notice that the mass of .17kg provided the largest percent error for each bungee length. However, at lengths of .43m, .35m, and .205m, the percent error in displacement is less than the percent error in the uncertainty.

While this does confirm that the CWE theorem seems somewhat appropriate for our bungee cord, we can also analyze our different linearized equations at each bungee length to explore relationships as the bungee cord length changes. This would give us another method of developing a prediction for other bungee lengths. So, we have the following table and graphs:

Length of Cord (m)	Linearized slope	Intercept
0.205	1.8	0.5469
0.35	1.62	0.751
0.43	1.339	0.949
0.5	1.296	0.9779





Interestingly, there is a clear relationship in the coefficients of our linearized equations. Since both of these equations are linear, we can proceed with a regression analysis to determine the validity of these relationships. However, for the linearized slope equation, we get a percent uncertainty of slope of 17% and percent uncertainty of the intercept of 5.6%. For the linearized slope equation, we get a percent uncertainty of slope of 13% and percent uncertainty of the intercept of 34%. While there does appear to be some sort of weak relationship between these values, we cannot conclude that there is a strong linear relationship given our data.

After calculating our data, we were able to compare our observed results with expected results using the CWE theorem. If our bungee does act like a spring, the equation $d^{1/2} = (2mgh/k)^{1/4}$ should be somewhat comparable to our results. After performing a regression analysis for each bungee lengths, and comparing expected to observed values, we were able to confirm that the percent error between observed and expected values was within the percent uncertainty for all bungee lengths except 0.5m.

DISCUSSION:

From our results, we see that the observed uncertainties for our bungee lengths are 16%, 13%, 8%, and 6% for bungee lengths of .205m, .35m, .43m, and .5m, respectively. We also calculated our expected values from the CWE theorem, and found the average percent errors of 5%, 9%, 8%, and 12%, for bungee lengths of .205m, .35m, .43m, and .5m, respectively. Surprisingly, the bungee length of .5m was the only bungee length that did not have a percent error within that of the uncertainty. However, the CWE theorem was not inaccurate for the smaller masses at .5m, and the percent error in the two larger masses skewed the average to be much larger. Also, the larger masses generally provided the largest percent errors for all bungee lengths, implying that as the masses get too heavy, our bungee fails to act like a spring.

For this lab, there were many sources of uncertainty. First, we had to tape the masses onto the hook, and the mass of all the tape was not accounted for. Also, we used a camera to estimate the measurements by looking at them from a distance, so there is likely some error in our measurements. Also, since we were just holding the camera by hand, there is likely some error due the angle of the camera and how the location of the mass was perceived. We also saw that larger masses provided larger percent error, so this implies that the force due to larger masses may have stretched our bungee too far, since the observed values of larger masses were always less than our expected values for the larger masses. Lastly, the masses on the lower end of the .11kg to .17kg generally had lower percent errors from expected to observed values, so the CWE theorem may be more accurate for smaller masses on our bungee.

Our results don't fully confirm that our bungee behaves like a spring in a CWE scenario for all masses and lengths, however, it does seem to appear to provide somewhat accurate estimates for the smaller masses that we tested. Therefore, to make a more solid confirmation of the CWE theorem and our bungee, it would require further testing with more appropriate masses.

CONCLUSION:

The goal of our experiment was to test the validity of the CWE theorem in application to our bungee cord. In theory, our bungee cord acts as a restoring force, so it should behave somewhat similar to spring. If, so we would be able to estimate the amount of elongation our bungee undergoes due to a falling mass. After statistical analysis of our experiment, it appears that the CWE theorem for a spring does appear to have some relation to the observed values of our experiment. We cannot confirm that this applies for every bungee length and any mass, however it does appear to be more accurate for smaller masses. The larger masses provided larger expected values than observed values, causing reason to believe that the larger masses may have stretched our bungee too far, to the point where the bungee wasn't able to continue stretching at that rate. Therefore, to develop a more solid conclusion, it would require further testing of more masses on lower end of the .11kg - .17kg range. Hence, our experiment hints that the CWE theorem may apply to our bungee, but we cannot fully confirm that at this point.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: **Theodore Bowie**