The Bungee Challenge

Abstract

The overall purpose of this experiment is to determine whether a bungee cord's behavior is linear enough to use Hooke's law (F_{spring}= kx) as a model to characterize the cord's movement. In order to properly characterize the behavior of a spring, five different masses were all hung at five different cord lengths. The changes in length of the cord for each mass were then recorded and plotted against force in order to quantify values for the spring constant *k* at different initial bungee lengths.This information illustrated that the cord's behavior is linear enough to be characterized using Hooke's Law. In addition, the values of k (indicated by the slope of each graph) were then plotted against the initial lengths. This indicated that as the length of the cord increased, the value of k decreased. This information will likely be of significant value in later experiments where specific characteristics of a bungee cord are necessary in order to create an ideal drop for a fragile egg.

Introduction

This report models the spring-like behavior of a bungee cord in order to determine the relationship between k (spring constant) and the length of the bungee (L). In order to quantify this relationship, Hooke's Law (F_{spring}= kx) was the primary focus of the lab set-up, where x is the extension of the cord after adding different masses. By calculating the force exerted on the bungee for different masses, the goal is to derive an equation (or equations) that will allow for an accurate prediction of the length that the bungee will stretch at any given height for a mass of 100-170 g (approximately the mass of an egg). The expectation is that k changes as a function of L, and so the primary purpose of this portion of the lab is to characterize the spring constant, k. The parameters of the bungee chosen based on this experimentation will eventually lead to the successful "jump" of an egg from the Great Hall. To get the best "jump" possible, the optimal level of deceleration must be reached, and the egg will come as close to the ground as possible without damage.

Methods

This report presents different characterizations of the spring constant (k) based on the Force experienced by the bungee (F = mg), and the length stretched by the bungee for every given mass. A range of masses from 100-170 kg were hung on a bungee cord at 5 different initial lengths (See Figures 2-6), and the force exerted by each mass was calculated using the equation F = mg, where g is the acceleration due to gravity. The force and displacement of the bungee cord were then graphed, and the slope of the line characterized k at that particular length. Then, the initial length of the cord was plotted against that spring constant, k, in order to depict the relationship between the two values.

Experiment 1: Characterizing spring constant k

STEP 1: Tied bungee securely to hanging apparatus

STEP 2: Tied a loop in the bungee at length 1 (repeated for lengths 2, 3,4, and 5), and then measured the length

STEP 3: Placed 5 different hanging masses on bungee, ranging between 100 and 170 kg

STEP 4: Recorded the change in length of the bungee (Δx in Figure 1)

STEP 5: Calculated the force due to gravity (mg) that the mass exerted on the bungee

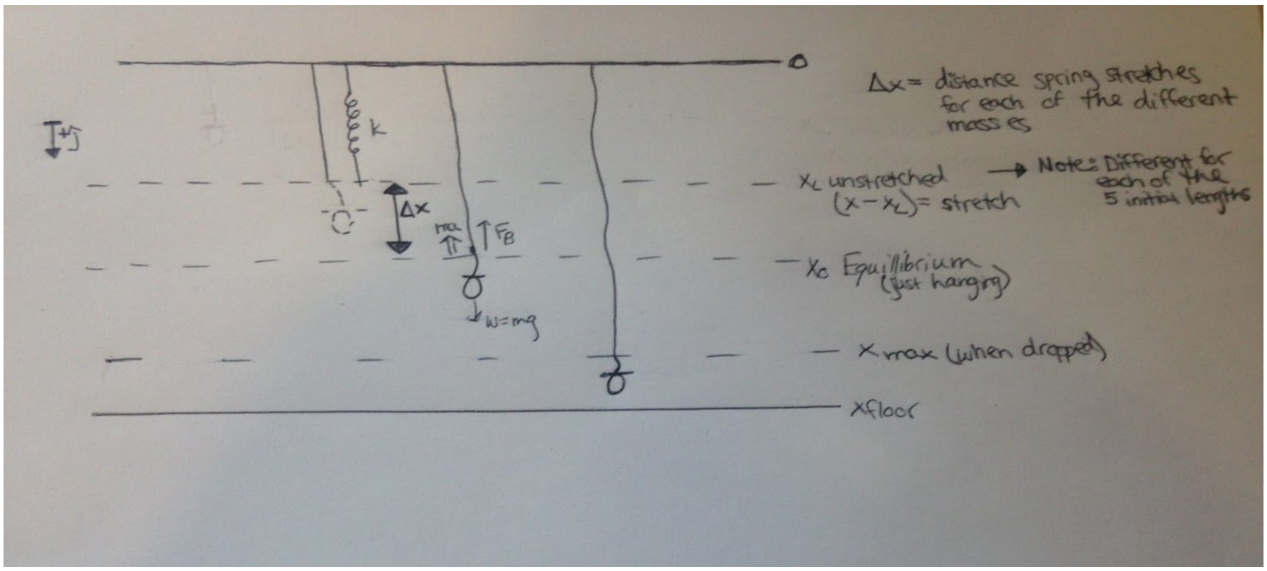


Figure 1: Diagram of experimental set-up, where Δx is the total displacement of the bungee after the hanging mass is added, and the force (F) on the bungee is equivalent to the mass times acceleration due to gravity (9.81 m/s^2).

Results:
Masses between 60 grams and 170 grams were attached to an elastic string mounted. The weight was tied to the string at varying heights. Displacement of the string was measured and the displacement was compared to the force exerted on the elastic string (varying mass* gravitational force). This relationship is shown below:

Tables 1-5: Displacement ($x_i – x_f$ = Δx) versus force at five initial heights (x_L unstretched) for the plotting of F and x to derive the spring constant (k, the slope of the graph).

The Bungee Challenge

Table 1:

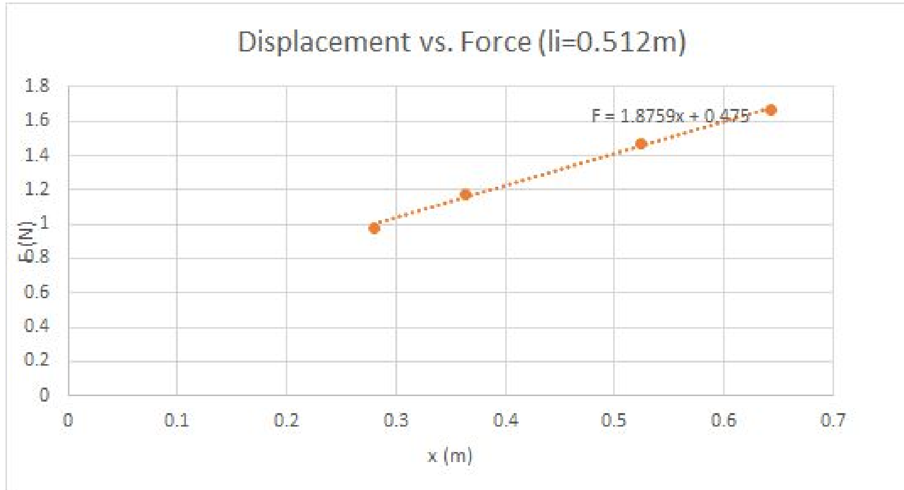
x_L unstretched =.512 m

Δx (m) (± 0.01m)	F (N) (± 0.01 kg)
0.28	0.98
0.36	1.18
0.52	1.47
0.64	1.67

Table 2:

x_L unstretched = 0.135 m

Δx (m) (± 0.01m)	F (N) (± 0.01 kg)
0.10	0.98
0.13	1.18
0.17	1.47
0.20	1.67



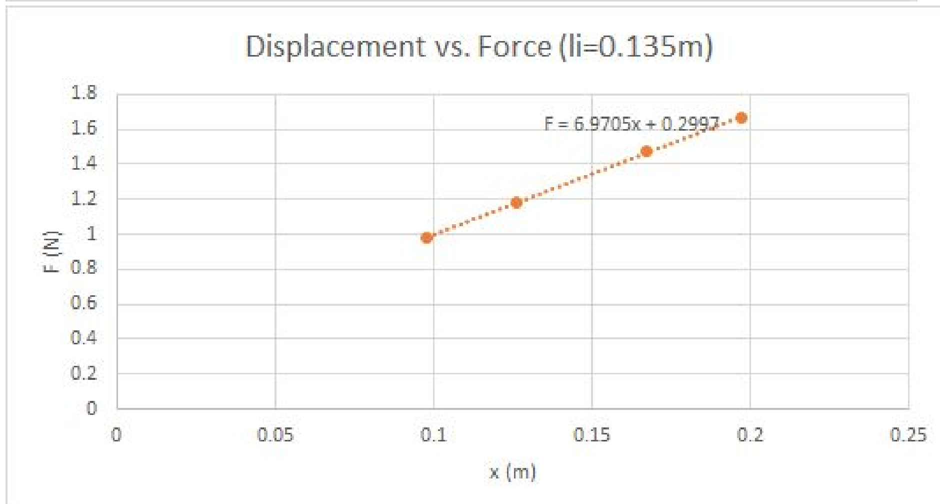


Table 3:

x_L unstretched = 0. 762 m

Δx (m) (± 0.01 m)	F (N) (± 0.01 kg)
0.45	0.98
0.59	1.18
0.83	1.47
0.93	1.57

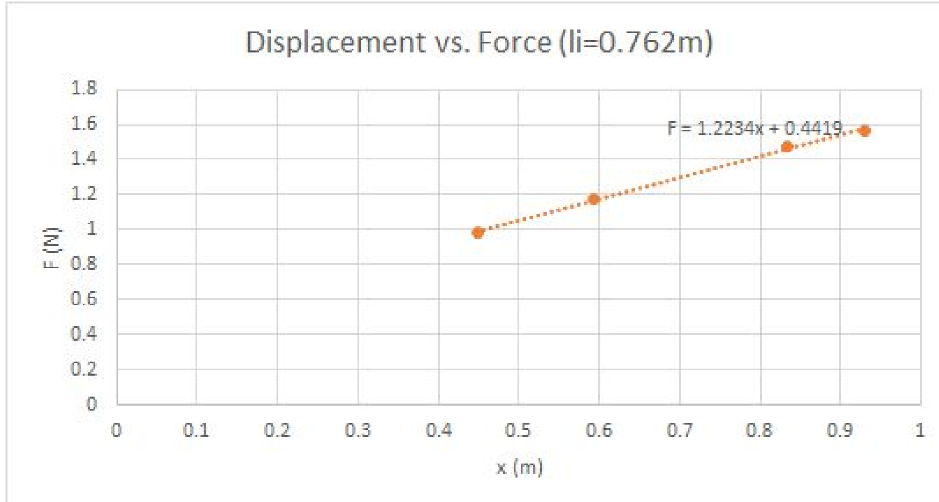


Table 4:

x_L unstretched = 1.067 m

Δx (m) (± 0.01 m)	F (N) (± 0.01 kg)
0.35	0.69
0.43	0.78
0.46	0.83
0.58	0.98

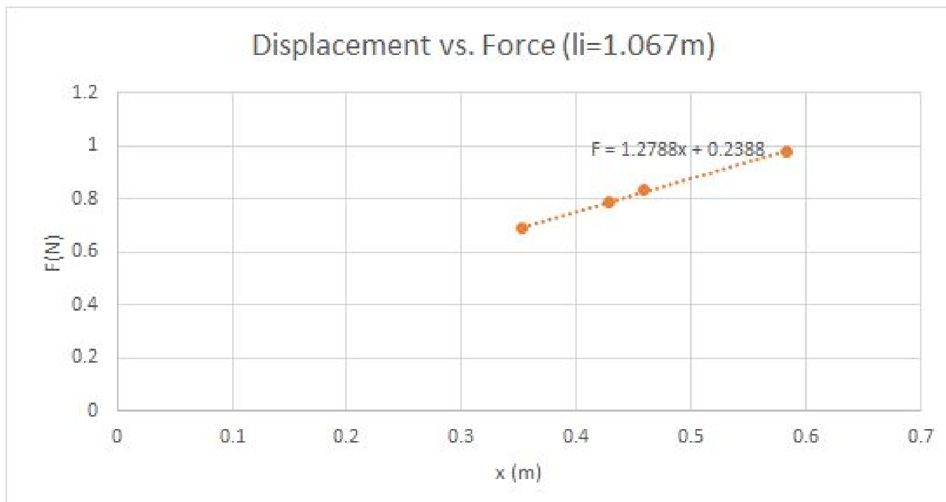
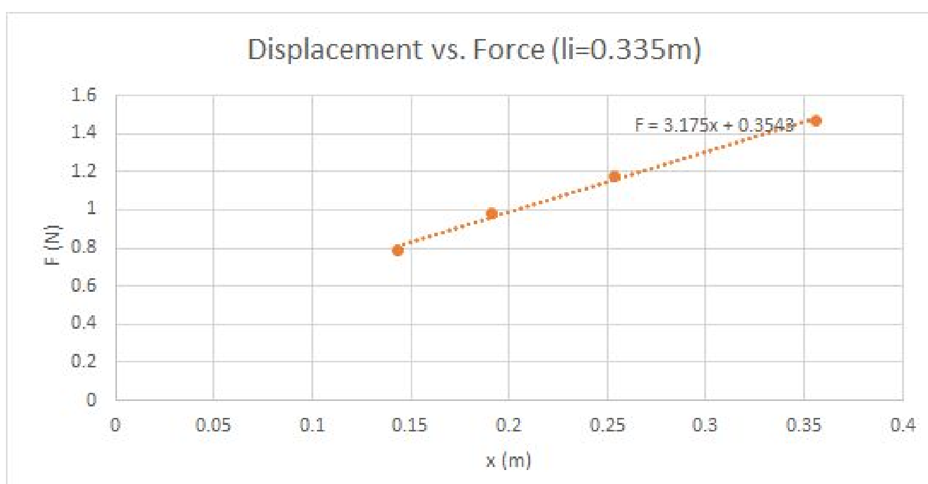


Table 5:

x_L unstretched= 0.335 m

Δx (m) (± 0.01 m)	F (N) (± 0.01 kg)
0.14	0.78
0.19	0.98
0.25	1.18
0.36	1.47



Figures 2-6: Displacement versus force of hanging masses; the slopes of each individual graph represents the spring constant (k) of the bungee

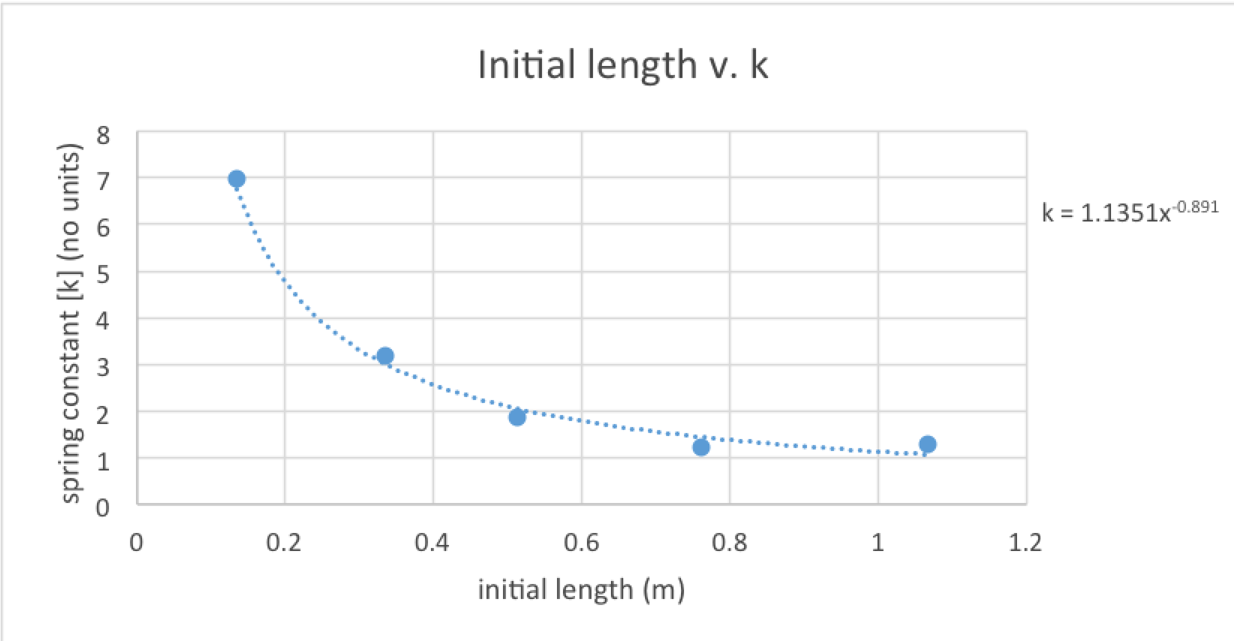


Figure 7: Initial length versus k; plotting of initial length and the spring constant (k) in order to show a relationship between the two variables.

Tables 6-10: Y-intercepts and slopes of the linearized graphs (Figures 2-6), in addition to the standard error for each of the values obtained through regression analysis.

x_L unstretched =.512 m

	Coefficients	*Standard Error*
Y Intercept	0.48	0.04
X Variable	2	0.1

x_L unstretched = 0.135 m

	Coefficients	*Standard Error*
Y Intercept	0.30	0.01
X Variable	7.0	0.1

x_L unstretched = 0. 762 m

	Coefficients	*Standard Error*
Y Intercept	0.4	0.03
X Variable	1.2	0.04

x_L unstretched = 1.067 m

	Coefficients	*Standard Error*
Y Intercept	0.24	0.02
X Variable	1.3	0.04

x_L unstretched= 0.335 m

	Coefficients	*Standard Error*
Y Intercept	0.35	0.04
X Variable	3	0.2

Discussion:

Overall, the experiment succeeded in deriving an equation that models k as a function of bungee cord length. The five different figures that plot displacement versus force (Figures 2-6) clearly illustrate a linear relationship for the cord's functional relationship between force and displacement. This is useful in that it will allow for the characterization of the spring using Hooke's law. In addition, the graph of initial length versus the spring constant (k), clearly indicates a relationship between the length of the bungee and the spring constant, although not a linear one. The trend line of this graph indicates an inversely related relationship between k and initial length-- as initial length increases, k decreases. This is valuable for the purposes of this experiment because the overall goal was to characterize k as a function of bungee cord length so that we can move on and start isolating other variables that could have an impact on the successful drop of an egg.

It is important to note that there are several sources of uncertainty that could have an impact on the accuracy of our bungee jump. For example, each loop that was tied in the bungee to hold the hanging mass had a short piece of bungee still hanging off at the end. This could have potentially affected measurements of the bungee's change in length, ultimately resulting in a slightly inaccurate k value. In addition, the knot securing the bungee to the top of the hanging apparatus was not particularly well fashioned, and is likely to have slipped a bit when the masses were added to the bungee. In order to minimize these uncertainties, the knot at the top of the cord could have been more securely tied, and the extra piece of cord hanging off the mass could have been reduced by more precisely looping the bottom of the cord.

Conclusion

The formula $k=1.1351x^{-0.891}$, derived through Hooke's law, is the model for the behavior of our specific elastic band. In the upcoming weeks, this model will be implemented in the design of an elastic cord system that will be expected to get a raw egg as close to the ground as possible without any damage. Knowing that elastic materials like rubber cords have the tendency to recover when they are released with weight on them, we plan on adapting our model for k to prepare for the egg drop.