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Experimentally modeling the behavior of a Bungee cord using Hooke's Law

ABSTRACT:

The main goal of this laboratory was to design and carry out an experiment that would characterize the behavior of a given bungee cord related to force and displacement. Hooke's law $F = -kx$ was used to determine the elasticity (spring constant *k*) at several lengths of bungee cord used by measuring the cord's displacement at rest when weight was added. This was done multiple times with different lengths of cord, and the *k* values plotted against the lengths to determine the model of k with respect to the length of the bungee. This model was linearized for convenience of calculation of both our *k* values and a theoretical Δx, from a rewritten Hooke's Law $\Delta x = \frac{-F}{k}$ Using a known *F* and calculated *k* from our equation, we can determine the displacement of the cord use in future experiments.

INTRODUCTION:

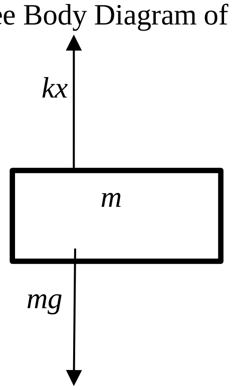
The purpose of this lab was to quantify the behavior of our given bungee cord. Elastics like the bungee are often modeled using Hooke's Law $F = -kx$, where their elasticity is given by the spring constant. However, we could not assume that this *k* value stays constant as the length of the bungee varied. Therefore, an experiment was designed to determine the relationship between this length *l* and spring constant *k*. We predict that shorter lengths of bungee cord will give us higher *k* values.

METHODS:

The bungee was attached to the top of an L- shaped bar, which itself was attached to a desk. The bungee cord was attached to the top by a knot looped around a bolt attached to the top of the bar, and hanging masses were added to the cord via another knot at the bottom of the bungee. The length of the bungee was measured with a ruler from the beginning of the top knot to the bottom of the bottom knot. Masses were changed by adding standard mass weights onto a hanging platform attached to the bungee.

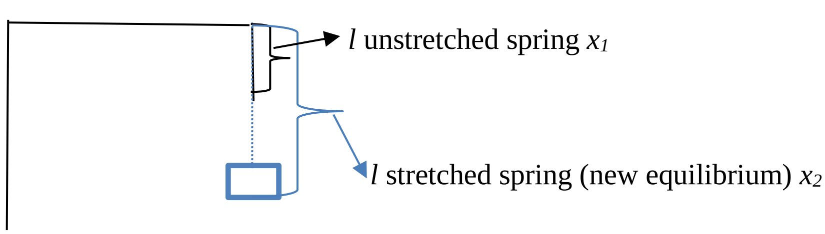
At rest, a vertical spring (bungee) system has a force *F* equal to $mg - k\Delta x$, where *m* is mass, *g* is gravitational force, *k* is spring constant, and Δx is the displacement from the length of the unstretched bungee to its length with a mass attached to it. At rest, $F = ma = 0$, so $mg = k\Delta x$. Rearranging, $k = \frac{mg}{\Delta x}$. Plotting change in weight versus change in Δx would give us the *k* in the form of the slope of the line. This was repeated multiple times, changing the length of the bungee cord, which changed our *k* values. The change in *k* values was plotted versus the change in length. The slope of this line would give us our relationship between *k* and *l* for the bungee.

Figure 1: Free Body Diagram of our Vertical Bungee system:



Where $mg = k\Delta x$ and $k = \frac{mg}{\Delta x}$

Figure 2: Diagram of vertical spring equilibria:



Where $\Delta x = x_2 - x_1$

RESULTS:

The data collected was the length (*l*) of the unstretched bungee (m), the weight of the hanging mass, obtained by multiplying its mass by local gravity (9.81 m/s^2), in Newtons, length of string at equilibrium, and the Δx, obtained by subtracting length of unstretched bungee from length at equilibrium, for each mass. Five lengths were used to determine different *k* values, those lengths being 0.746 m, 0.530 m, .374 m, 0.242 m, and 0.155m.

Table 1: Raw Data collected for bungee of length 0.746 m

Length (*l*) of unstretched Cord (m) (±0.01m)	Masses (kg)	Weight (N)	Length of string at equilibrium (m)	Δx (m)
0.746	0.05	0.491	0.897	0.151
0.746	0.07	0.687	0.987	0.241
0.746	0.1	0.981	1.15	0.402
0.746	0.12	1.18	1.28	0.534
0.746	0.15	1.48	1.50	0.749

Graph 1: Weight versus Δx for bungee of length 0.746 m. The slope, *k*, equals 1.629 (N/m).

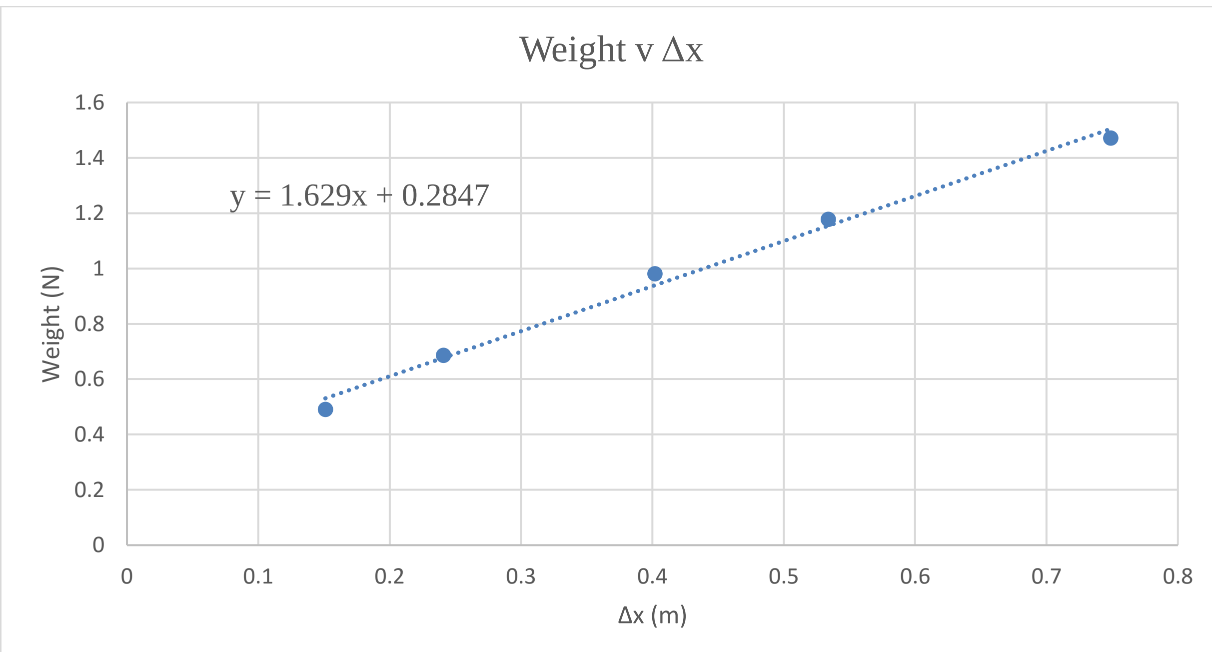
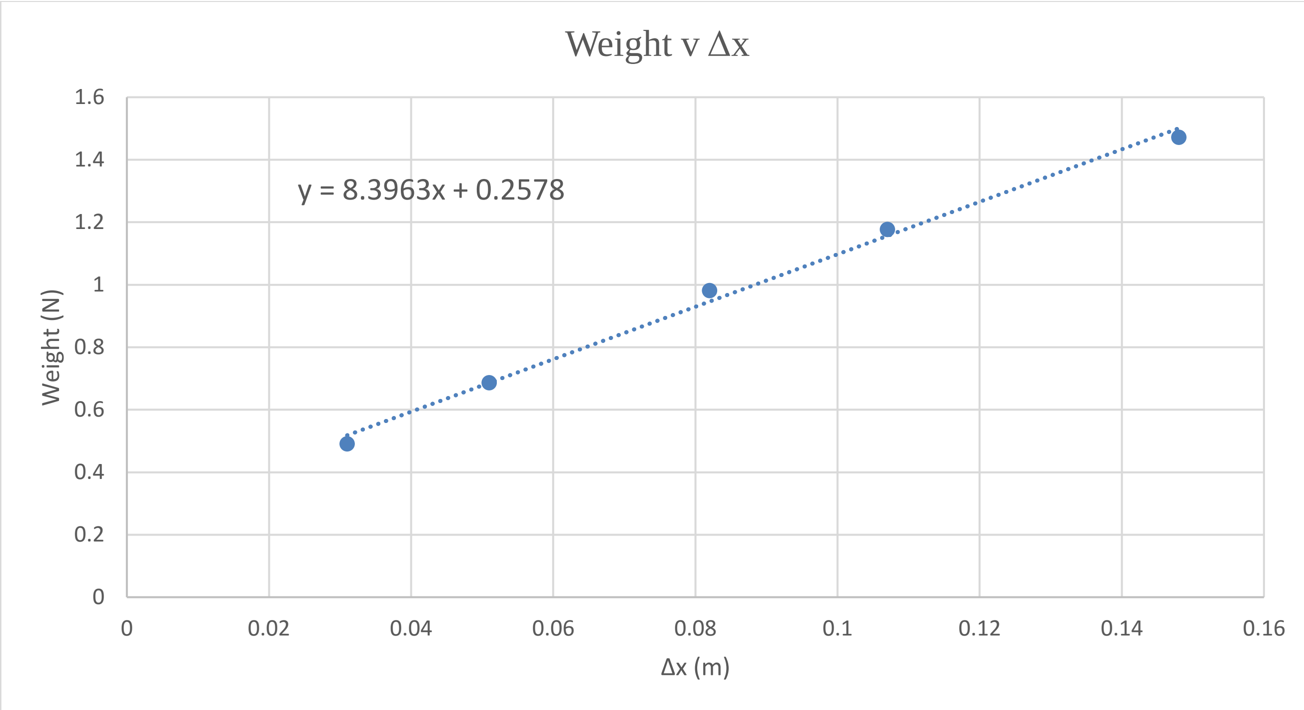


Table 2: Raw data collected for bungee of length 0.155 m.

Length (*l*) of unstretched Cord (m) (±0.01m)	Masses (kg)	Weight (N)	Length of string at equilibrium (m)	Δx (m)
0.155	0.05	0.491	0.186	0.031
0.155	0.07	0.687	0.206	0.051
0.155	0.1	0.981	0.237	0.082
0.155	0.12	1.18	0.262	0.107
0.155	0.15	1.48	0.303	0.148

Graph 2: Weight vs Δx for bungee of length 0.155 m. The slope, *k*, equals 8.3963 (N/m).



Equations of the Curve-Fit:

Graph 1: Weight = 1.629(*k*) + 0.2847
Graph 2: Weight = 8.3963(*k*) + 0.2578

These are two out of the five tables and graphs for the measurements done with varying lengths of bungee cord. Observe that the equation of the best fit for each curve is linear, and per the relationship $k = \frac{mg}{\Delta x}$, since $mg = k\Delta x$ when the system is at rest (Figures 1 and 2), the slope from this equation gives us our *k* spring constant values. The uncertainties in our calculated *k* values for our five lengths of bungee cord come from the R^2 values of the equation of the curve-fit, which represents how linear the relationship between our weight and Δx is, since our curve-fit was linear. The percent uncertainties were determined by multiplying these R^2 values by 100 and subtracting them from 100%.

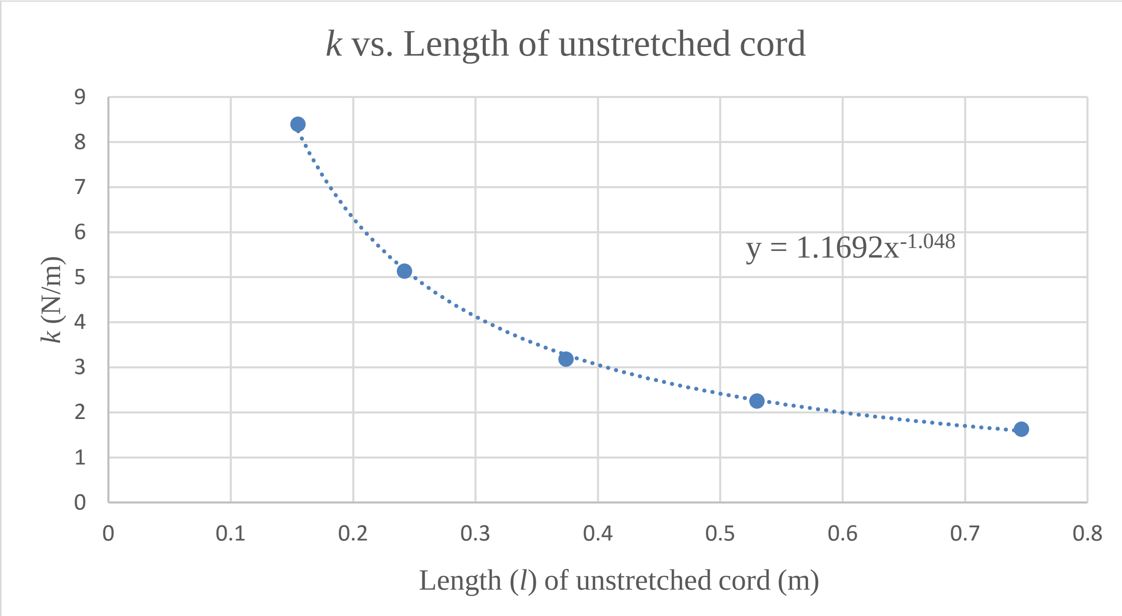
However, the goal was to determine the relationship between the length of the bungee cord and its spring constant, so we had to plot *k* versus the length of the bungee cord, and the equation of the curve-fit for this graph would give us our relationship between *k* and length *l* of the bungee cord.

Table 3: Calculated *k* values for lengths *l*, and their uncertainties

k (N/m)	Length (*l*) of unstretched Cord (m) (±0.01m)	Uncertainty (R^2)	Percent Uncertainty
1.629	0.746	0.9917	.83%
2.2523	0.530	0.9919	.81%
3.1863	0.374	0.9923	.77%
5.1404	0.242	0.9899	1%
8.3963	0.155	0.9946	.54%

From this table, we can plot our calculated *k* values versus the length of the unstretched cord, and use its curve-fit equation as the model for the relationship between length of the bungee and its spring constant.

Graph 3: *k* value versus Length of unstretched bungee cord



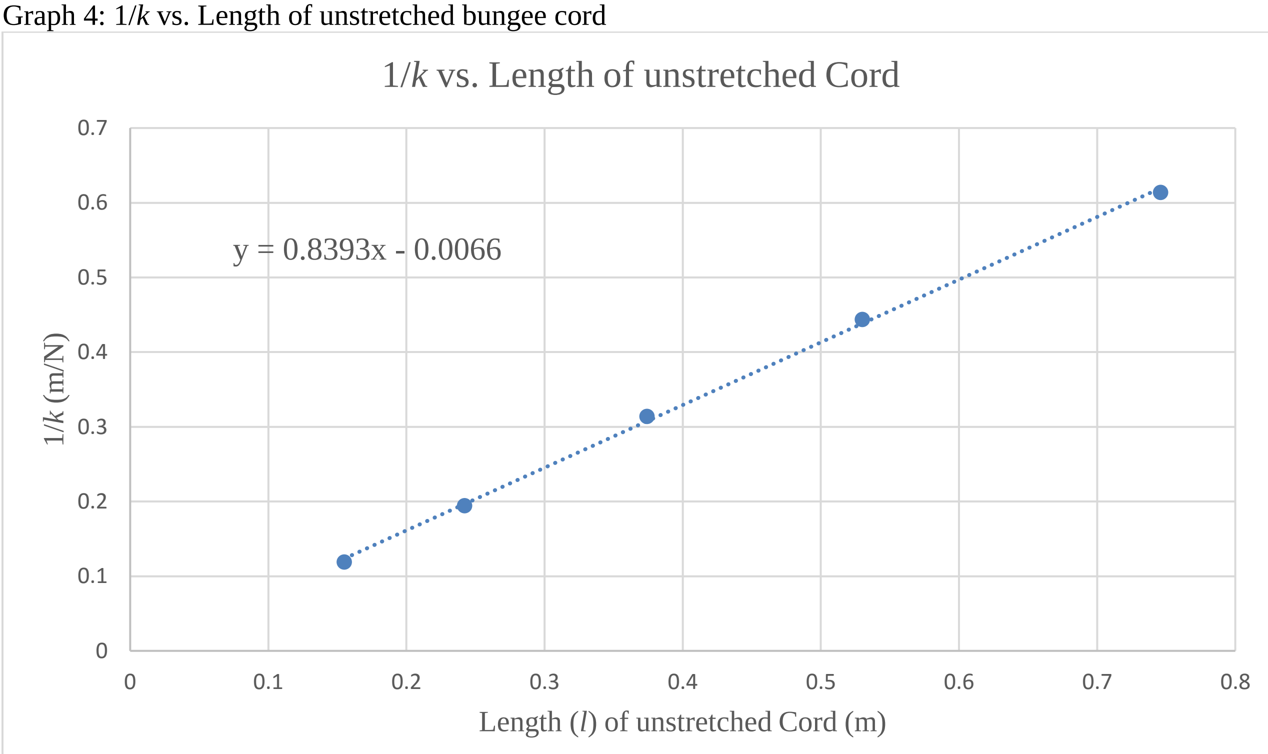
Here, our equation of curve-fit is:

$k = 1.1692(length)^{-1.048}$, which gives us our relationship between length of bungee and its spring constant. However, due to this equation's exponential nature, this graph was linearized, and a new equation of curve-fit was found for convenience in calculation of the appropriate spring constant in further experiments.

Table 4: Linearized *k* values, obtained by dividing 1 by *k*, and the appropriate length of unstretched cord.

1/*k* (m/N)	Length (*l*) of unstretched Cord (m) (±0.01m)
0.614	0.746
0.444	0.530
0.314	0.374
0.195	0.242
0.119	0.155

Graph 4: 1/*k* vs. Length of unstretched bungee cord



From our linearized graph, our now linear equation is: $\frac{1}{k}$ = 0.8393*(length)* – 0.0066 which relates our k value to our length in a linear equation, which is convenient for calculation.

The uncertainties for this model of k and length are given by a regression data analysis on the values in Table 4. The percent uncertainty is calculated as $100 * \frac{Standard\ Error\ of\ 1/k}{Coefficient\ of\ 1/k}$, where both Standard Error and Coefficient(s) are given in the regression data analysis.

Table 5: Coefficients, Standard Error, and Percent Uncertainty from Excel Regression Test on Linearized values.

	Coefficients	*Standard Error*	*Percent Uncertainty*
Intercept	-0.00656	0.006461	-
1/ k	0.839349	0.014033	1.7%

In summary, we measured the length of the unstretched bungee cord, added a mass to it (whose weight was recorded), measured the new length of the bungee cord at equilibrium, found the change in position (Δx) and plotted the weight versus the Δx. From Figures 1 and 2, we can use the relationships $mg = k\Delta x$, and $k = \frac{mg}{\Delta x}$ to find k, which would be the slope of the graph of weight versus Δx. Then, we perform this measurement five times, using different lengths of bungee cord. Each length has a corresponding k value, so we plotted the k values against the length of the cord, to determine how the spring constant changed as different lengths of bungee cord were used. The equation from the curve fit gave us this relationship, but the graph was linearized for ease of interpretation and calculation.

DISCUSSION:

Since the purpose of this experiment was to quantify a relationship between the spring constant and the length of the bungee cord, the equations $k = 1.1692(length)^{-1.048}$ and $\frac{1}{k}$ = 0.8393*(length)* – 0.0066 were what we were looking for. This experiment produced purely experimental values, and as such, we only have a percent uncertainty related to the linearized form of the graph (1/k). A percent uncertainty of 1.7% characterizes the uncertainty of the linear relationship between 1/k and the length. Our experiment was based off Hooke's Law, $F = -kx$, which is inherently a model of a linear spring, to characterize the elasticity of our bungee cord. Therefore, this relatively low linear percent uncertainty can be determined as deeming our linearized equation "acceptable" as a model of our bungee cord's behavior.

To test this, we must rearrange the equation $F = -kx$, or $F = -k\Delta x$, where Δx is the displacement of the cord, but this time, the displacement between the cord's equilibrium and maximum stretch distances. Rearranging this, we can determine what the theoretical Δx should be, as $\Delta x = \frac{-F}{k}$, and perform an experiment where a mass on the cord is dropped from rest, and its equilibrium distance and maximum stretch distances measured. This experimental displacement's uncertainty can be compared to the theoretical uncertainty (1.7%), and therefore we can determine the accuracy of our linearized equation. Observe that the linearized version of k is used in the Δx equation above, and provides another reason for the linearization of our equation for k with respect to l.

Potential sources of uncertainty could result from differences in our spring constants due to changes to the bungee. The bungee stretches differently where the knots are tied, and this can lead to small changes in our total spring constant. Furthermore, the bungee loses its elasticity over time and use, so testing heavy weights, especially at longer lengths, could have stretched our bungee between measurements and between lengths.

Finally, the results that we obtained support our hypothesis that a shorter length of bungee cord would result in a larger spring constant.

CONCLUSION:

This experiment revealed the relationship that exists between the length of a spring, or in this experiment's case, a bungee cord, and its spring constant value, k. With our given equation to figure out the value of k for any length of cord l, we can now determine the theoretical displacement that will occur when the bungee cord is stretched with any mass on it, which is important for further experimentation, where we use the bungee cord to minimize the distance between a falling mass on the cord and the floor.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Daniel Monteagudo