Michael Shields (Scott Philips) Monday 11/2

**Mass and $X_{max}$ of Elastic String**

**Abstract**: We attempted to find the relationship between the length of the elastic string and the total stretch it went through with various masses hanging from it. This experiment was chosen because if a model could be developed for these two variables, the length of the string where the mass would drop without hitting the ground could be easily predicted. Three different masses were tested with 5 varying string lengths for each mass. The mass was dropped from a resting position and a video was taken and allowed us to find the maximum distance the mass dropped. The $X_{max}$ of the hanging mass was plotted against the length of the string for the 3 masses. We then took the slope of each graph and plotted slope against the three varying masses. This gave the equation of **slope=19.57m + 1.16.** The mass of our objects will be known so the slope can be found easily. From this, the slope can be plugged into **$X_{max}$= Slope($X_L$)** which will give the $X_{max}$.

**Introduction:** The relationship between the distances dropped of the hanging mass ($X_{max}$) and the length of the elastic string ($X_L$) were explored. Hooke's law says that **F= -kx** and while this was not explicitly investigated, the slope value explored is similar to the value of k. K is known as the spring constant and is different for different strings and springs. A model was created for this by changing hanging masses and recording the distance the string was stretched giving us total stretch. This model is related to Hooke's law but instead of finding Force, it gives us $X_{max}$. Both this equation as well as Hooke's uses the variable of x. We thought that changing the length of the string would lead to a non-proportional increase of the total stretch with the total stretch being increased greater than the change in the length of the string.

**Methods:** We varied three different masses and 5 different elastic string lengths in order to derive an equation that plots stretch max vs. string length. For each string length, there were 3 different masses, in order to give us the most amount of data

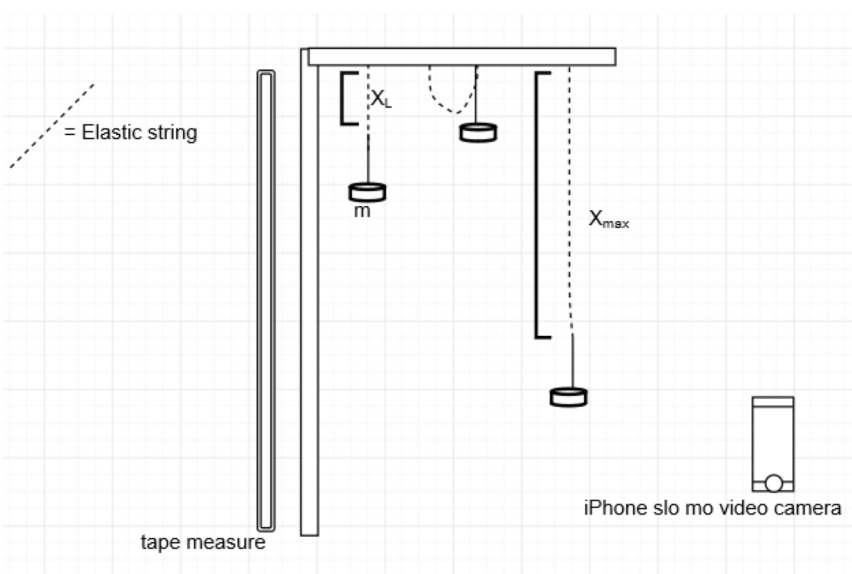


**Figure 1: Methods Diagram.** Basic setup with variables and important details.

- Used 3 different masses of .100 kg, .135 kg, .170 kg
- Varied the length of the string 5 different times between .18 m and .48 m.
- Tied string on top of pole and attached the hanging mass on the bottom of string
- The mass was dropped from where the knot is connecting the string to the mass holder (a .05 kg stand that holds extra masses).
- The camera was started as the mass was dropped and recorded the lowest point the knot in the string connecting the mass went. This was recorded in slow motion. We dropped the mass twice, averaging the total distance dropped.

**Results:** We collected 30 total data points and took the average of each trial to give us 15 total points. With this data, we were able to collect another 3 data points. We measured the total stretch of the string for 5 different lengths of the string. We did this for 3 different weights. Once we got our graphs, we then used the slopes of each graph and got 3 new data points for a new equation and graph.

| 100 g mass | |
|---|---|
| Length of Elastic String $X_L$ (m ± 0.01 m) | Average Total Stretch $X_{max}$ (m ± 0.01 m) |
| 0.19 | 0.59 |
| 0.25 | 0.77 |
| 0.33 | 1.02 |
| 0.42 | 1.3 |
| 0.48 | 1.52 |
| 135 g mass | |
| Length of Elastic String $X_L$ (m ± 0.01 m) | Average Total Stretch $X_{max}$ (m ± 0.01 m) |
| 0.18 | 0.67 |
| 0.25 | 0.92 |
| 0.31 | 1.16 |
| 0.4 | 1.51 |
| 0.48 | 1.88 |
| 170 g mass | |
| Length of Elastic String $X_L$ (m ± 0.01 m) | Average Total Stretch $X_{max}$ (m ± 0.01 m) |
| 0.19 | 0.86 |
| 0.25 | 1.12 |
| 0.33 | 1.46 |
| 0.42 | 1.89 |
| 0.48 | 2.17 |

**Figure 2: Length of Elastic String and Average total stretch for three different masses.** For each mass, there were 10 different measurements, two at each length of the string, and then they were averaged.

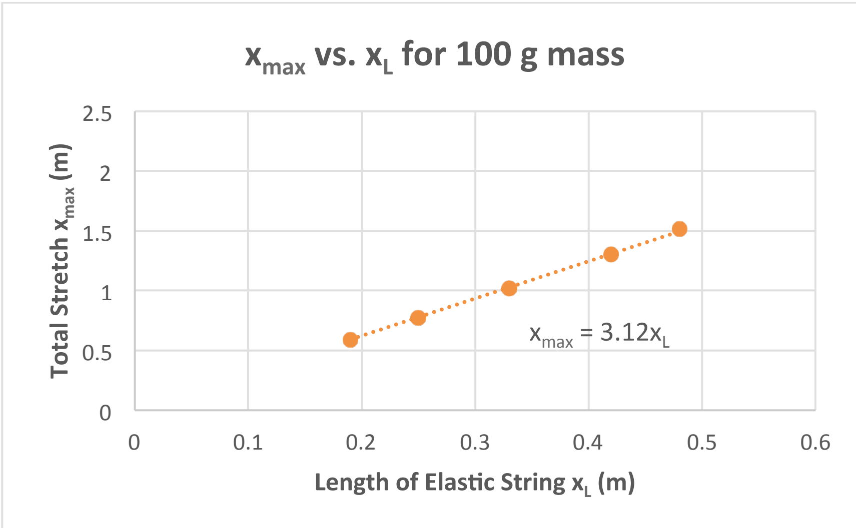


**Figure 3: Graph of $X_{max}$ vs. $X_L$ for the 100 g mass.** The "fall factor" is 3.12.

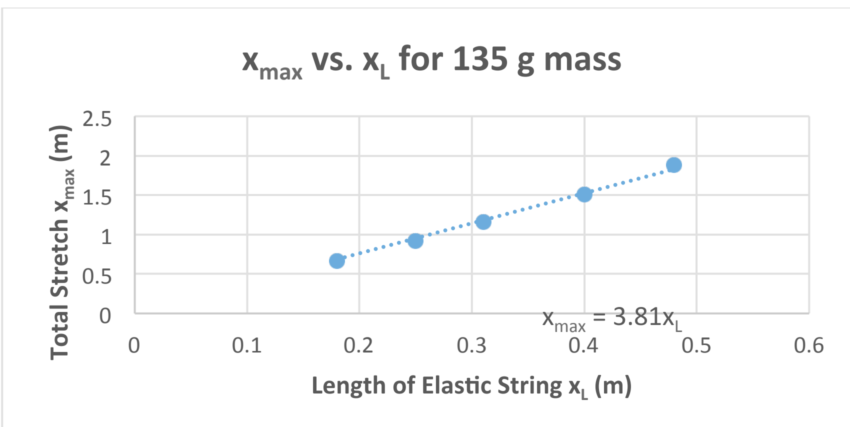


**Figure 4: Graph of $X_{max}$ vs. $X_L$ for the 135 g mass.** The "fall factor" is 3.81.

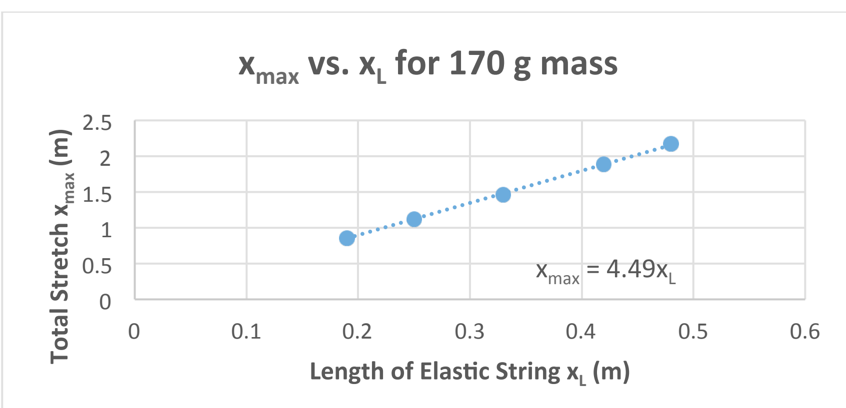


**Figure 5: Graph of $X_{max}$ vs. $X_L$ for the 170 g mass.** The "fall factor" is 4.49

These three graphs show the linear relationship between the length of the string and the total stretch of the string.

| Mass (kg ± 0.001 kg) | Slope of $X_{max}$ vs. $X_L$ |
|---|---|
| 0.1 | 3.12 |
| 0.135 | 3.81 |
| 0.17 | 4.49 |

**Figure 6: Mass and the slope of the $X_{max}$ vs. $X_L$ graphs.** The slopes are from each of the previous graphs. These are what we called the "fall factors".

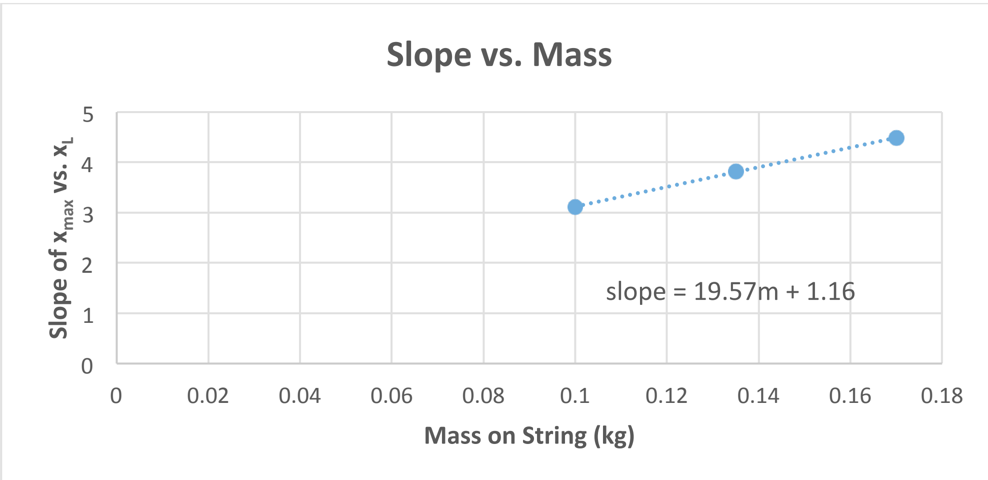


**Figure 7: Slope of the $X_{max}$ vs. $X_L$ graphs vs. Mass.** These graphs allow us to find the total stretch by plugging in slope to other equation.

Ultimately, the experimental value of interest is the "fall factor". This is the slope of the $X_{max}$ vs. $X_L$ graph. Knowing this value, allows us to accurately calculate the length the string will be stretched based on how long it is. This value is dependent on mass and can be calculated using the two experimental equations of interest. On the $X_{max}$ vs. $X_L$ graph, we got the equation **$X_{max}$= Slope($X_L$)**. For the slope vs. mass graph, we got the equation **slope=19.57m + 1.16.** The two equations can be condensed into **$X_{max}$= (19.57m + 1.16)($X_L$).** Any mass can be plugged in to find the $X_{max}$ for a certain length of string using the two equations. This equation lets any mass and any length of string be used and the $X_{max}$ can be calculated very easily.

The uncertainty for this equation and the values included is .01m. This was the raw uncertainty for measuring the distance stretched and this value carried forward to the final equation, as all of the equations used were linear. The trendline was linear as it was, so there was no need to linearize it. Having this equation makes it very easy to predict where the mass will be stretched as we can control both variables involved.

**Discussion:** There are no accepted values for the "fall factors" and the other constants in the equations. Because of this, we cannot find any percent error or be able to compare the uncertainty to the percent error. The uncertainty is .01 m for the equations and this remains true for all of the equations. This value is considered acceptable as a 1 cm uncertainty is very accurate.

We did not have time to test this but what we could have done is measured out a length of string and picked a mass somewhere between 100g and 170g, and plugged these values into the equation and see what the distance we should get. We then could drop the mass and see how accurate our equation is.

One major potential source of error was how tight the knot was that connected the string to the top of the pole. If this knot is loose, then the knot will stretch as well, adding to our total stretch and skewing the data. If the knot is tight, then there will not be the extra stretch, making the equation much more accurate. Another source of error could be how the mass fell. If the mass did not fall in a straight line down, then it would have travelled a longer distance than we recorded. It may not be extremely important on a small scale, but when dropping an egg in the great hall, every few centimeters count.

Our results did not exactly support our hypothesis. We were correct in predicting that as the length of the string increased, the total stretch would increase but we failed to predict that it would be proportional, linear increase. For the most part, our results were in support with our expectations.

**Conclusion:** This experiment gave a great starting point for determining how we want to approach the bungee jump. We wanted to discover a model for our string specifically where we can find out how far the string will stretch and this was accomplished. An equation was found that appears to be accurate for our purpose. In the future, this equation will need to be tested with different masses in between the values already tested to ensure accuracy. More importantly, longer strings will need to be tested as we extended our starting length of the string past 0.5 meters. It will be much longer than this for the bungee challenge and it will be necessary to ensure that our equation remains accurate on a larger scale.

I pledge my honor that I have neither given nor received aid on this report.

*Michael Shields*