

**TITLE:**

Using Hooke's Law to approximate the k value of a bungee cord

**ABSTRACT:**

Hooke's Law states that the force of a spring is a result of some spring constant,  $k$ , multiplied by the displacement,  $x$ , of some mass. We predict that we can model a bungee cord system using Hooke's law to get a linear relationship between the force of the spring and the displacement. We assume that our bungee cord acts like a spring. In this experiment, we measured the displacement of the bungee cord by measuring the change from an un-stretched equilibrium ( $x_L$ ) to a stretched equilibrium ( $x_0$ ), when varying masses were added. The system in our experiment was the bungee cord and the added hanging mass. Then, we plotted the spring force and the displacement, which gave us an estimate of  $k$  (the slope of the linear equation). Our equation was  $2.772x + 0.453$ , so our  $k$ -value was 2.772. This  $k$  value can then be applied to our final egg bungee jump because we can use it to determine the maximum force possible on the egg, given its mass, without breaking the egg.

**INTRODUCTION:**

Purpose or question: We will calculate the value of the spring constant,  $k$ , from the displacement caused by adding various masses to our bungee cord. The  $k$  value is important because we can use it in our final bungee jump with the egg to determine the maximum force allowed on the egg without breaking it.

Relevant equation(s), identifying variables:  $F_s = -kx$ , where  $F_s$  is the force of the spring,  $k$  is the spring constant, and  $x$  is the displacement

For our experiment:

$F_s = -kx$  and  $W = mg$        $F_{total} = ma$   
Because acceleration ( $a$ ) = 0,  
 $mg = -kx$

where  $m$  = mass (kg),  $g$  = acceleration due to gravity ( $m/s^2$ ),  $k$  = spring constant,  $x$  = displacement (m)

Basis or brief theoretical background: Hooke's law can be used to model an elastic system, like the bungee cord, when the relationship between force and displacement is linear.

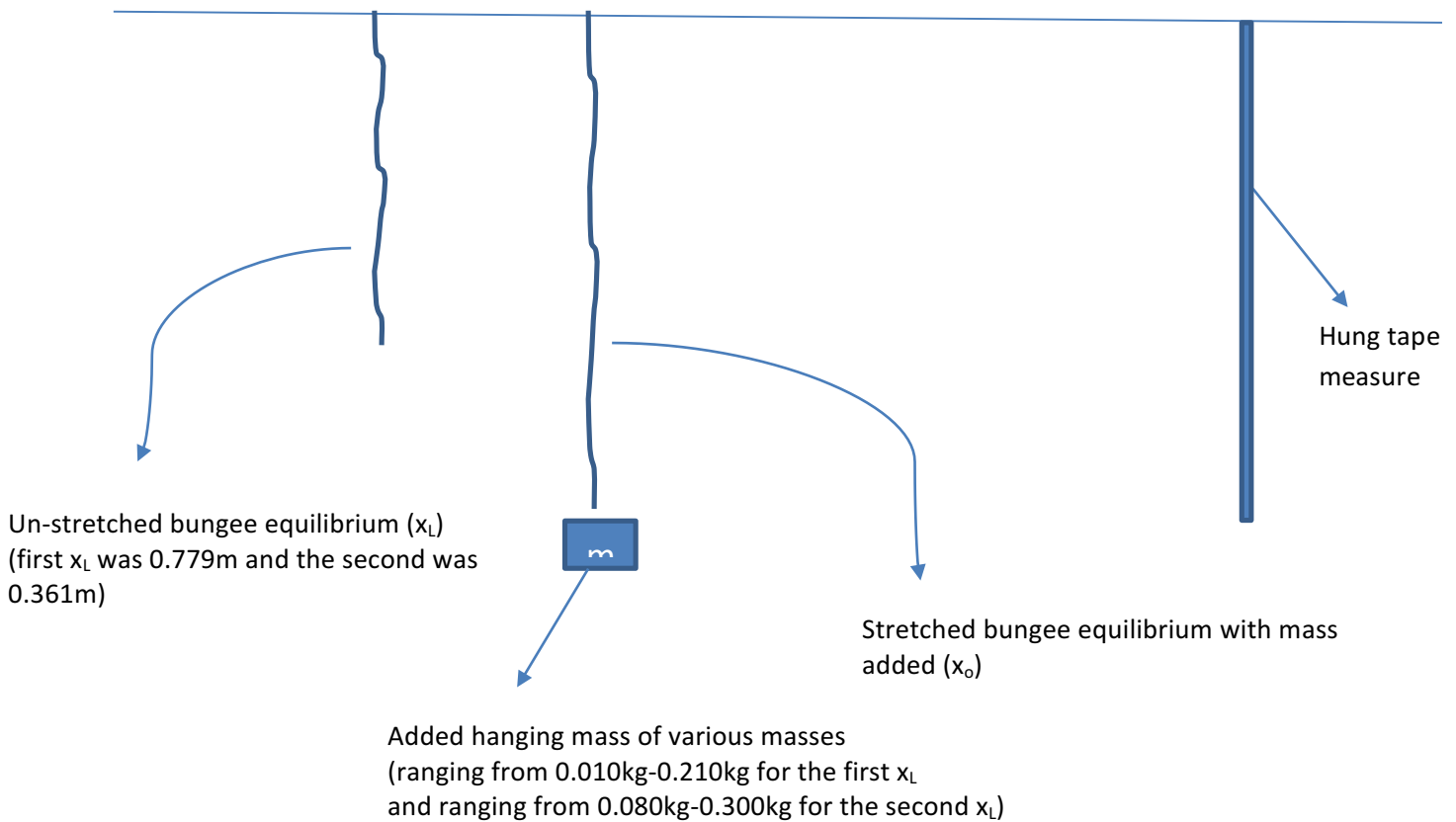
Hypothesis (or expectations): We predict that our bungee cord system will behave, and can thus be modeled, like an oscillating spring.

**METHODS:**

Describe the overall method and its rationale in a sentence or two:

We added various masses (21 for the first bungee length and 17 for the second bungee length) to our bungee cord to measure the displacement from an un-stretched equilibrium ( $x_L$ ) to a stretched equilibrium ( $x_0$ ). We then used Excel to graph the results and predict  $k$ .

Diagram, identifying *all* items, variables and/or measurements--use *Word* (Insert-shapes-drawing canvas), a drawing program, or *at least* use a ruler and blank paper and scan it in:



Describe setup and procedure, *including relevant or significant details* (may be bullets):

- Tied two small loops of equal sizes in bungee cord
- Attached one end of the cord to the stand
- Measured the distance from the top of the loop to the top of the other loop in the bungee to record  $x_L$  (the first  $x_L$  was 0.779m)
- Added various masses to the other end of the cord (the other loop)
- Measured the distance from the top of the loop to the top of the other loop in the bungee to record  $x_0$
- Subtracted  $x_L$  from  $x_0$  to find the displacement
- Repeated the steps above for the second, shorter  $x_L$  (0.361m)

### **RESULTS:**

In a sentence or so, **introduce the Results section** to give the reader context—data collected, and how it is analyzed to get the relevant result:

**We measured the displacement of the bungee cord using various hanging masses for two different un-stretched equilibria lengths ( $x_L$ ) to predict a value of the spring constant ( $k$ ).**

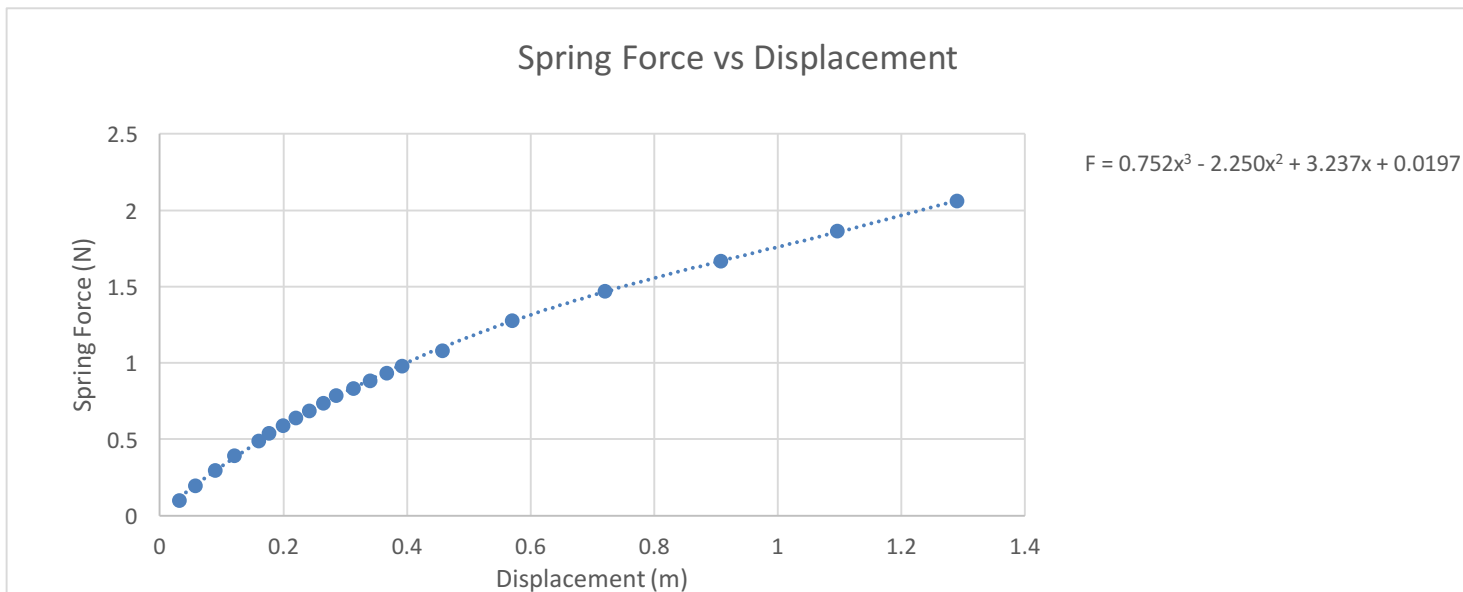
**Table 1: Displacement and spring force for a bungee cord un-stretched equilibrium length ( $x_1$ ) of 0.779m(+/- 0.001m).** A total of 21 different masses were added to the bungee cord. The force is a result of the weight ( $m \cdot g$ ) added to the cord.

Mass added (kg)	$X_0$ in m (+/-0.001m)	$X_0 - X_1$ (displacement) in m (+/-0.001m)	Force (N)
0.010	0.811	0.032	0.098
0.020	0.837	0.058	0.196
0.030	0.869	0.090	0.294
0.040	0.900	0.121	0.392
0.050	0.939	0.160	0.491
0.055	0.956	0.177	0.540
0.060	0.978	0.199	0.589
0.065	0.999	0.220	0.638
0.070	1.021	0.242	0.687
0.075	1.044	0.265	0.736
0.080	1.064	0.285	0.785
0.085	1.092	0.313	0.834
0.090	1.119	0.340	0.883
0.095	1.146	0.367	0.932
0.100	1.171	0.392	0.981
0.110	1.236	0.457	1.079
0.130	1.349	0.570	1.275
0.150	1.500	0.721	1.472
0.170	1.687	0.908	1.668
0.190	1.875	1.096	1.864
0.210	2.069	1.290	2.060

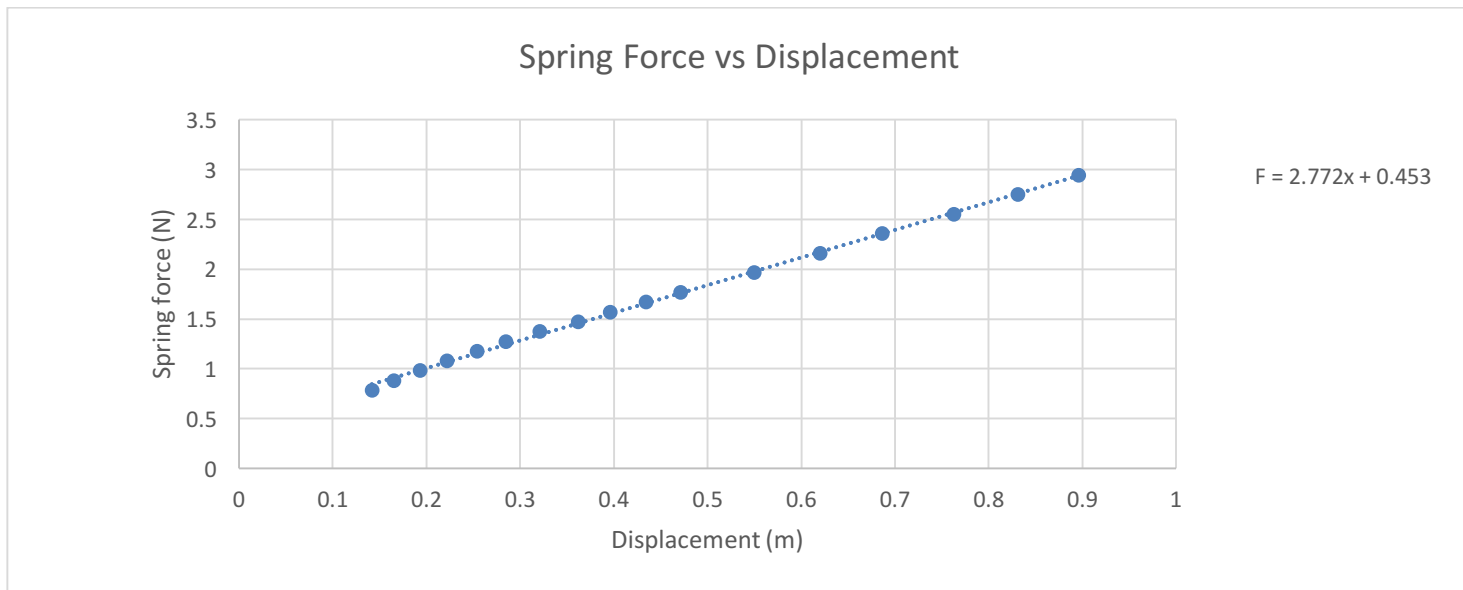
**Table 2: Displacement and spring force for a bungee cord un-stretched equilibrium length of 0.361m (+/-0.001m).** A total of 17 different masses were added to the bungee cord. Again, the force is a result of the weight added to the cord.

Mass added (kg)	Xo in m (+/-0.001m)	Xo-Xl(displacement) in m (+/-0.001m)	Spring Force (N)
0.080	0.503	0.142	0.785
0.090	0.526	0.165	0.883
0.100	0.554	0.193	0.981
0.110	0.583	0.222	1.079
0.120	0.615	0.254	1.177
0.130	0.646	0.285	1.275
0.140	0.682	0.321	1.373
0.150	0.723	0.362	1.472
0.160	0.757	0.396	1.570
0.170	0.795	0.434	1.668
0.180	0.832	0.471	1.766
0.200	0.911	0.550	1.962
0.220	0.981	0.620	2.158
0.240	1.047	0.686	2.354
0.260	1.124	0.763	2.551
0.280	1.192	0.831	2.747
0.300	1.257	0.896	2.943

**Graph**, if applicable, inserted from *Excel*, **formatted and labeled according to “Formalities”** document, and with **curve-fit** (also known as “trendline” in *Excel*, this could be a linear or non-linear fit):



**Figure 1: Spring force vs Displacement for an un-stretched equilibrium ( $x_l$ ) of 0.779m.** A linear fit was not appropriate for this data as it did not fit the data well, so we included an equation that best fit the data presented. However, although the data is not linear for smaller forces (smaller masses), this should not hinder our results as the egg mass will be in a range between 100-170 g and we will therefore use the k value found in Figure 2.



**Figure 2: Spring force vs Displacement for an un-stretched equilibrium of 0.361m.** Here, the forces (and masses) are larger and are much more linear. Therefore, this range is applicable to our egg bungee jump, so we can presume this data to be linear.

**Equation** of the curve-fit from the graph, if applicable (*substitute* your variables for *Excel's*  $y$  and  $x$ , and round coefficients and powers appropriately—also within graph):

$F = 0.752x^3 - 2.250x^2 + 3.237x + 0.0197$  (for Figure 1)

$F = 2.772x + 0.453$  (for Figure 2)

Use **Excel regression analysis** on any graph that has a **linear** fit only (see EG), to obtain: USED data from figure 2

uncertainty for slope= 0.0251                      % uncert= 0.905%

uncertainty for y-intercept= 0.0127              % uncert= 2.804%

**Identify experimental value(s) of interest**, why it is of interest, and how/from where obtained, briefly:

**Because  $F=-kx$ , we experimentally found  $k$  from our linear equation in Figure 2.**

value obtained =  $k=2.772$

uncertainty of experimental value(s) = 0.0251              % uncert= 0.905%

**Add** any other pertinent info for the reader (who may not have done this experiment) to follow along: We found that  $k$  is not constant for small forces (masses) added to the bungee cord. However, this should not affect the results of using the  $k$  derived from the linear data, because the egg will have a mass that is in the linear range of our data.

**Summarize Results (just the facts)**—give the important, relevant results, and why/how they are relevant to the purpose, in a sentence or two, including main equations and quantitative results:

We found that for larger masses (larger forces),  $k$  is constant and varies linearly with displacement. Therefore, we should be able to apply our experimental  $k$  value to our egg bungee experiment. We will use the  $k$  value of 2.772 from our linear graph (figure 2) in our egg bungee jump. Because  $F=-kx$  (Hooke's law), we could say that  $k$  is the slope of our linear equation found in Figure 2.

### **DISCUSSION:**

If no values are available for comparison, **determine “acceptability” of uncertainty in your value(s)** according to your needs. **AND determine a test** of your value(s) for “error” -- e.g. use your result to predict something, and then measure it (if time permits), or briefly describe how you would test it:

Our data provided a linear graph (Figure 2 and the larger force ranges of Figure 1), which is expected from the Hooke's law equation (where the force is proportional to  $k$  and  $x$ ). We could further test the acceptability of our experimental value of  $k$  by experimentally comparing it to a bungee cord with a known  $k$  value. This would allow us to see how well the experiment worked. We could then find the percent error in our experimental design by comparing our experimental value to the known (or theoretical) value of  $k$ .

**Sources of uncertainty** and their relative significance (PLEASE don't say “human error.” Identify *specific* sources of “error”—think of things that may add uncertainty or skew data, rather than “bad” things or “mistakes”):

Because this experiment is not an ideal situation, other factors, such as air resistance, motion due to the air, friction, may have impacted how this situation can be modeled (i.e. it is not exactly an ideal simple harmonic motion occurrence). The bungee cord may also have gotten irreparably stretched throughout the experiment, which would have caused a larger displacement than expected, however; we believe this is a minor contributor to uncertainty. We were also limited in our experiment by the masses added, because  $k$  was not constant at smaller masses (forces), as shown in Figure 1. We were also limited by how large our masses were, as we were not physically tall enough to make the stand taller (we could not use any larger masses because they hit the floor).

**Any further observations** or extenuating circumstances that aid in interpretation or evaluation:

We are basing our  $k$  value and true model of Hooke's law off of the data shown in Figure 2.

In a couple sentences, **describe whether your main results support your hypothesis.** How well were the results in agreement with theory, expectations, or otherwise deemed “acceptable”? Why/how so, or not?

Because we found that larger masses (larger forces) vary linearly with displacement,  $k$  is constant and we can model this bungee cord system with Hooke's law.

### **CONCLUSION:**

Clearly and definitively state the experimental outcome(s) in terms of your question or purpose:

We were able to estimate the value of the spring constant,  $k$ , from the displacement caused by adding various masses to our bungee cord.

Implications of these conclusions (e.g. the significance to larger questions), or next steps proposed:

We can use Hooke's law to model this bungee system as simple harmonic motion. Therefore, we can use our experimental spring constant,  $k$ , when we do the egg bungee experiment.

**On my honor, I have neither given nor received any unacknowledged aid on this assignment.**

***Pledged: Morgan Trimas***