Does k vary with Length?

Section: 02 **Date:10/24/16**

TITLE: The investigation of whether k, our spring constant, varies for different lengths of our Bungee cord.

ABSTRACT:
We wanted to investigate in this experiment whether k varies with the length of our Bungee cord. We hypothesized that it does. Specifically, we sought to come up with a model to predict k given any length of the Bungee cord, so that eventually we could model the forces acting on our Bungee jumper. To do this, we hung our bungee cord vertically, chose different lengths, and for each length, plotted force vs. Δ x (the change in the length of the cord when the mass is hung from it). The slope of these graphs was be our k value for each different length. We then plotted k vs length, to find an equation for our k value, given length. In conducting this experiment, we found an equation for our k value given the length to be $K = 20.44L^2 – 26.072L + 9.9685$. This result means that the k value for our bungee cord does change with length, and that the higher the length, the lower the corresponding k value. This result confirms our hypothesis.

***INTRODUCTION**:*

We want to investigate whether k varies with the length of our Bungee Cord. This is important to us, because eventually we want to be able to attach an egg to our bungee cord, and give it a wonderful bungee jump experience. We will want the egg to come as close to the ground as possible without being injured. In order to safely give an egg this experience, we will need to know the exact spring constant k of the bungee cord for a given length. This information will allow us to model the spring force acting on the egg against the weight of the egg at all times during the fall. Finally, if we know the forces acting on the egg at all times, we can model the motion of the egg, and choose a length of the bungee cord that will give us a k value, which will allow the egg to drop as low as possible to the ground without being injured.

Our main equation we work with is $F = k * \Delta x$.

This equation is Hooke's law, which states that the force F needed to extend or compress a spring by some distance Δ x is proportional to that distance by some k value, which we call the spring constant. For our purposes in this experiment, Δ x represents the length of the cord when a mass is hanging from it (stretching it), and the unstretched length of the cord. Using Newton's second law F=ma, we realize that when the hanging mass is at rest, the magnitude of the spring force equals the magnitude of the weight of the mass. We can thus plot F vs. Δ x, and from our equation $F = k * \Delta x$, we can see that k will be the slope of this graph. We can then plot the k value vs. the length to see how k changes (if at all) with a change in length.

We expect that k will change with different lengths. Moreover, we anticipate that longer bungee cord lengths will have lower k values, so the k vs length graph should be downward sloping.

METHODS:

Conceptually, we know that we can find the k value for a given length by taking the slope of the F vs. Δ x graph. We will take the k value for each length, and then model an equation based on our data to predict the k value based on a given length of our bungee cord.

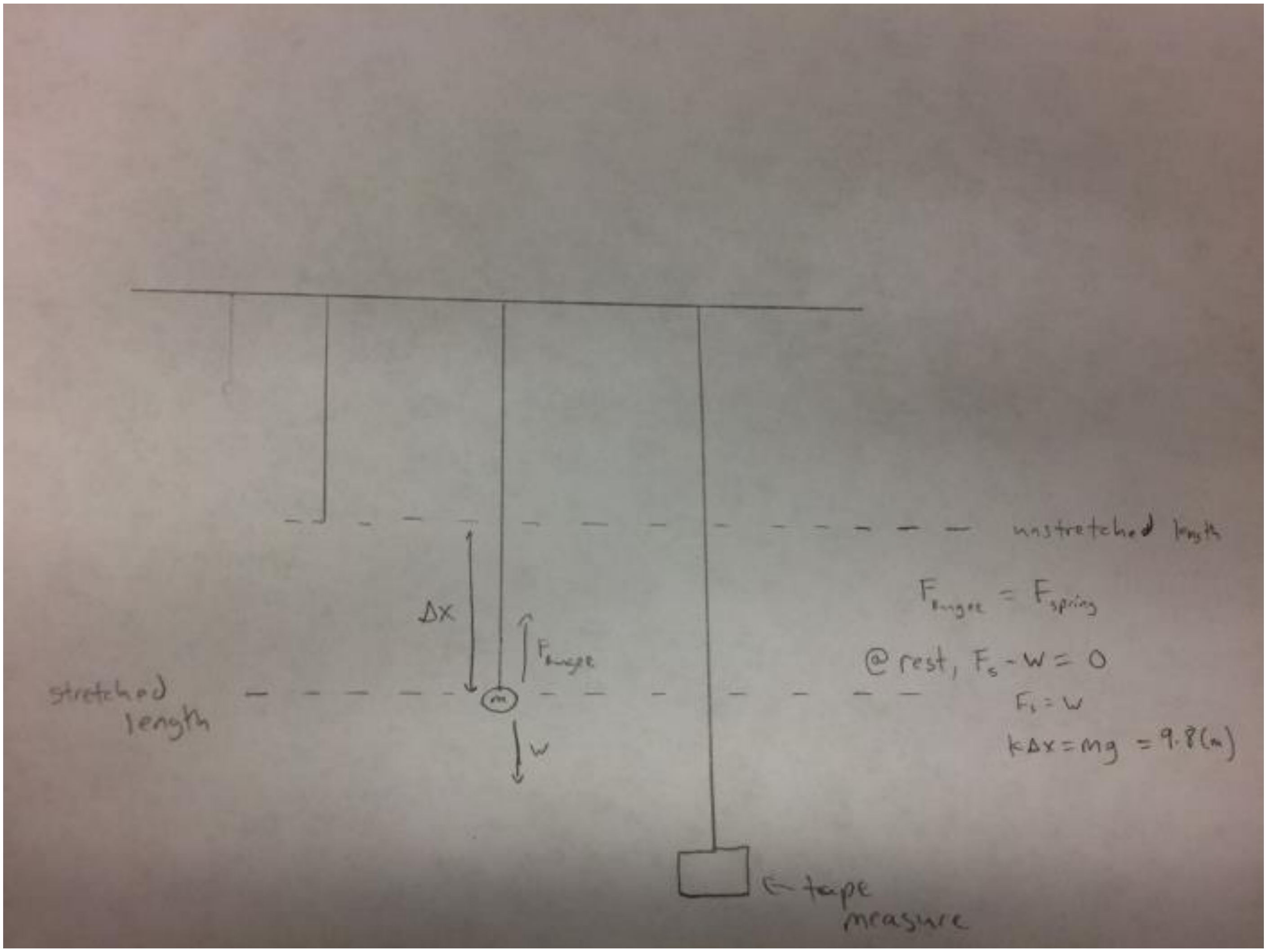

Figure #1, Diagram of our experimental setup

We had a horizontal hanger, with clamps from which we hung the bungee cord and a tape measurer right next to it, measuring from the top of the hanger to the floor. We tied an approved loop in our bungee cord, and put the loop around the clamp, so that we had a length of the bungee cord hanging from the clamp. We never moved or adjusted this loop. To use different lengths, we tied approved loops on the hanging part of the bungee cord, undoing them and redoing them at different spots to get different lengths. We hung the different masses from these loops

Describe procedure

- We chose 6 different lengths to find k values for.
- For each length, we measure the unstretched length from the clamp where the cord was secured, to the middle of the loop marking the length.
- For each length, we tested 5 different masses, and measured the Δ x (the stretched length minus the unstretched length).
    - We also calculated the force of the bungee cord, which as we have said earlier equaled the hanging mass times gravity.
- We thus plotted 5 different data points in a force vs. Δ x graph for each length, and then took a linear equation of the graph, the slope of which was our k value for that length.

- Once we finished hanging masses for each length, we measured the unstretched length again, to ensure that the masses hadn't permanently stretched the bungee cord, making our Δ x's inaccurate. For each length, the cord had not been permanently stretched, and the unstretched length was the same as before we hung the masses, so we did not need to worry about this problem.
- Also, we carefully measured each Δ x, and since they lined up linearly, in our F vs. Δ x graph, we did not need to take two trials for any Δ x.
- We conducted the above steps for each length, and finally plotted our k vs. length points, and fit an equation to predict the k value of the bungee cord for a given length.

RESULTS:

We plotted all our data points for each length in a force vs. Δ x graph, and found a linear equation for each length, the slope of these graphs being the k value for that given length. We then took each k value and plotted it against its length, and fit an equation to this data to predict a k value given a length of our bungee cord. Below, we have the data as well as the graph for each different length. Each table lists mass, unstretched length, stretched length, Δ x, and weight (mass times gravity). Each graph is force vs. Δ x for a given length.

<u>Length 0.315m</u>

mass (kg)	X(l)	x(o)	delta x	weight
0.2	0.315	0.79	0.475	1.96
0.18	0.315	0.75	0.435	1.764
0.15	0.315	0.65	0.335	1.47
0.1	0.315	0.497	0.182	0.98
0.05	0.315	0.385	0.07	0.49

Figure #2, mass, Δx, and weight with length 0.315m

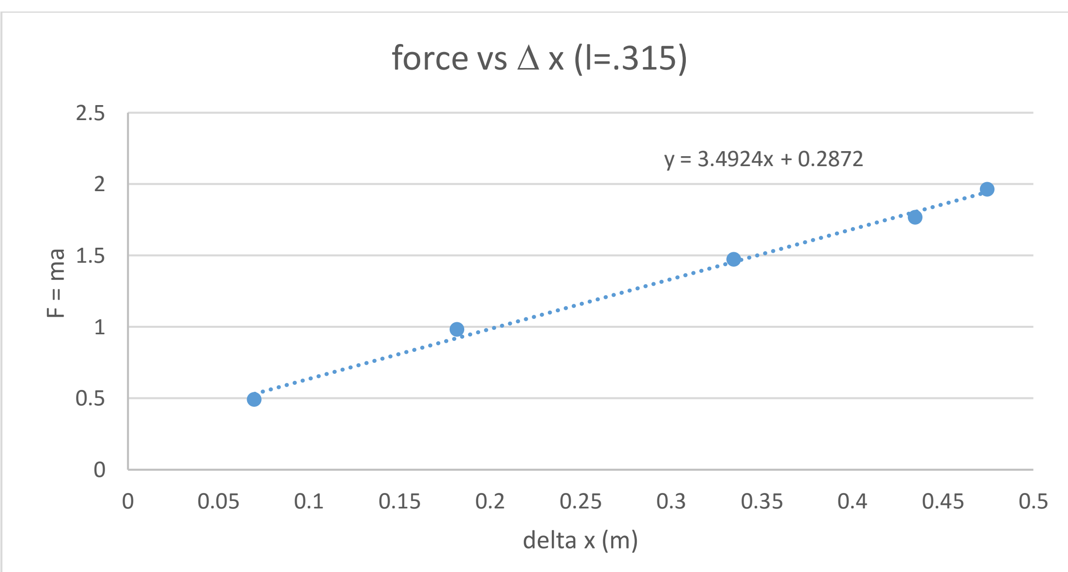

Figure #3, Force vs. mass, for length 0.315m

Linear Equation of the curve-fit from the graph

F=3.4924Δx + 0.2872

Regression analysis:

uncertainty for slope= 0.14 % uncert= 4%
uncertainty for y-intercept= 0.05 % uncert= 17%

Experimental value(s) of interest

We are interested in the slope F vs. Δx for this length, because as shown earlier, this is our k value for this length.

value obtained = 3.4924

uncertainty of experimental value(s) = 0.14 % uncert= 4%

Uncertainty was obtained using Excel regression analysis.

Length 0.443m

mass (g)	x(l)	x(o)	delta x	weight
0.2	0.443	1.115	0.672	1.96
0.18	0.443	1.072	0.629	1.764
0.15	0.443	0.934	0.491	1.47
0.1	0.443	0.715	0.272	0.98
0.05	0.443	0.553	0.11	0.49

Figure #4, mass, Δx, and weight with length 0.443m

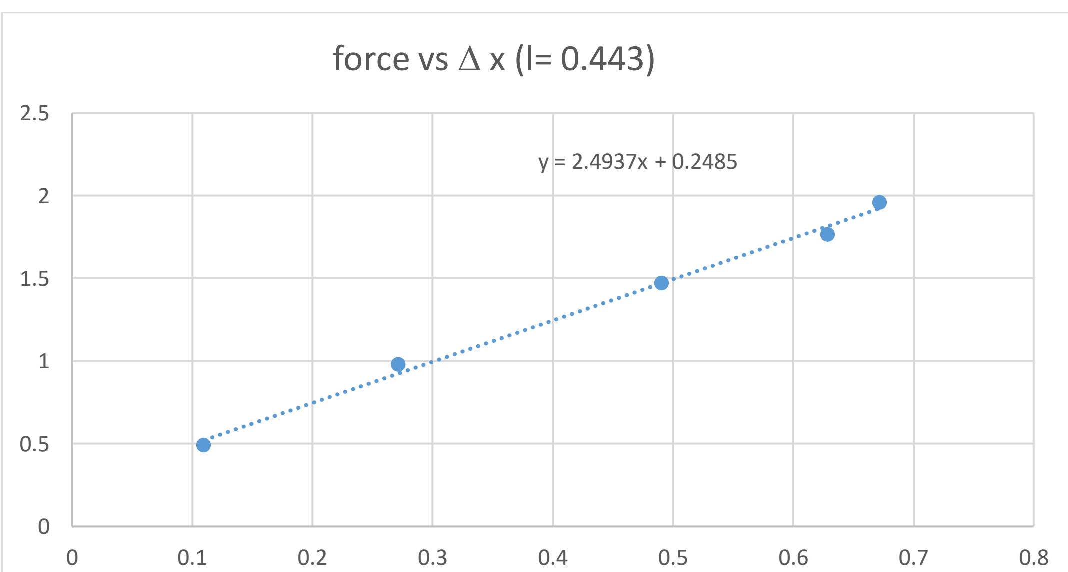

Figure #5, Force vs. mass, for length 0.443m

Equation of the curve-fit from the graph
F=2.4937Δx +0.2485

Regression analysis

uncertainty for slope= 0.11 % uncert= 4%
uncertainty for y-intercept= 0.05 % uncert= 20%

Experimental value(s) of interest

We are interested in the slope F vs. Δx for this length, because as shown earlier, this is our k value for this length.

value obtained = 2.4937

uncertainty of experimental value(s) = 0.11 % uncert= 4%

Uncertainty was obtained using Excel regression analysis.

<u>Length 0.485m</u>

mass (g)	x(l)	x(o)	delta x	weight
0.2	0.485	1.292	0.807	1.96
0.18	0.485	1.21	0.725	1.764
0.15	0.485	1.055	0.57	1.47
0.1	0.485	0.87	0.385	0.98
0.05	0.485	0.625	0.14	0.49

Figure #6, mass, Δx, and weight with length 0.485m

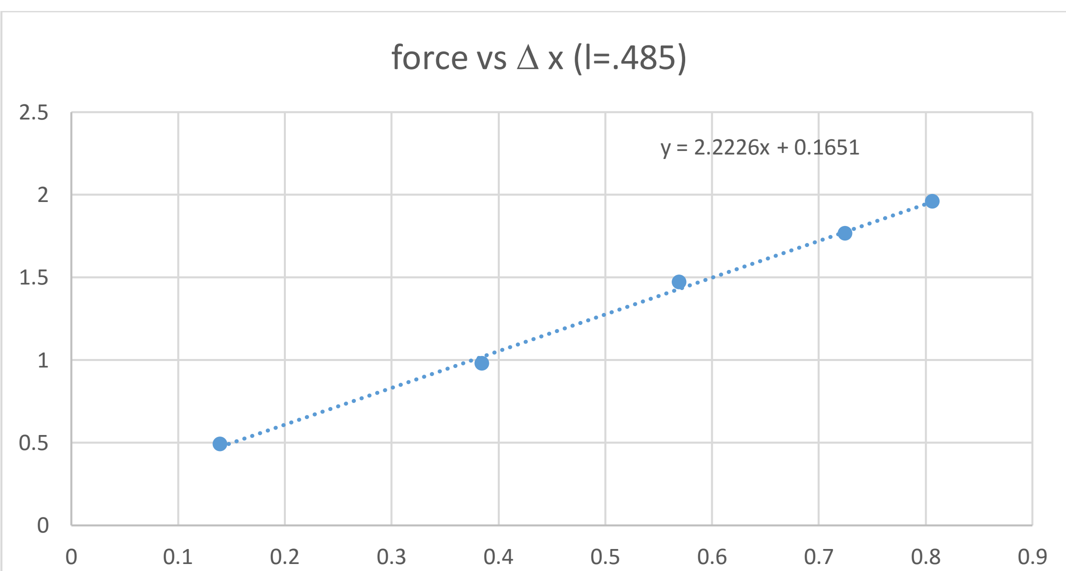

Figure #7, Force vs. mass, for length 0.485m

Equation of the curve-fit from the graph
$F = 2.2226\Delta x + 0.1651$

Regression analysis

uncertainty for slope= 0.06 % uncert= 3%
uncertainty for y-intercept= 0.04 % uncert= 18%

Experimental value(s) of interest

We are interested in the slope F vs. Δx for this length, because as shown earlier, this is our k value for this length.

value obtained = 2.2226

uncertainty of experimental value(s) = 0.06 % uncert= 3%

Uncertainty was obtained using Excel regression analysis.

<u>Length 0.57m</u>

mass (g)	x(l)	x(o)	delta x	weight
0.2	0.57	1.47	0.9	1.96
0.18	0.57	1.37	0.8	1.764
0.15	0.57	1.199	0.629	1.47
0.1	0.57	0.912	0.342	0.98
0.05	0.57	0.705	0.135	0.49

Figure #8, mass, Δx, and weight with length 0.57m

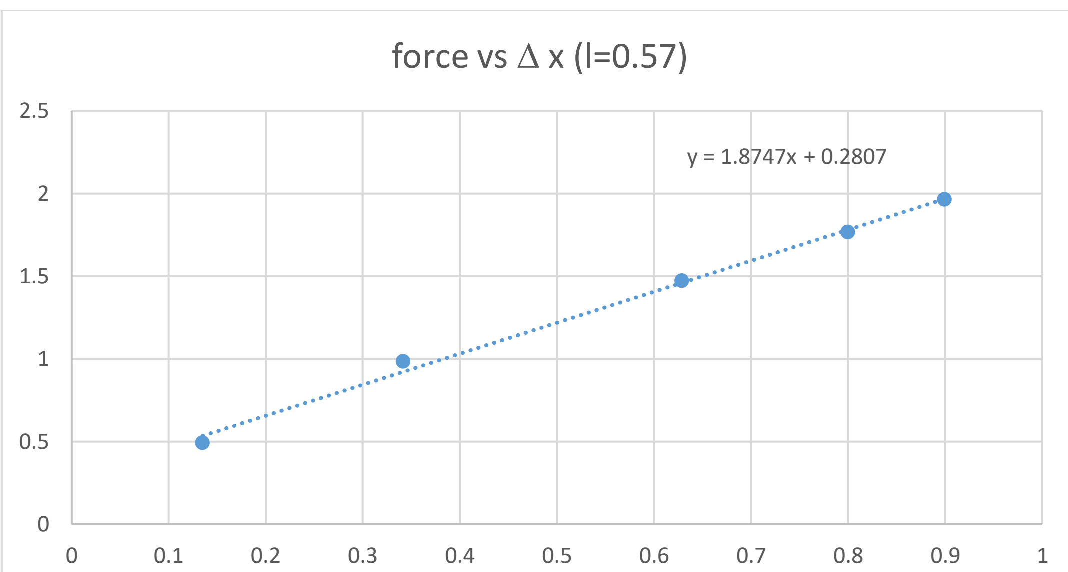

Figure #9, Force vs. mass, for length 0.57m

Equation of the curve-fit from the graph
$F = 1.8747\Delta x + 0.2807$

Regression analysis

uncertainty for slope= 0.07 % uncert= 4%
uncertainty for y-intercept= 0.04 % uncert= 14%

Identify experimental value(s) of interest

We are interested in the slope F vs. Δx for this length, because as shown earlier, this is our k value for this length.

value obtained = 1.8747

uncertainty of experimental value(s) = 0.07 % uncert= 4%

Uncertainty was obtained using Excel regression analysis.

<u>Length 0.662m</u>

mass (g)	x(l)	x(o)	delta x	weight
0.2	0.662	1.705	1.043	1.96
0.18	0.662	1.66	0.998	1.764
0.15	0.662	1.395	0.733	1.47
0.1	0.662	1.06	0.398	0.98
0.05	0.662	0.82	0.158	0.49

Figure #10, mass, Δx, and weight with length 0.662m

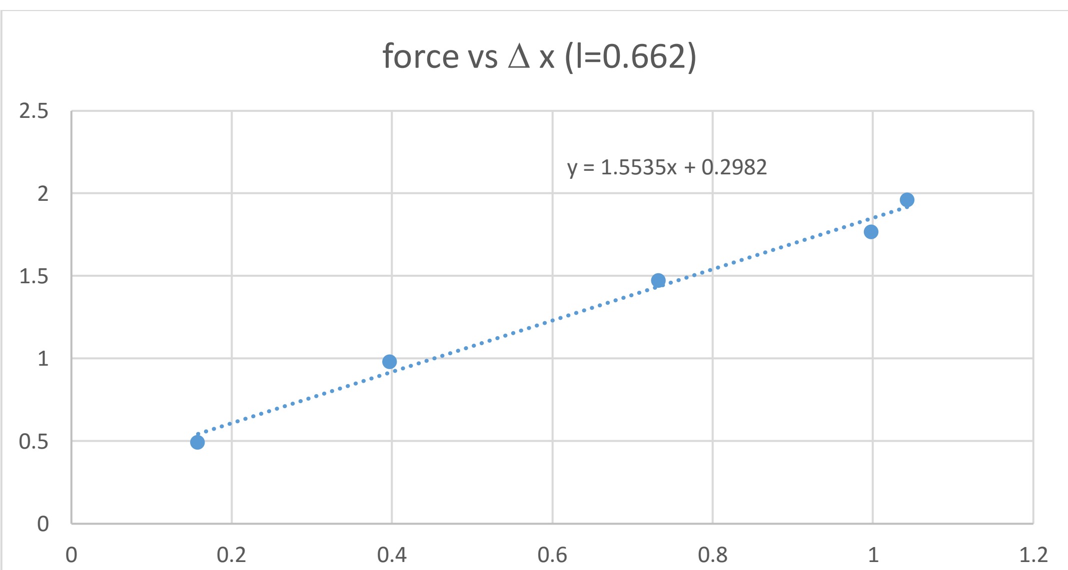

Figure #11, Force vs. mass, for length 0.662m

Equation of the curve-fit from the graph
$F=1.5535\Delta x + 0.2982$

Regression analysis

uncertainty for slope= 0.10 % uncert= 6%
uncertainty for y-intercept= 0.07 % uncert= 23%

Experimental value(s) of interest

We are interested in the slope F vs. Δx for this length, because as shown earlier, this is our k value for this length.

value obtained = 1.5535

uncertainty of experimental value(s) = 0.10 % uncert= 6%

Uncertainty was obtained using Excel regression analysis.

<u>Length 0.191m</u>

mass (g)	x(l)	x(o)	delta x	weight
0.2	0.191	0.48	0.289	1.96
0.18	0.191	0.452	0.261	1.764
0.15	0.191	0.395	0.204	1.47
0.1	0.191	0.305	0.114	0.98
0.05	0.191	0.236	0.045	0.49

Figure #12, mass, Δx, and weight with length 0.191m

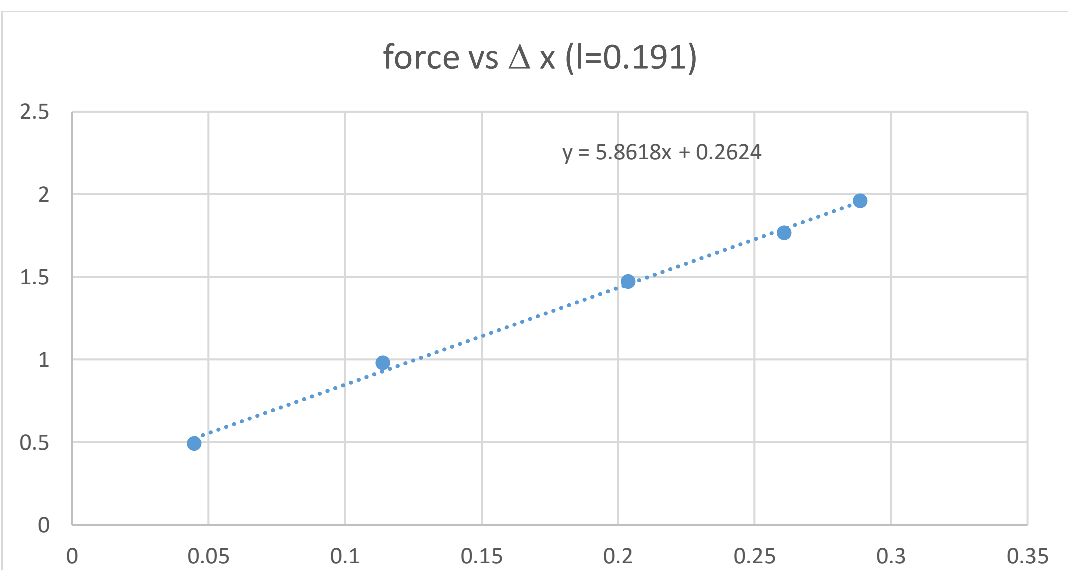

Figure #13, Force vs. mass, for length 0.191m

Equation of the curve-fit from the graph
F=5.8618Δx + 0.2624

Regression analysis

uncertainty for slope= 0.19 % uncert= 3%
uncertainty for y-intercept= 0.04 % uncert= 15%

Experimental value(s) of interest

We are interested in the slope F vs. Δx for this length, because as shown earlier, this is our k value for this length.

value obtained = 5.8618

uncertainty of experimental value(s) = 0.19 % uncert= 3%

The uncertainty was obtained using Excel Regression analysis.

At this point, we had all our k values for each different length represented in the following table.

length(m)	k-value
0.315	3.4924
0.443	2.4937
0.485	2.2226
0.57	1.8747
0.662	1.5535
0.191	5.8618

Figure #14, the k-value for each different length

We finally plotted this data in a k vs. length graph, and found an equation to fit the data.

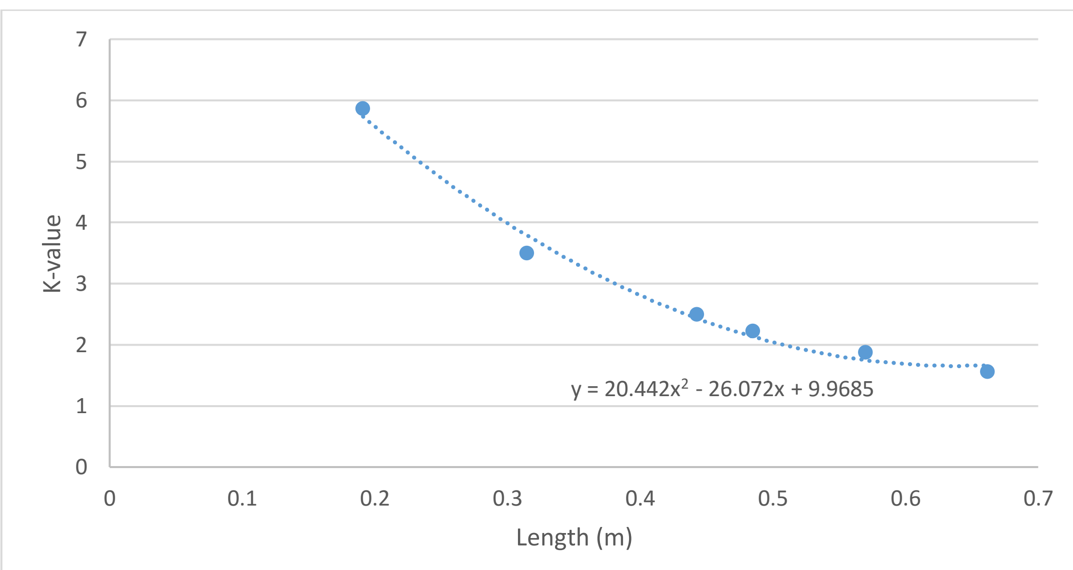

Figure #15, K vs. Length

Our model to predict the k-value for a given length is the equation $K=20.442L^2 - 26.072L + 9.9685$.

Summarized Results

Our main result from this experiment is the equation we found $K=20.442L^2 – 26.072L + 9.9685$ to predict the k-value for a given length. Finding this model was the entire purpose for this experiment, since in the future we will need to know the k values for different lengths of the Bungee cord, so that we can ensure our precious egg the safest and most thrilling jump.

DISCUSSION:

Our experimental equation for k is $K=20.442L^2 – 26.072L + 9.9685$. This is a downward sloping and slightly L shaped graph. This makes perfect sense intuitively, when we realize the fact that from our data, the same mass had a larger Δx for longer lengths, and a smaller Δx for shorter lengths. If we think about our $F=k\Delta x$ equation, which is $mg=k\Delta x$, we realize that mg doesn't change for the same mass, and Δx gets larger for longer lengths. So, for the equation to still hold, k must be getting smaller as the length of the bungee cord gets longer. The L shape aspect of our graph also makes sense, since the length will never be 0, and the k value will never be 0. Our graph reflects these ideas, and leads us to believe that our equation is quite accurate.

As far as our individual k values are concerned, we calculated our uncertainty for each k value in the results section above. Unfortunately, we have no "accepted" value to compare our k values to. However, the highest % uncertainty for any of our k values was 6%. This is a relatively low uncertainty, so we determine that our experimental values for each individual k are "acceptably" close to the actual value of each k.

One way to test our k values would be to do the experiment for each length again, this time using completely different masses, and then seeing if the slope of F vs. Δx is the same.

Sources of uncertainty

Potential sources of uncertainty are potential air draft in the room, possible slippage in the measuring tape, or simply inaccurate measurements.

Our main result supports our hypothesis that the k value for our bungee cord does vary with length, and that longer lengths will have lower k values. The results are in agreement with our theory and expectations, due to the fact that our $K=20.442L^2 – 26.072L + 9.9685$ equation gives us a function that is downward sloping and L shaped, which is exactly what we would expect from theory/reason, and it is consistent with findings from our data.

CONCLUSION:

Our experimental outcome put simply is that the k value of our bungee cord does vary with the length of it, and that a longer length of the bungee cord will have a lower respective k value. We now have an equation $K=20.442L^2 – 26.072L + 9.9685$ that can reasonably predict the k value for a given length of our bungee cord. This is an important finding, because we will be able to use this equation to move forward with our overall plan to have a bungee jumping egg fall as close to the ground as possible without being injured.

Report Outlines are *individual assignments*. Cite any work not your own, acknowledge any aid, and pledge the report:
On my honor, I have neither given nor received any unacknowledged aid on this assignment.