Justin Pusztay

TITLE:

Does the elastic cord that will be used in the Bungee experiment follow Hooke's law? Can the spring constant of the elastic cord be found?

ABSTRACT:

This lab is to verify our hypothesis that the elastic cord will follow a model of Hooke's law. We are testing this for the greater Bungee Experiment. We had to modify Hooke's law from F_{spring}= -kx to mg= -k(x_0-x_L) to fit our system in a better way. We set the unstretched length of the cord, with no mass hanging from it, as x_l. This value is kept constant throughout all our trials. We had to set x_o as the new length of the cord when masses were hung from it. We performed three trials with three different initial cord lengths. However, we maintained each unstretched cord length constant throughout the experiment. We added a range of masses onto the end of the cord. The new stretch was measured to calculate displacement. Our results show that all our trials follow a linear regression when the weight is plotted against the displacement. We were able to obtain three spring constants from each of our trials. Our first trial gave us a spring constant of 5.502 (±0.300) N/m, our second trial had a spring constant of 3.788 (±0.100) N/m, and our third trial had a spring constant of 1.836 (±0.080) N/m. We believe that our data is accurate since after running the excel data analysis we obtained an uncertainty of 5% for our first trial, 3% for our second trial and 4% for our third trial. This uncertainty is small enough for us to make a reasonable conclusion that our regression is accurate and the elastic cord follows a linear model of Hooke's law thus affirming our hypothesis. We then used our spring constants to calculate the force exerted by the cord. We used the displacement from our mass of 0.20 kg. Ideally, we should have gotten 1.962 N. But our first trial gave us a force of 2.013 N, our second trial gave us 2.061 N, and our third trial gave us 2.012 N. All these results gave a percent error of about 4%. We conclude that since the elastic cord follows a model of Hooke's law, it confirms our hypothesis, as well as being able to calculate a spring constant for a unstretched cord length and will be able to calculate the spring constant and have confidence the constant is accurate.

INTRODUCTION*: Gives the purpose and conceptual or theoretical context.*

The purpose of this lab is to decide if the elastic cord has properties that model Hooke's Law. If the cord does, then the spring constant of the cord must be found. The desire to find the properties of this string emerges from a larger project creating a quality bungee cord experience that will allow an egg to come close to the ground without breaking. This lab is to determine if the cord that will be used for the bungee jump follows Hooke's law.

Relevant equation(s) specific to this experimental purpose or setup, identifying variables:

F_{spring}= -kx
X_L= Unstreched Cord length
X_o=Equlibrium length of cord
X_{max}=Maximum length when dropped
m= mass of hanging mass
mg=weight of hanging mass
(x_0-x_L)= Displacement
mg= -k(x_0-x_L)

Basis or brief theoretical background, providing enough context that the reader understands where the equation(s) are from:

The force of a spring on a mass is always pointing towards the equilibrium position of the spring. The spring constant, k, is how much applied force is needed to stretch or compress the string a certain length. The mass of the object does not contribute to the rate of oscillation. Also, in simple harmonic motion, the mass experiences its maximum speed at the equilibrium point during oscillation. The mass also experiences the maximum magnitude in its acceleration when the object is at the ends of its amplitude. We can assume that the spring force is equal to the weight of the hanging mass because when the mass reaches its equilibrium point, where the mass is hanging and not oscillating, where the acceleration is 0 m/s^2. Thus, mg-kx=0 is true, according to Newton's second law. Therefore our equation is mg=kx, which is a resemblance of Hooke's law.

Hypothesis (or expectations):

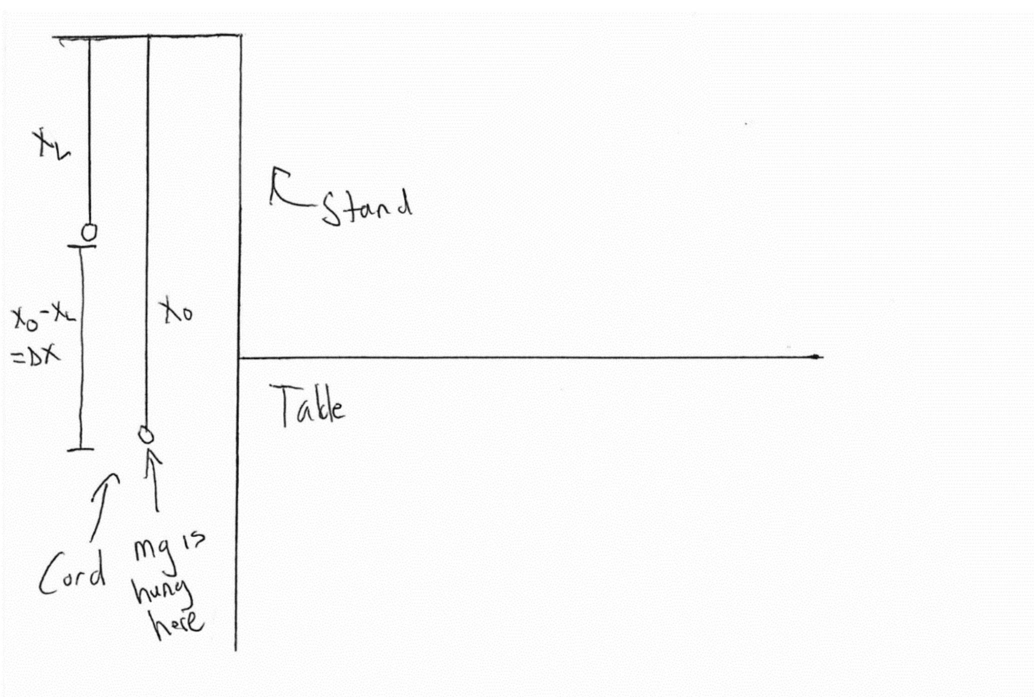
We expect that the elastic cord will follow a linear model of Hooke's law to characterize its behavior.

METHODS:

Describe the overall method and its rationale in a sentence or two:

We used a different cord length fo reach trial, however, we used the same length within each trial. We maintained a constant unstretched cord length for our trials, but we changed the hanging mass to find the equilibrium point. We measured the new length of the cord when the cord was stretched to use the displacement and the force to find the spring constant.

Figure 1: Diagram of the set up



Describe setup:
We have a stand attached to the end of a table. The hanging mass is attached to the end of the cord. There is a knot at the other end to hang the hanging masses.

Describe procedure, *including relevant or significant details* (may be bullets):

We set up a stand on the edge of table that extended in the vertical direction. At the top of the stand the cord was tied to be incapable of sliding when you pull on the cord. Rather the cord would only stretch. There was a knot created at the end of the cord that was hanging off the stand. This knot was created to be able to hang the hanging mass from. A tape measure was placed adjacent to the cord, and hung off the top of the stand to the floor. This tape measure was to measure the length of the cord. We set the unstretched cord length at an arbitrary value and measured this length. This was the length of the unstretched cord for our first trial. We hung our first mass of .05 kg to the end of the cord and measure the new length of the cord. We continued this procedure for 9 different masses, increasing the mass by .025 kg each time. Afterwards, we ran two more trials with a different initial unstretched cord length each time. After finding the new cord length, we found the displacement and ran linear regression on each of trials. This is how we obtained our spring constant to analyze the behavior of the cord by using Hooke's law.

RESULTS:

We measured the stretch of the cord when hanging different masses on it. We then analyzed the displacement of the cord with the weight that was hung to determine if the cord follows Hooke's law.

Table 1: Hanging masses from an initial stretch of .230 m (±0.002 m) with displacement and weight hung. Trial 1

Hanging Mass (kg) (±0.001 kg)	**Stretch (m) (±0.002 m)**	**Displacement (m) (±0.002 m)**	**Weight (N) (±0.001 N)**
0	.230	0	0
0.050	.275	0.045	0.491
0.075	.308	0.078	0.736
0.100	.345	0.115	0.981
0.125	.394	0.164	1.226
0.150	.463	0.233	1.471
0.175	.527	0.297	1.722
0.200	.596	0.366	1.962
0.225	.656	0.426	2.207
0.250	.720	0.490	2.452

Table 2: Hanging masses from an initial stretch of .316 m (±0.002 m) with displacement and weight hung. Trial 2

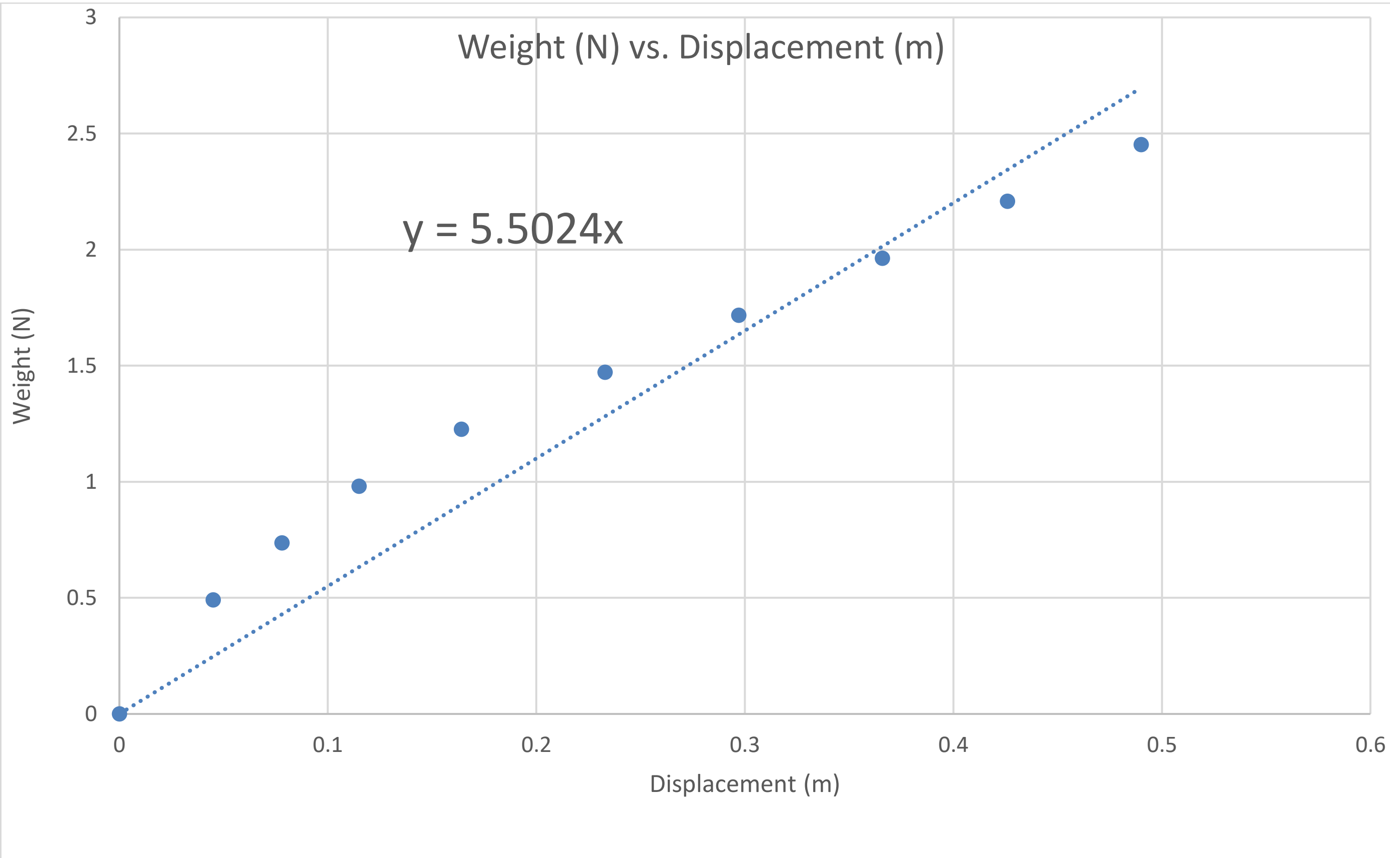
Hanging Mass (kg) (±0.001 kg)	**Stretch (m) (±0.002 m)**	**Displacement (m) (±0.002 m)**	**Weight (N) (±0.001 N)**
0	.316	0	0
0.05	.386	0.07	0.491
0.075	.439	0.123	0.736
0.100	.504	0.188	0.981
0.125	.588	0.272	1.226
0.150	.678	0.362	1.471
0.175	.774	0.458	1.722
0.200	.860	0.544	1.962
0.225	.932	0.616	2.207
0.250	.983	0.667	2.452

Table 3: Hanging masses from an initial stretch of .654 m **(±0.002 m)** with displacement and weight hung. Trial 3

Hanging Mass (kg) (±0.001 kg)	**Stretch (m) (±0.002 m)**	**Displacement (m) (±0.002 m)**	**Weight (N) (±0.001 N)**
0	.654	0	0
0.05	.781	0.127	0.4905
0.075	.882	0.228	0.73575
0.1	.101	0.359	0.981

0.125	.119	0.531	1.22625
0.15	.137	0.72	1.4715
0.175	.157	0.917	1.71675
0.2	.175	1.096	1.962
0.225	.195	1.291	2.20725
0.25	.208	1.423	2.4525

Graph 1: The regression of the Weight (N) vs. the Displacement (m) from Table 1 (Trail 1). The slope 5.502 (±0.300) N/m is the spring constant, k, of the elastic band at that unstretched length.

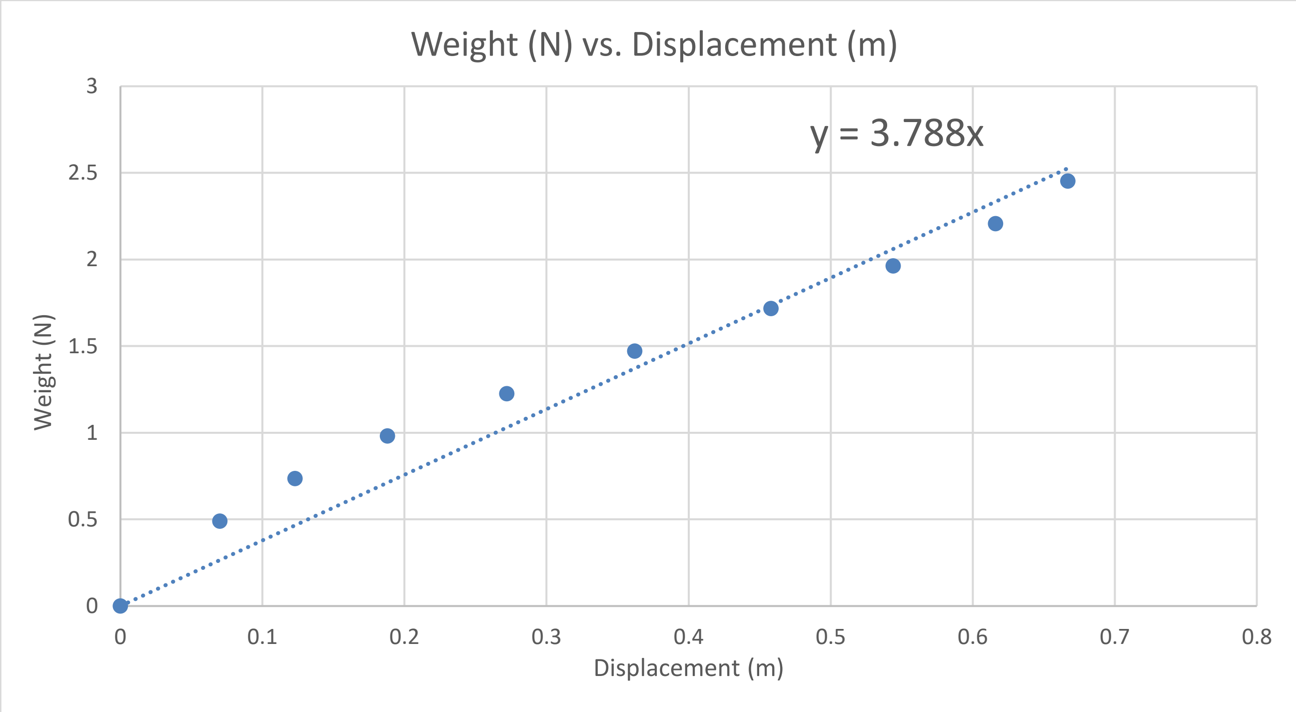


$F_{spring}=5.502(x_o-x_L)$

Use *Excel* regression analysis on any graph that has a **linear** fit only (see EG), to obtain:

uncertainty for slope= ± 0.300 N/m   % uncert= 5%

Graph 2: The regression of the Weight (N) vs. the Displacement (m) from Table 2. The slope, 3.788 (±0.100) N/m is the spring constant, k, of the elastic band at that unstretched length.

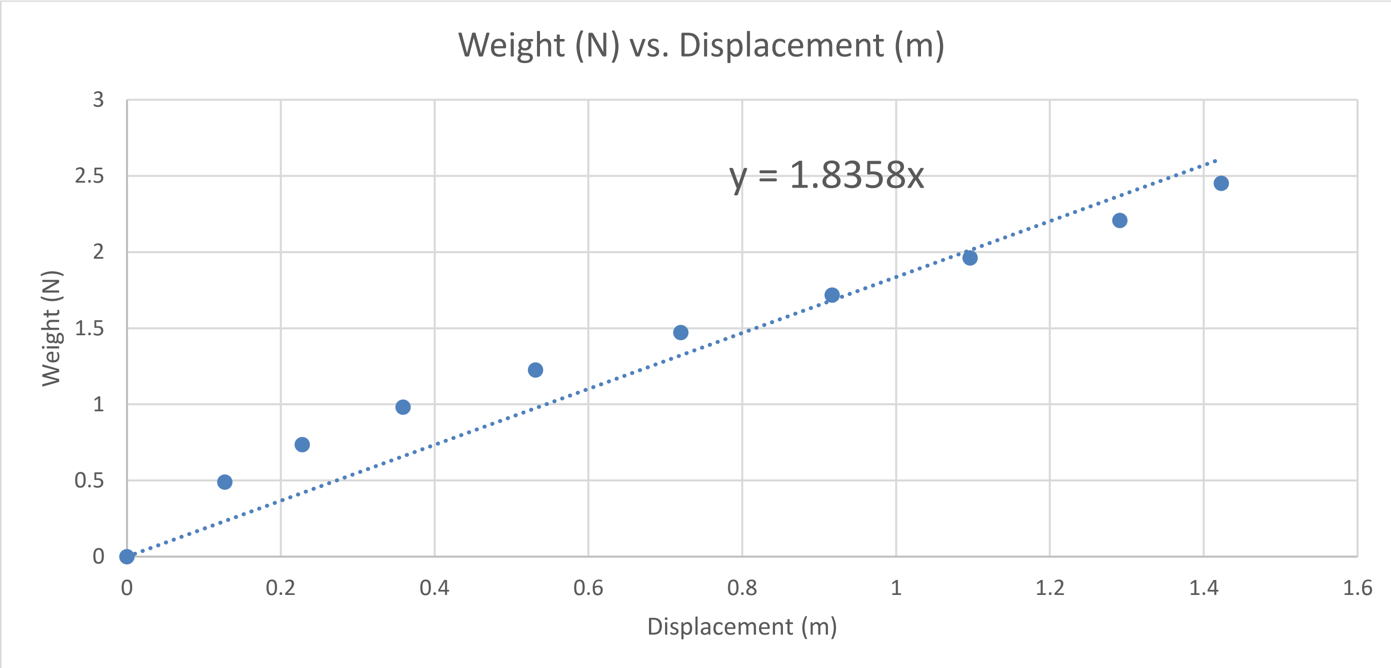


$F_{spring}=3.788(x_o-x_L)$

Use *Excel* regression analysis on any graph that has a **linear** fit only (see EG), to obtain:

uncertainty for slope= ± 0.100 N/m % uncert= 3%

Graph 3: The regression of the Weight (N) vs. the Displacement (m) from Table 3. The slope, 1.836 (±0.080) N/m is the spring constant, k, of the elastic band at that unstretched length.



$F_{spring}=1.835(x_o-x_L)$

Use *Excel* regression analysis on any graph that has a **linear** fit only (see EG), to obtain:

uncertainty for slope= ± 0.080 N/m % uncert= 4%

Identify experimental value(s) of interest, why it is of interest, and how/from where obtained, briefly:

The slope of each graph is of interest. This value is the spring constant, k, of the elastic band. This value was obtained by hanging different masses off the band and measuring the new equilibrium point. We then ran a regression of each set of data, by plotting the weight (N) vs. displacement (m), to obtain the k constant for each initial cord length with no hanging mass.

value obtained =

From Table 1= 5.502 (±0.300) N/m
From Table 2= 3.788 (±0.100) N/m
From Table 3= 1.836 (±0.080) N/m

uncertainty of experimental value(s) =

From Table 1= (±0.300) N/m
From Table 2= (±0.100) N/m
From Table 3= (±0.080) N/m

% uncert=

From Table 1= 5%
From Table 2= 3%
From Table 3= 4%

This uncertainty was obtained through a Data Analysis on Microsoft Excel.

Add any other pertinent info for the reader (who may not have done this experiment) to follow along:

We decided to use the regression that ran through the origin for two reasons. The first is that ideally, if there is no displacement, there should be no F_{spring}. Also since the graph is pinned the origin, there is less uncertainity in our results. We believe that groups that did not set the regression line through the origin are neglecting these two major ideas.

Summarize Results (just the facts)—give the important, relevant results, and why/how they are relevant to the purpose, in a sentence or two, including main equations and quantitative results:

The slope of each linear regression is the spring constant, k. These regressions demonstrate that the relationship between the force and the displacement of the cord is linear, which approximately follow Hooke's law. This is relevant to the purpose since we could see that the cord does follow Hooke's law and we could obtain spring constants for various initial lengths.

DISCUSSION: *What do you make of your results? Evaluate them.*

We are confident in our spring constants since the uncertainty we obtained from our excel data analysis is low for each trial. The spring constant with the uncertainty for trial 1 is 5.502 (±5%) N/m. The spring constant with the uncertainty for trial 2 is 3.788 (±3) N/m. The spring constant with the uncertainty for trial 3 is 1.836 (±4) N/m. Since the percentage uncertainty is low for our trials, we predict that our spring constants are accurate. Therefore, to obtain a percentage error in our model we took a mass of 0.2 kg and used the measured displacement for each trial. Then we took each displacement and multiplied it by the appropriate spring constant for that trial. This calculation gives the restoring force of the spring. In theory, this must be equal to the weight of the mass that we hung on the end of the cord.

Table 4: Theoretical Calculations for each of our cord lengths

Trial 1: $mg=k(x_o-x_L)$ mg=((5.502 (N/m))(0.366 (m)) mg= 2.013 N	Trial 2: $mg=k(x_o-x_L)$ mg=(3.788 (N/m))(0.544 (m)) mg= 2.061 N	Trial 3: $mg=k(x_o-x_L)$ mg= (1.836 (N/m)(1.096 (m)) mg= 2.012 N

If the calculated spring constant is the actual spring constant, then this should have been the weight applied to the cord.

Uncertainty vs. error, or % uncertainty vs. % error, for each value of interest from Results section:

value obtained for spring constant =

From Trial 1= 5.502 (±0.300) N/m
From Trial 2= 3.788 (±0.100) N/m
From Trial 3= 1.836 (±0.080) N/m

uncertainty of experimental value(s) =

From Trial 1= (±0.300) N/m
From Trial 2= (±0.100) N/m
From Trial 3= (±0.080) N/m

% uncert=

From Trial 1= 5%
From Trial 2= 3%
From Trial 3= 4%

For Trial 1 at a displacement of 0.366 m

$$\frac{2.013\ N - 1.962\ N}{1.962\ N} \times 100 = \%error$$

= 3%

For Trial 2 at a displacement of 0.544 m

$$\frac{2.061\ N - 1.962\ N}{1.962\ N} \times 100 = \%error$$

=5%

For Trial 3 at a displacement of 1.096 m

$$\frac{2.012\ N - 1.962\ N}{1.962} \times 100 = \%error$$

=3%

Sources of uncertainty:

A source of uncertainty in our calculations is the way that the knot was tied where our hanging mass was hanging from. The cord would stretch more as the masses got bigger, meaning that the displacement calculation was off by a small factor of about 0.001 m, which is accounted for in our uncertainty. We believe that our lack of sufficient trials, of using different cord lengths, results within a conclusion that needs more evidence. The percent error that we obtained in our trials, from the difference of the calculated force and actual force is all within the percentage uncertainty of our spring constant.

Any further observations or extenuating circumstances that aid in interpretation or evaluation:

We began to form a ratio between the spring constant and the initial stretch without a hanging mass. We multiplied the k constant by the unstretched length to obtain a ratio. We find our results interesting:

Trial 1: (5.502)(.230) = 1.265552
Trial 2: (3.788)(.316) = 1.19008
Trial 3: (1.836)(.654) = 1.20061
We notice that all the ratios are close to a value of 1.20. We believe this ratio affirms that the cord follows a model of Hooke's law since there is some correlation to the unstrechted distance, which decides the spring constant, with the spring constant itself. Nonetheless, we would like more trials to affirm our beliefs.

In a couple sentences**, describe whether your main results support your hypothesis.**

We believe that our main results support our hypothesis, meaning the cord follows a model of Hooke's law, since our percentage error is within our percentage uncertainty, where the uncertainty is small enough, at under 5% for all our trials, that we can assume it will not affect the linear nature of the graph significantly. This means that the linear regression of our data is accurate, thus a linear model of Hooke's law can be followed.

CONCLUSION:

Through the experimental values the purpose of the lab is fulfilled since the elastic cord follows a model of Hooke's law. We found the spring constant, k, for three different cord lengths where k=5.502 (±0.300) N/m, for trial 2 k= 3.788 (±0.100) N/m and for trial 3 k= 1.836 (±0.080) N/m. Since our uncertainty is small enough to not affect the linear nature of our regression, the linear model of our regression is the best fit, thus the cord follows a linear model of Hooke's law. The purpose is met since spring constants were calculated as well as affirming the cord follows Hooke's law.

Implications of the conclusion:

We can calculate the spring constant for any length, since we know that the force will be equal to the weight of the mass hung from the end of the cord. In the greater Bungee project, this will be the weight of the egg. Since the cord follows a linear model of Hooke's law we will be able to calculate the spring constant of the cord for the un-stretched length that will be used. The spring constant will allow us to find ω and the position equation, along with velocity and acceleration, of the egg.