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How the k Constant of the Elastic Bungee Cord Changes with Length

ABSTRACT:

Knowing the behavior of the elastic bungee cord is essential in order to predict how it will act during the final egg drop. In order to learn more about the elastic cord and subsequently be able to model its behavior, the purpose of this experiment was to determine how the *k* constant of elastic proportionality changed as the length of the cord itself changed. To find this *k* constant, the dynamic case of the cord was observed as the cord was in motion. Three different masses were dropped from five different lengths, and the behavior of the elastic cord was modeled through use of the Conservation of Work Energy (CWE) theorem. The relationship between the *k* value and longer length was shown to be decreasing and nonlinear, but the relationship between the inverse *k* value and length was linear. The coefficient of the slope for *k* was (.705), which is then multiplied by the length of cord to find the *k* value. This means that longer lengths of cord yield lower *k* values, and are therefore more "stretchy." This information will be critical in being able to successfully and safely drop the egg later, as it tells us how the elastic cord will behave at longer lengths.

INTRODUCTION:

Purpose: The purpose of the experiment was to understand how the *k* constant of proportionality for a bungee cord would change when the length of the cord changed.
Specific Question: How will the *k* constant change as the length of the bungee cord changes?

The overarching goal is to determine how the elastic cord behaves under different conditions (varying mass, varying height, etc.). One of the best ways to analyze the behavior of the elastic bungee cord is to model its energy throughout its entire fall. The conservation of work and energy (CWE) theorem was the fundamental formula used throughout this experiment, which was applied as follows:

$$PE_{top} + KE_{top} = PE_{bottom} + KE_{bottom}$$

(We consider the bungee cord system to be isolated. This approximation means that there are not any major external forces acting on the system, such as air resistance or friction.)
Furthermore, the CWE theorem can be broken down at key points during the fall:
At the instant that the mass is released, all the energy in the system is potential energy. Potential energy at the point of release is described as follows:

$$PE_{top} = mgh$$

Where ***m*** is the mass on the elastic bungee cord (the mass of the cord itself is negligible)
g is the acceleration due to gravity (9.8 m/s²)
h is the height of the fall (which is equivalent to the maximum distance the bungee cord stretches (x_{max}))

At the time during which the mass is at the very bottom of its descent, or when the mass stretches the elastic cord to (x_{max}), all of the energy now is transferred to Kinetic Energy in the elastic cord. The elastic cord itself is very close to behaving like a spring, and the Kinetic Energy of a spring is described as follows:

$$KE_{bottom} = \frac{1}{2}kx^2$$

Where ***k*** is the spring constant of the elastic cord at that length.

> (Note, the spring constant *k* will vary based on the length of cord. Also, it is important to know that the elastic bungee cord does not always behave entirely like a spring, which was confirmed in our last Lab Report Outline, but its behavior during this period of the fall is very similar to, and can be modeled as, a spring.)
>
> ***x*** is the distance over which the elastic cord acts on the mass ($x_{max} - x_{unstretched}$)
>
> > The bungee cord does not act on the mass during the time of the fall when it is still unstrethced. Once the elastic cord begins to stretch however, it then begins acting on the mass. This length of elongation then is what we call *x*.

The total mechanical energy in a system is the sum of its Potential Energy and Kinetic Energy. During the fall, the Potential Energy is transferred into Kinetic Energy, but the total energy in the system is always constant and conserved (CWE) theorem. Knowing this, the total Potential Energy at the top must be equivalent to the total Kinetic Energy at the bottom:

$$mgh = \frac{1}{2}kx^2$$

Since we know the mass (*m*), gravitation acceleration (*g*), maximum distance (*h*), and the length of elongation of the elastic cord (*x*), we can rearrange and solve for the spring constant *k* at each length of elastic cord:

$$k = \frac{2mgh}{x^2}$$

So, the primary characteristic that we are seeking to find in this experiment is how the *k* constant changes over different lengths.

Hypothesis: Prior to the experiment, my partner and I predicted a few things.
We predicted that longer lengths of elastic cord, with constant mass, would yield longer elongations and displacement values - thus altering the *k* constant. (Mathematically speaking, the elongation value (*x*) is squared in the above formula, whereas the height (*h*) is not. Therefore, the elongation value (*x*) has a much more significant effect on the *k* value.)
As for how the *k* spring constant itself changed with different lengths, we predicted that it would not change linearly. Rather, we predicted that the *k* constant would increase substantially over smaller lengths, but would gradually approach a threshold at which it leveled off.

METHODS:

In this particular dynamic experiment, a mass was attached to the end of the elastic bungee cord and released from the same initial height every time. The maximum distance that the bungee cord stretched for each trial was measured and recorded by slow-motion camera on an IPhone 6+.
The same three different masses (50 grams, 75 grams, and 100 grams) were dropped from five different lengths (17.3cm, 27.4cm, 43.0cm, 62.3cm, and 91.8cm).

Side Note: This experiment served a dual purpose, of sorts, in that it allows us to look at how constant mass affects elongation at different lengths, and also how the varying lengths themselves affect the subsequent total elongation. All of this information is potentially very useful and will help us find a *k* constant of proportionality later. However, in this specific experiment, we only looked at the *k* value for each length.

Procedure:
The following procedure was performed in order to obtain the best possible results while still maintaining consistency:

- Set the ring stand firmly on the edge of the lab table by tightening the C-clamp.
- Attach the tape measure to the arm of the ring stand hanging over the table. Pull the tape measure down to the floor and make sure it remains as vertical as possible.
- Tie a knot in the elastic bungee cord. This is what the hanging mass will be attached to.
    - As for where to tie the knot on the elastic cord itself, my partner and I just chose distances of appreciable difference.
- Attach the elastic cord alongside the tape measure on the arm of the ring stand.
- With the elastic cord now dangling, use the tape measure to find how long it is.
    - This is the unstretched length of elastic cord (x_0).
- The hook of the hanger is then attached through the knot in the bungee cord.
    - The mass of the hanger was 50 grams, so 25 grams and 50 grams were added to make total masses of 75 grams and 100 grams, respectively.
- The hanging mass was then raised to be equal to the beginning of the tape measure at the arm of the ring stand.
- The mass was released and the bungee cord subsequently stretched a maximum distance while falling.
    - The maximum distance was carefully videotape and recorded through use of the Slow-Motion camera on an IPhone 6+. The lowest point is the maximum distance (x_{max}).
- The rebounding mass was caught as quickly as possible after the fall so as to not compromise the elasticity of the cord unnecessarily.
- The Slow-Motion video was analyzed. The tape measure served as the marker, and the point of the knot in the bungee cord was always used for measuring purposes.
- Data was recorded in Excel. Masses of 50 grams, 75 grams, and 100 grams were all used for lengths of 17.3 cm, 27.4 cm, 43.0 cm, 62.3 cm, and 91.8 cm.
    - My partner and I wanted to collect a wide range of data, so we made sure to vary the masses and lengths substantially.

My partner and I did our best to remain consistent throughout the entire experiment. For example, the person releasing the mass stayed the same, and the person measuring and recording also stayed the same.

Figure 1: ***A Diagram of the Bungee Cord at Different Stages.*** Three different bungee cords were not all hung at once, rather, the diagram shows the same bungee cord at different stages. The following letters correspond to the following components: **A:** The lab table **B:** The ring stand attached to the edge of the lab table by a C-clamp. The arm of the ring stand was where the elastic cord was hung from **C:** The unstretched elastic cord (x_0) **D:** The equilibrium point of the elastic cord when a mass was gently attached and allowed to hang freely. This equilibrium point is not of use in this particular analysis, but is potentially useful in other applications **E:** The elastic cord after the mass was dropped, as shown by the large stretch **F:** The tape measure **G:** The floor

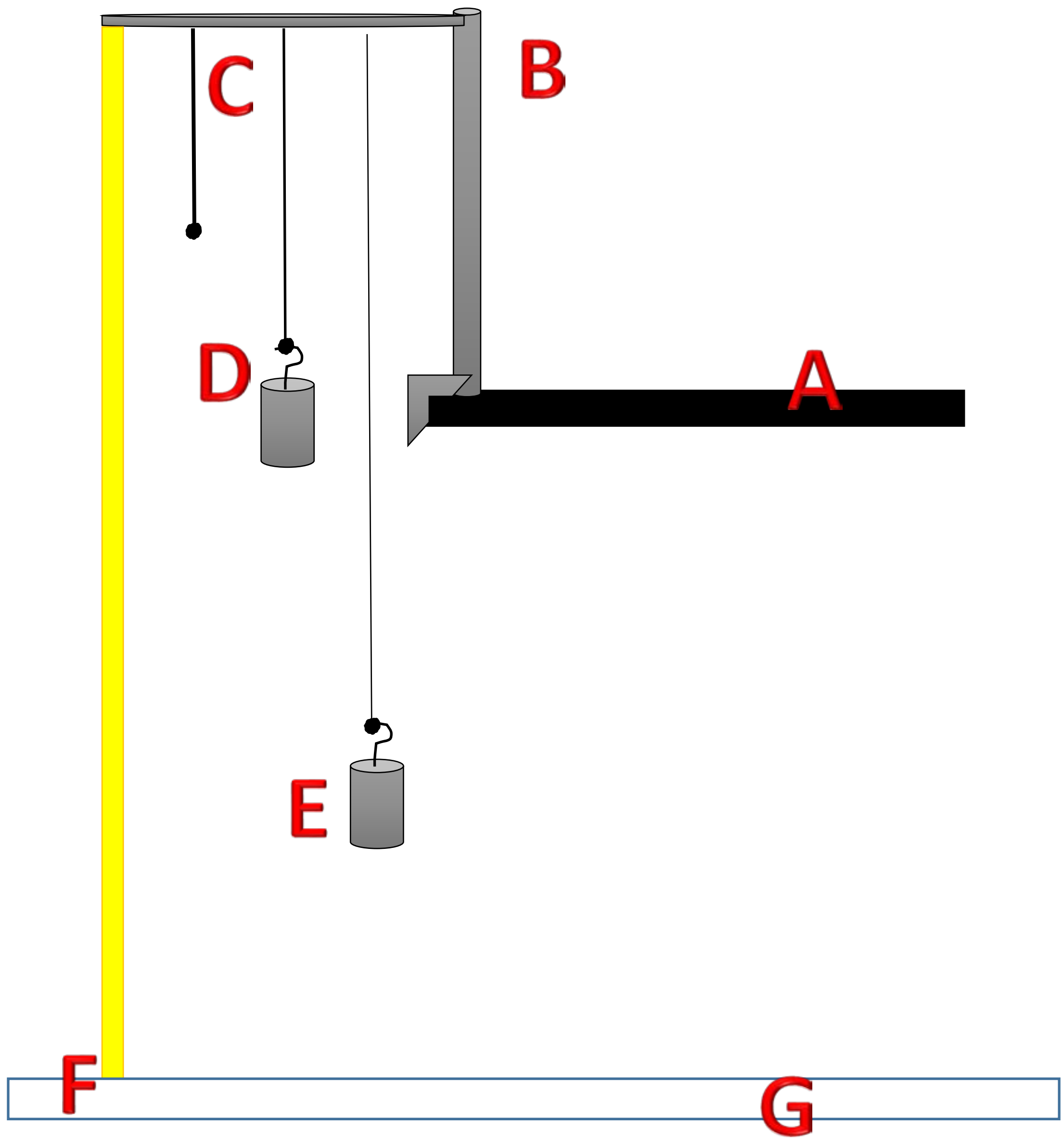
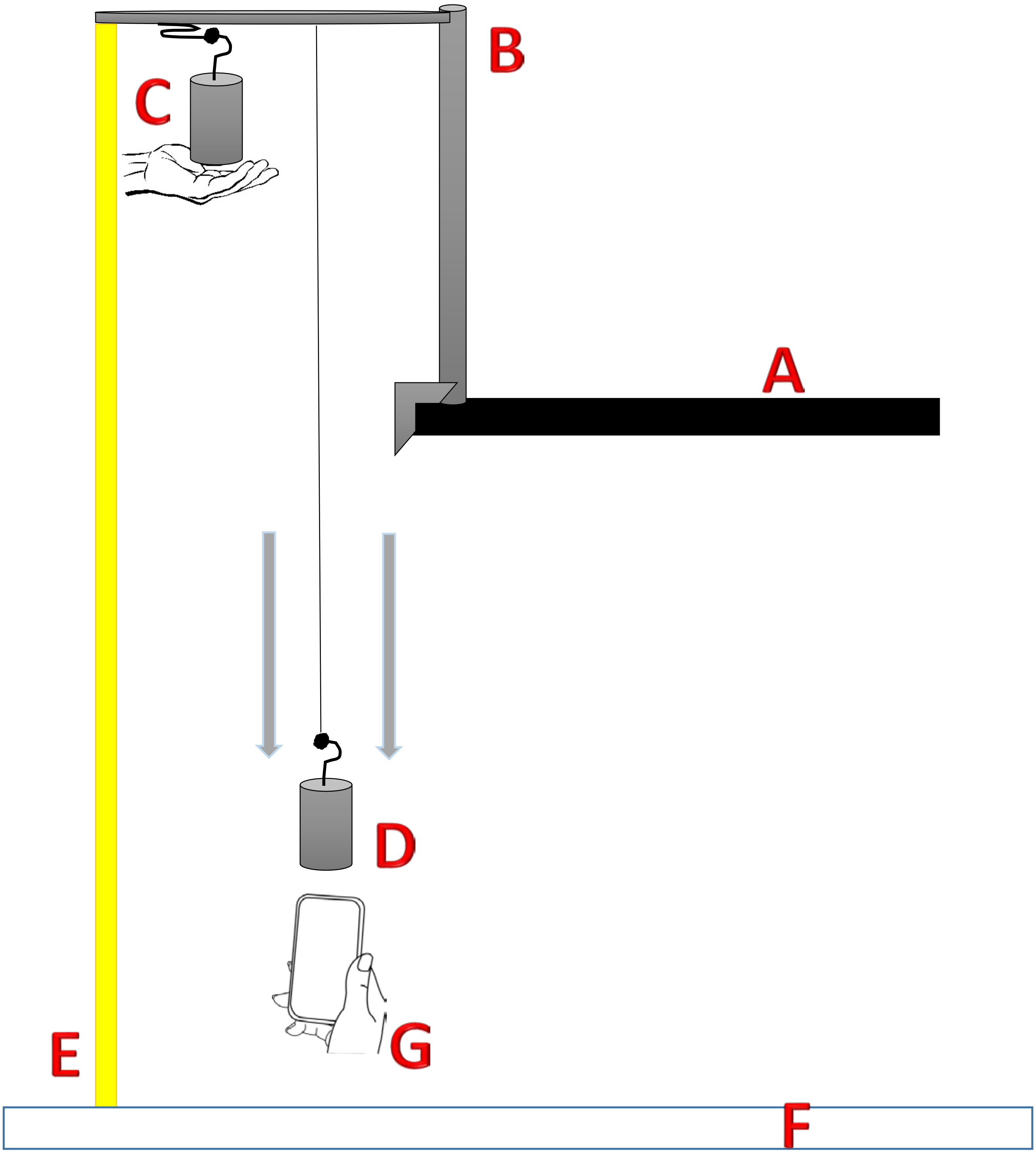


Figure 2: ***Diagram of the Setup.*** This illustration depicts a diagram of the setup used during the experiment. The following letters correspond to the following components **A:** The lab table **B:** The ring stand attached to the edge of the lab table by a C-clamp. The arm of the ring stand was where the elastic cord was hung from **C:** The system before being dropped. The mass was raised to this same initial height every time **D:** The system after being dropped. The elastic bungee cord has stretched and the mass has fallen to its maximum distance **E:** The tape measure **F:** The floor **G:** The IPhone that was used to vieotape and record the maximum distance. The fallen distance was measured by looking over to the nearby tape measure.



RESULTS:

The results showed that longer lengths of cord yielded lower *k* constant values, and vice versa. Utilizing the Potential and Kinetic Energy equation, as stated above in the Introduction above, *k* was found as follows:

$$k = \frac{2mgh}{x^2}$$

Keep in mind that a large *k* constant implies greater "stiffness," or a higher resistance to stretch.

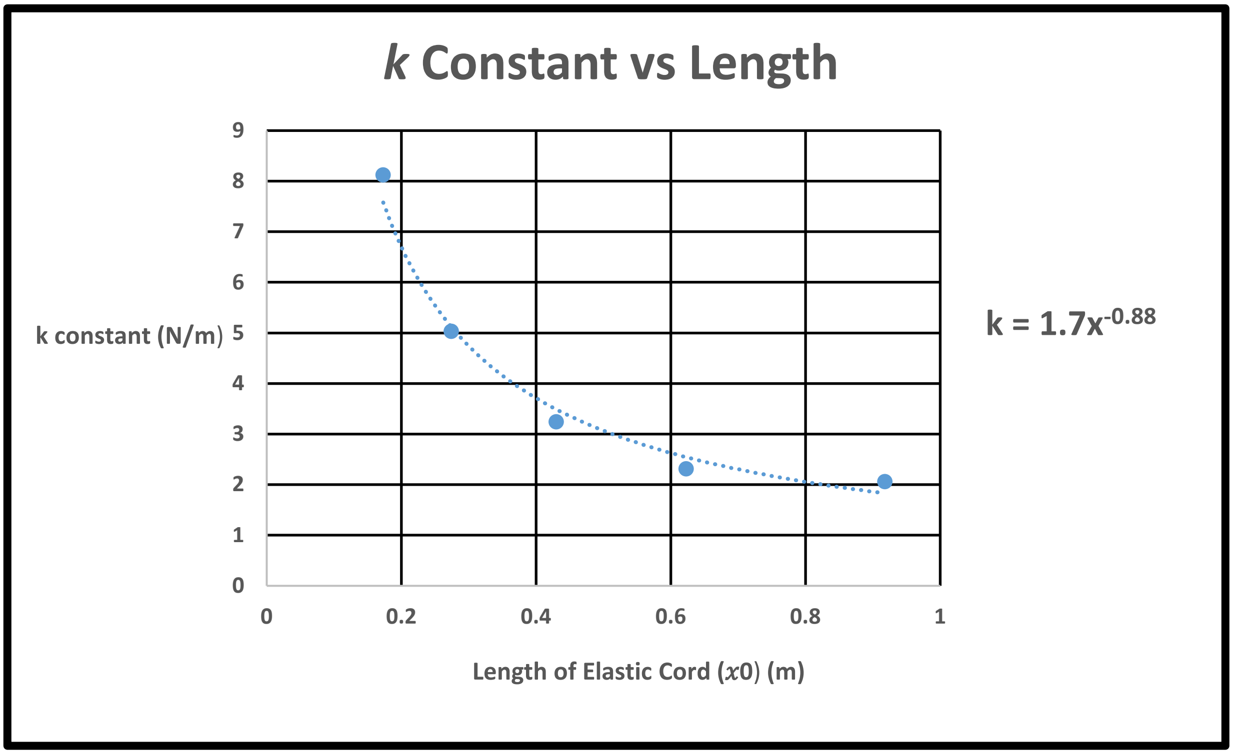
Table 1: ***Data Table of Relevant Data Collected and Calculated.*** The *k* constant was calculated using the above derived formula, and the rest of the data was recorded during the experiment.

Mass (***m***) (kg) ± .01	Unstretched Length (x_0) (m) ± .001	***X*** amount of stretch (m) ± .006	***k*** constant (***k***) (N/m) ± .05
0.05	0.173	0.181	10.60
0.075	0.173	0.27	8.94
0.1	0.173	0.358	8.13
0.05	0.274	0.297	6.35
0.075	0.274	0.435	5.51
0.1	0.274	0.575	5.01
0.05	0.43	0.441	4.39
0.075	0.43	0.688	3.48
0.1	0.43	0.895	3.25
0.05	0.623	0.649	2.96
0.075	0.623	0.953	2.55
0.1	0.623	1.268	2.31
0.05	0.918	0.94	2.06
0.075	0.918	N/A	N/A
0.1	0.918	N/A	N/A

This table may seem very large, and arguably unnecessary, but it has been included to display the data collected as a whole. It is also very helpful in finding progressions, and is not very complicated when looked at closely. The masses in the far left column are the same three masses, and there are only five different lengths to consider.

It is also of interest to note that at the longest length of .918m, the heavier two masses hit the floor and the distance was thus not able to be definitively found.

Figure 3: ***Graph of the k constant as length changes but mass is constant.*** The calculated *k* value was simply plotted against its respective length of cord and the relationship is shown.



The relationship is certainly not linear, which is evident by the curve. This indicates that as length gets longer the *k* constant decreases, but not by the same rate.

Also, for simplicity sake, this graph is the one for the constant mass of 100 grams. The other two graphs for the other two masses (50 grams, 75 grams) are very similar in relationship, but not included.

The relationship between the *k* constant and the length of the cord has been found:

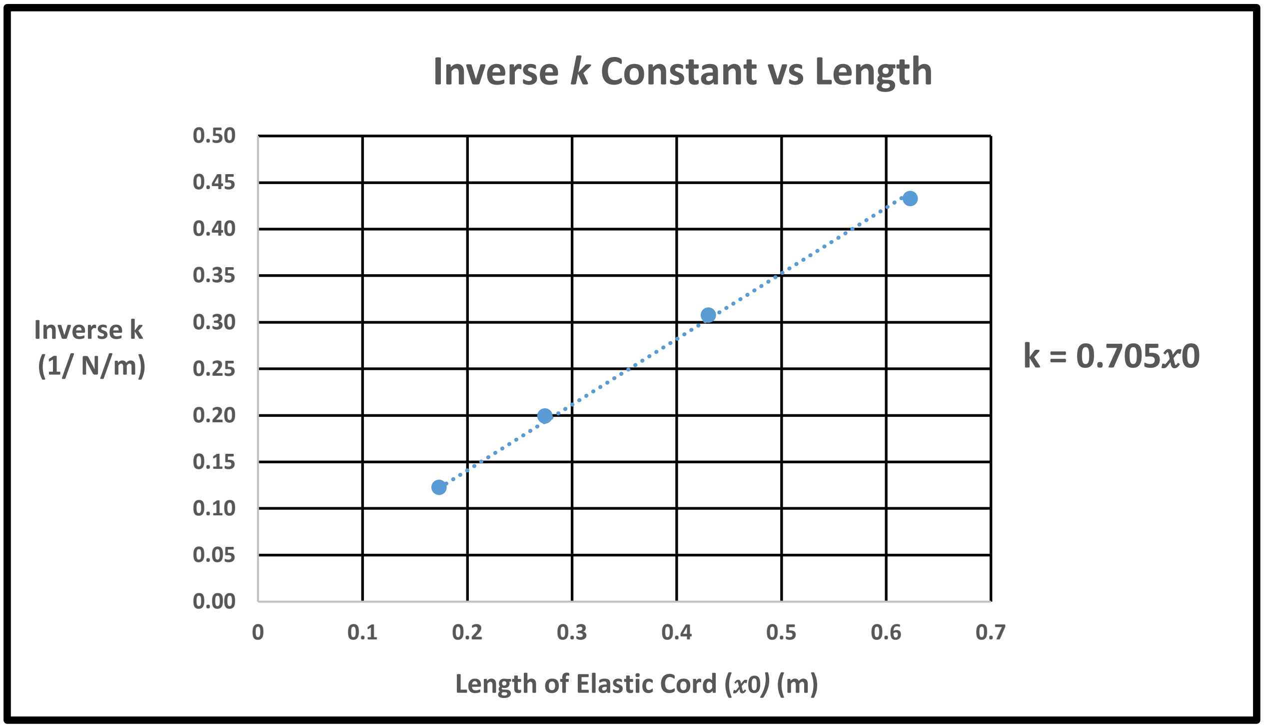
$$\boldsymbol{K = 1.7x_0^{-.88}}$$

Uncertainty for Slope= .05 Percent Uncertainty = 3%

Uncertainty for y-intercept= N/A Percent Uncertainty = N/A

The uncertainty for the coefficient of the slope was found by using the propagation of uncertainty method. A y-intercept was not found, and therefore its uncertainty cannot be found either.

Figure 4: ***Graph of Inverse k vs. Length.*** The inverse of the *k* constant was found and then plotted at its respective lengths.



The inverse *k* value was determined by taking the *k* value in the far right column of Table 1 and finding its inverse.

This graph now displays a linear relationship between the inverse of the *k* value and the length of the cord x_0:

$$K = .705x_0$$

Uncertainty for Slope= .007 Percent Uncertainty= 1%

Since this line is indeed linear, the Regression Analysis on Excel could be used to find the uncertainty.

Experimental Value of Interest:

The whole purpose of this experiment was to find how the *k* constant changed over different lengths, and the linear slope of the inverse line in Graph 2 is very important in displaying that relationship. The coefficient shows that relationship:

Value Obtained = .705

Uncertainty of Experimental Value = .007 Percent Uncertainty = 1%

Again, the Regression Analysis on Excel was able to determine the uncertainty here. It should also be emphasized that the .705 coefficient is the relationship between the *inverse k* value and the length of cord, so ensuing calculations must be made with this in mind.

Taking the CWE theorem and rearranging the Potential Energy and Kinetic Energy formulas gave the equation to find the *k* constant at each length. The *k* constant at each length was then plotted against the lengths of the cord, and a decreasing relationship was found. The inverse of the *k* value was plotted against the lengths, which revealed a linear relationship there. This relationship is .705.

DISCUSSION:

This experiment was more inquiry and discovery-based rather than attempting to show or confirm a principle or theory. Thus, the experimental nature of this experiment means that there are not any definitive known values to compare with.

Looking at the uncertainties, it can be seen that they are relatively low. There was not any error greater than 3%, which indicates sound results. (We shall see how sound these results actually are when the egg is at stake!)

One very intuitive, and frankly quite simple, test to determine acceptability of these results would be to predict a certain height that the egg would be dropped at. Knowing the *k* constant of the bungee cord and its attached mass would make it very simple to predict the maximum distance that it should fall to. Carrying out this experiment and seeing how close the predicted distance was to the actual distance would show how good the results are.

Sources of uncertainty were certainly not absent, and could have manifested as follows:

- Uncertainty in measuring the maximum distance the mass traveled. Even with Slow-Motion cameras and dogged scrutiny, the reported distances are certainly not perfect.
    - To combat this, the same person recorded and watched the videos to ensure that at least the same method of measurement was used.
- Uncertainty when recording other measurements, such as how long the unstretched cord length was. Also, uncertainty in dropping the mass uniformly.
    - To combat this, the same person tied the bungee knots and released the mass to ensure consistency.
- Uncertainty in the elastic bungee cord itself. Due to the elastic nature of the bungee cord, it is possible that the cord was stretched or that the elasticity changed over the course of the experiment.
    - To combat this, my partner and I switched the ends that we tied the knots from and also stretched the elastic cord before official use.

Referring back to the hypothesis, my partner and I were correct in that longer lengths of cord would yield longer distances. This only makes sense, as there is more elastic cord to be stretched.
My partner and I were incorrect, however, in predicting how the *k* constant would quantitatively change over time. We did correctly predict that the relationship would be nonlinear, but we were mistaken in that longer cords have lower *k* values whereas shorter cords have much higher *k* values.

CONCLUSION:

The primary purpose of the experiment was to gather a wide range of data about how the elastic bungee cord behaves, and one of the best indicators of behavior is the value of its *k* constant. The *k* constant will prove to be vital later during the final egg drop, as it will help my partner and I determine how much resistance the bungee cord will provide.
The results show that the *k* constant is very high for short lengths, but decreases significantly as length becomes longer. Additionally, this relationship is not linear. The *k* value initially decreases rapidly as length increases slightly, but the rate slows and begins to level off at longer lengths. However, the relationship between the inverse *k* constant value and the length of the cord is linear! This makes calculations later much more simple and provides a sense of how the *k* value responds to different lengths.

Implications:
My partner and I do not want to give too much information on our final design, but there are a few things that will now be considered after looking at these results.

- We will most likely use more than just one single strand, since a greater number of strands would lead to less abrupt force.
- We will most likely integrate static string into the design. We performed an additional side experiment about how static string affects force exerted and stretch of the elastic cord.

This data, along with all the other data obtained from this experiment, will put us on the correct path to finding the correct formula and procedure for keeping our egg safe from harm.
The relationship that we found with the inverse *k* constant and length of the cord will help us greatly in the final egg drop by rearranging and working backwards using the formula:

$$k = \frac{2mgh}{x^2}$$

Which is equal to:

$$x = \sqrt{\frac{2mgh}{k}}$$

We will know the mass of the egg, gravitational force (constant 9.81), and height of the drop in the Great Hall.
The formula $k = 1.7x^{-.88}$ will be used to find our *k* constant,
By combining all this information, we will be able to determine the amount of stretch our egg will endure.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Joseph Jast