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Section: 06

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TITLE: Bungee Cord Characteristics: How the Spring Constant Changes with Length

ABSTRACT: Bungee cords that stretch and retract behave similarly to springs. They possess some spring constant k, and this constant changes depending on the length of the bungee cord. The purpose of this experiment was to create a model that accurately predicts the spring constant k with any given length of the bungee cord. In order to determine this model, several weights were placed at the end of a bungee cord from several different lengths. The length of the resulting displacement from each weight was then modelled on a force vs. displacement graph, where the slope produced the constant k at that length. When multiple values for k were calculated, they were all graphed versus the corresponding unstretched bungee length at each value, and the resulting graph modelled how the spring constant varied with the length of the bungee. With the equation of this graph, any bungee length could be inputted as x, and the resulting y value would be the estimated spring constant k of the cord at that length. This information is critical in later determining the best way to drop an egg on the same bungee cord without the egg breaking by impacting with the ground.

INTRODUCTION*:* Bungee cords behave differently depending on material and length of the bungee cord. In this experiment, we are concerned with finding more information about how our particular bungee cord moves in different circumstances. The most notable quality of the bungee would be its spring constant, k. Since the material of our bungee cord is known and constant, we will focus on how changing the length of our bungee cord impacts the spring constant. With the k in mind, it becomes possible to predict the bungee's displacement when any given mass is attached to it. For the purposes of the egg drop experiment, it is first essential that this spring constant k is found. With the spring constant, we can accurately predict how close the bungee will get to the ground, and prevent the egg from ever making contact with the ground and breaking. K changes depending on bungee length, so the end goal of this experiment is to find some way to model these two variables.

The critical equation of this lab report is $F=kx$, where F is the force applied to a spring, k is the spring constant unique to the spring, and x is the resulting displacement. Since we are dealing with bungee cords, everything is in the vertical direction, and the F for the most part refers to the weight of whatever object is attached, and the x is the resulting vertical displacement.

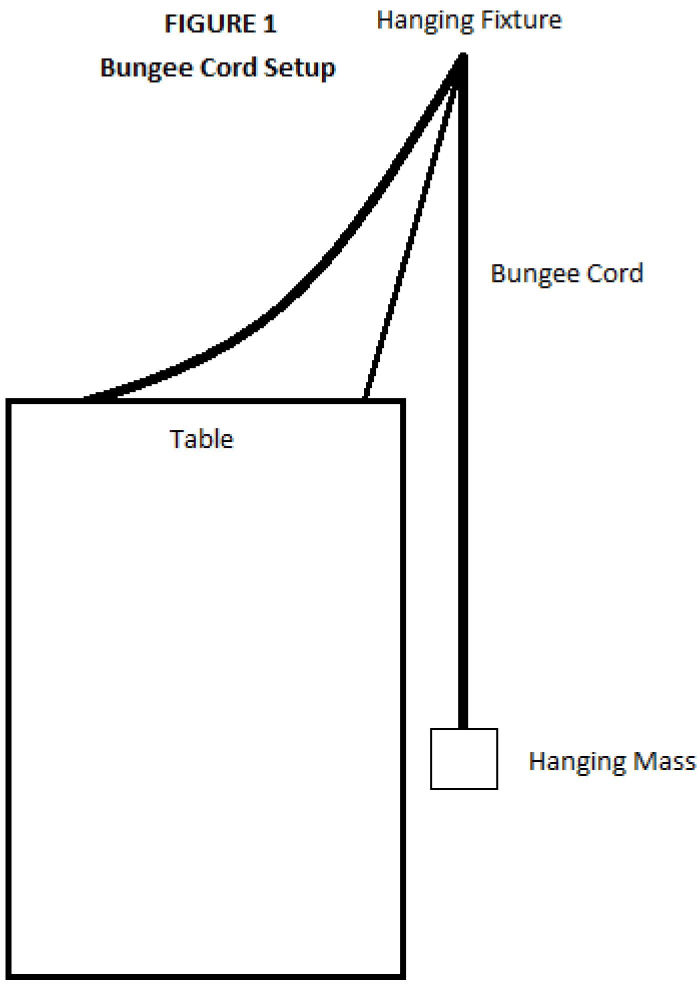
If we observe how the bungee reacts to several different weights at a particular length, we can find the spring constant k by graphing the two variables, F (weight) and x (displacement). The spring constant will be the slope of these graphs. If we repeat this process several times, and find multiple values for k, then we can make another graph that illustrates the spring constant k versus the length of the unstretched bungee used to determine each corresponding constant. The equation of this graph is what we need.

Though we do not know precisely what the end result will be, we predict that the spring constant k will likely be much larger for smaller lengths and smaller for longer bungee cords.

METHODS: The setup requires the bungee cord to be completely vertical, so that the weight of an attached mass is the only thing affecting its movement. The bungee remains stable via a special loop that has negligible effect on the bungee's performance. An identical loop was placed at the bottom of the bungee to hang masses from. A diagram of the setup can be found in Figure 1 below:

FIGURE 1

Bungee Cord Setup



At different lengths, the cord either gets longer or shorter. There is enough slack coming from the other end of the bungee and affixed onto the table that it should not interfere with the experiment.

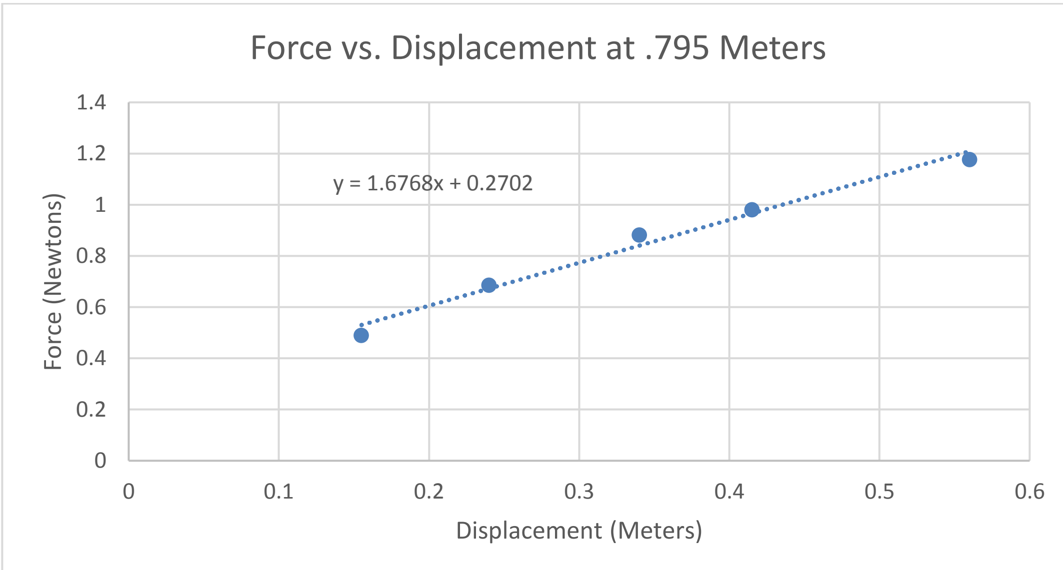
Using the above setup, we proceeded as follows:

-The unstretched length of the bungee was measured and recorded.
-A mass was placed on the end of the bungee and the resulting displacement was measured by subtracting the old length from the new length. To ensure consistency, the bungee was measured only when it was completely still and in its new resting position.
-This was repeated for five different weights. The resulting weight versus displacement data was graphed, and the slope of this graph was recorded. This is the spring constant for that particular length.
-The bungee was then shortened to a new length and the above was repeated again for the new length. This process continued for six unique lengths, and on each graph the slope was recorded.
-The resulting six slopes provide six spring constants for the given bungee at six different lengths. With this in mind, a new graph was produced that modelled the spring constant k versus the bungee's length at each position.
-The equation of this graph provided a model for bungee length and spring constant.

RESULTS: The results of these tests are presented below. First, the resulting table of weight applied and displacement, followed by a graph of each table. The uncertainties for each graph were found using propagation of uncertainty tools available under Excel's data analysis tool.

Test 1: Spring Length at .795 Meters

Weight (N)	Displ. (m)
0.4905	0.155
0.6867	0.24
0.8829	0.34
0.981	0.415
1.1772	0.56



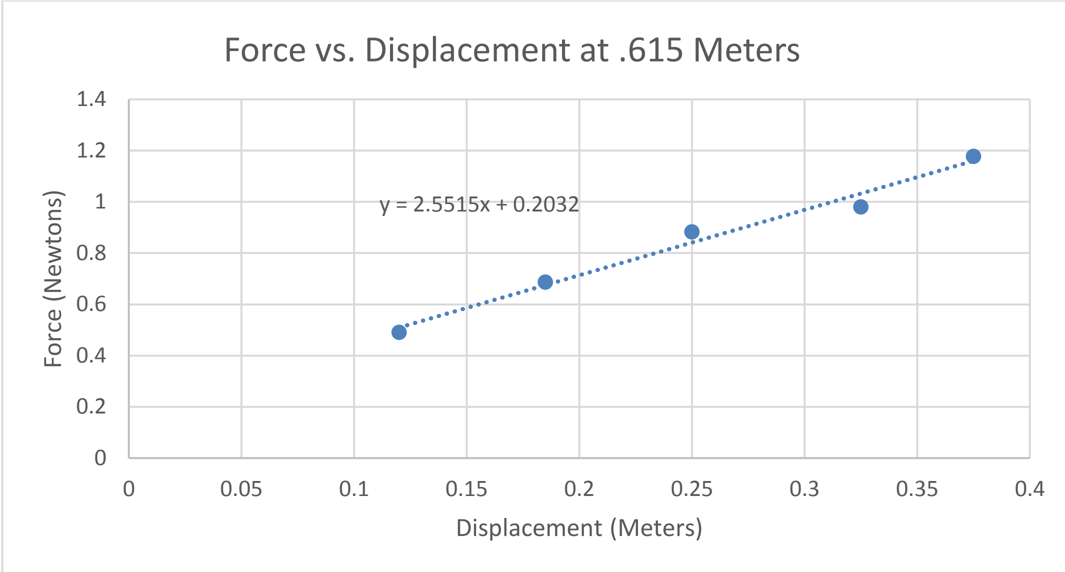
Equation of line: F = 1.6768x + .2702

Uncertainty for slope= .128 % uncert= 7.6%

Uncertainty for y-intercept= .047 % uncert= 17.4%

Test 2: Length at .615 Meters

Weight (N)	Displ. (m)
0.4905	0.12
0.6867	0.185
0.8829	0.25
0.981	0.325
1.1772	0.375



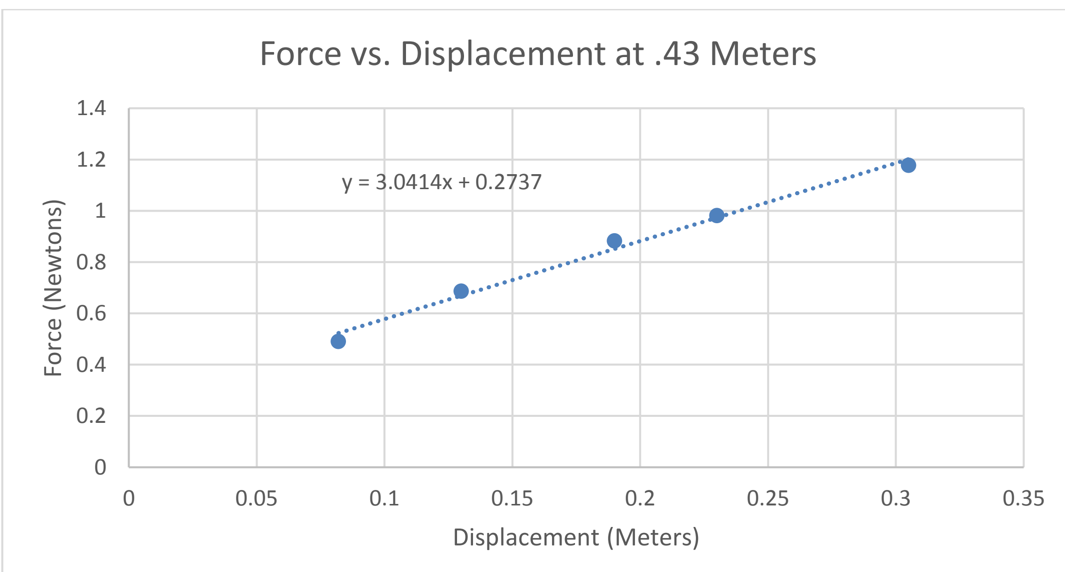
Equation of line: F = 2.5515x + .2032

Uncertainty for slope= .202 % uncert.= 7.9%

Uncertainty for y-intercept= .054 % uncert.= 26.57%

Test 3: Length at .43 Meters

Weight (N)	Displ. (m)
0.4905	0.082
0.6867	0.13
0.8829	0.19
0.981	0.23
1.1772	0.305



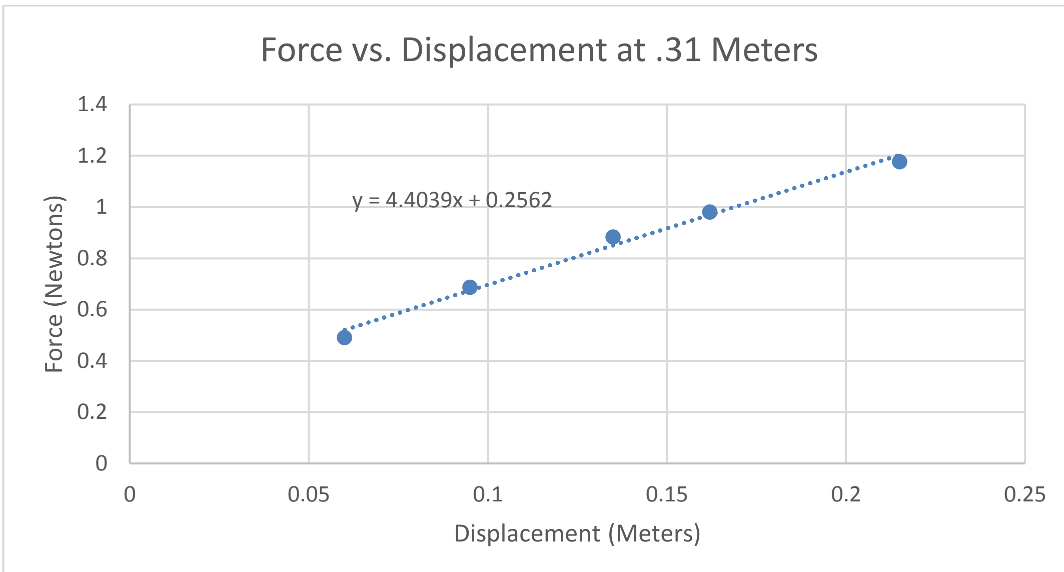
Equation of line: F = 3.0414x + .2737

Uncertainty of slope= .182 % uncert.= 6.0%

Uncertainty of y-intercept= .037 % uncert.= 13.52%

Test 4: Length at .31 Meters

Weight (N)	Displ. (m)
0.4905	0.06
0.6867	0.095
0.8829	0.135
0.981	0.162
1.1772	0.215



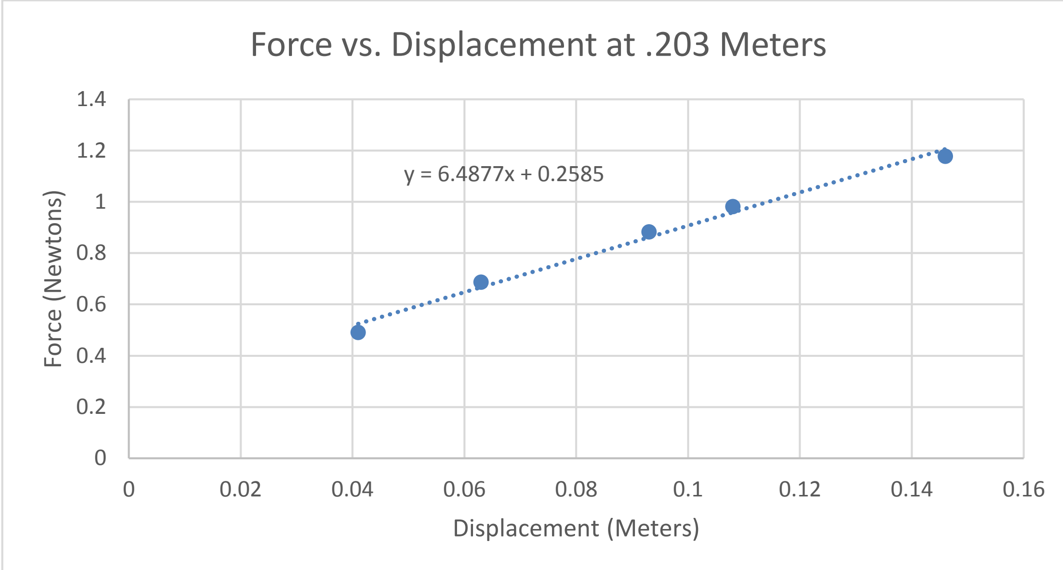
Equation of line: F = 4.4039x + .2562

Uncertainty of slope = .259 % uncert.= 5.88%

Uncertainty of y-intercept = .037 % uncert.= 14.44%

Test 5: Length at .203 Meters

Weight (N)	Displ. (m)
0.4905	0.041
0.6867	0.063
0.8829	0.093
0.981	0.108
1.1772	0.146



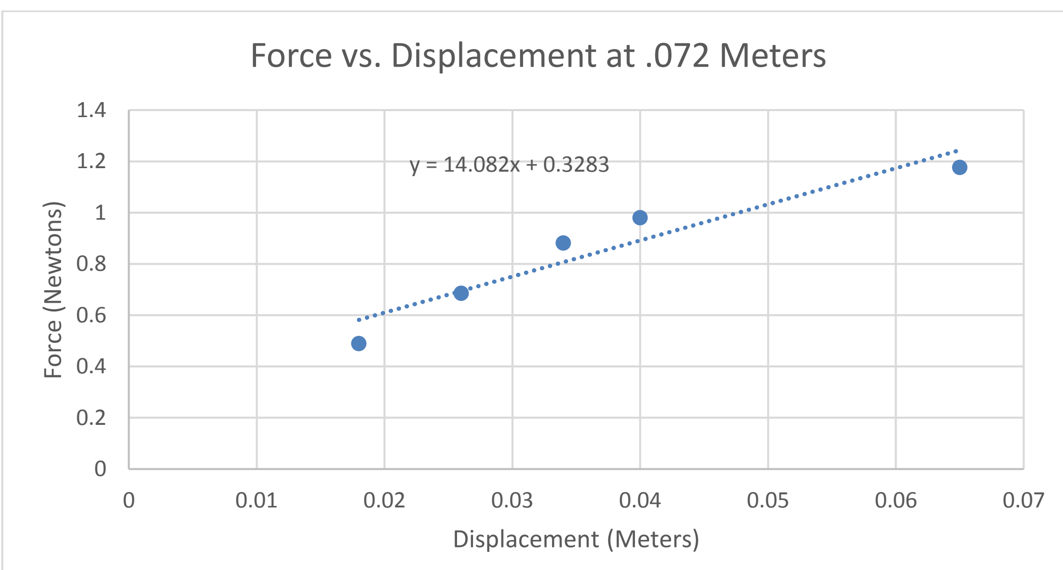
Equation of line: F = 6.4877x + .2585

Uncertainty of slope = .406 % uncert.= 6.26%

Uncertainty of y-intercept= .040 % uncert.= 15.47%

Test 6: Length at .072 Meters

Weight (N)	Displ. (m)
0.4905	0.018
0.6867	0.026
0.8829	0.034
0.981	0.04
1.1772	0.065



Equation of line: F = 14.082x + .3283

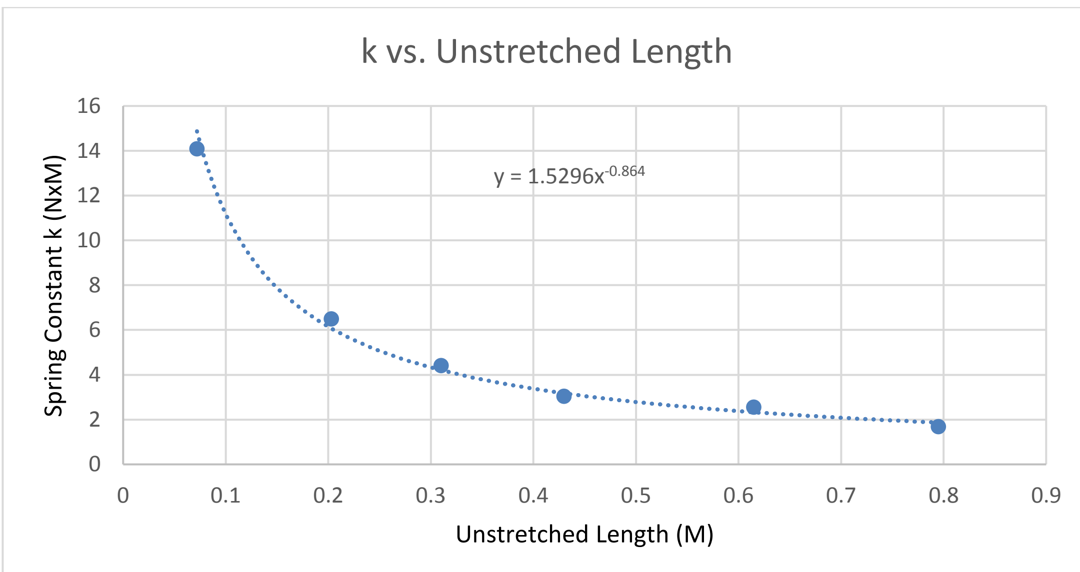
Uncertainty of slope= 2.63 % uncert.= 18.68%

Uncertainty of y-intercept= .105 % uncert.= 31.99%

The important values here are the slopes for each graph, which indicate the spring constant, k. The spring constant of each graph was taken and graphed against the bungee's unstretched length at each test, yielding the following results:

Unstretched Length	Spring Constant
0.795	1.6768
0.615	2.5515
0.43	3.0414
0.31	4.4039
0.203	6.4877
0.072	14.082

A graph of the above data yields the following graph:



The equation for this graph is: $K = 1.5296x^{-.864}$

Since this graph was made using the slopes of the last six graphs, the uncertainties of those slopes will be reflected in the uncertainty of this equation. Using propagation of uncertainty, the uncertainty of this equation would be about 24.01%.

The above equation is the model for spring constant versus spring length for this particular bungee cord. The constant is the y variable, and the bungee length is the x variable. By inputting any bungee length into this equation, the result would give you the spring constant of the bungee when positioned at that exact length.

This equation is the first step in understanding how this particular bungee cord moves and operates. With this information, we can more accurately predict how the bungee will react if a certain mass is hung from a certain length. Knowing this equation will be invaluable in the egg drop experiment.

DISCUSSION: In order to test the above result, simply calculate the spring constant k from some bungee length, hang a weight from the bungee, and see if the resulting displacement is the same that would be predicted from the equation F=kx.

The individual uncertainty of each of the first six graphs is not particularly high, save for perhaps the final graph where the unstretched length is .072 meters. For this graph, because the bungee cord length was already so small, even minute changes in displacement produced higher uncertainties. There was a smaller margin for error in this test than in prior tests, and the closer proximity of the end knots used to hold the bungee in place could have come into play. When combined, the multiple uncertainties create the admittedly large uncertainty of 24.01%. The multiple different bungee lengths and weights across six experiments create a higher than normal uncertainty for the ending equation.

It is possible that some of this uncertainty comes from the wear and tear applied to the bungee cord as the experiment progressed. The cord took a lot of abuse during the initial testing phase, and was stretched almost to the floor when too much weight was applied. This may have altered the bungee's behavior and, by the time of the last experiment, the bungee been changed enough that the uncertainty was noticeably higher than the previous five tests.

The end results are still useful. The power function model was an appropriate result for the situation at hand, and the rapid fluctuation of the spring constant depending on length made sense given our understanding of the procedure and the science behind our methods. With a shorter length, the bungee became more responsive to applied weight. When the bungee was already fairly long, it was not as affected by the same weight. As the length of a bungee cord increases, the resulting spring constant rapidly approaches zero. As the limit of the unstretched bungee cord approaches zero, the spring constant of the bungee will get infinitely high. This relationship is best illustrated with a power function, which alone is a key discovery in understanding the qualities of the bungee cord, rather than the relationship being illustrated through a linear or exponential function.

CONCLUSION: The equation of this graph discovered in this experiment offers a valuable model comparing unstretched length and spring constant. Now the spring constant can be derived from anywhere, and this spring constant can then be used as a helpful tool in further predicting how the bungee will behave in very specific circumstances. This is especially useful for the egg drop experiment, where the spring constant will be invaluable in trying to determine how close the egg will get to the ground when the bungee is dropped from a certain height. By determining the k, we can then find out how far the bungee will stretch downward towards the ground after it unfurls with an egg attached to the end. With this model, the egg drop experiment gets a little bit simpler to resolve.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Matt Reichel