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***TITLE:*** Bungee Jump: How does displacement vary with mass?

## *ABSTRACT:*

We modeled the relationship between the force applied to a bungee cord of set length and how much it is stretched as a result of that force. Variable amounts of mass were added to a hanging mass attached to the end of a set length of cord, which was attached to a fixed point over the ground. We found the displacement of the hanging mass for each amount of mass added. Using the model of total energy for an ideal Hooke's law system, $mg(x_{max})= ½ k(x_{max}-x_L)^2$, we found a linear model for our data by graphing $mg(x_{max})$ over $k(x_{max}-x_L)^2$. This was done in order to simplify later calculations. The equation of our line of best fit was mg(xmax)= 1.056(xmax-x¬L)2 + 0.148, the slope of which represents ½ the value of k for the spring. Thus, our experimental k value for our cord was 2.112 ± 1%. When tested, our model had a percent error less than that of the total percent uncertainty for our system, and thus can be considered an accurate model.

## *INTRODUCTION:*

Purpose: To determine if the functional relationship between the force applied to a bungee cord of constant length and its displacement can be modeled as an ideal Hooke's law system.

Relevant equation(s), identifying variables:

- $mgh= ½ kx^2$ where:
    - m is the mass of the object dropped, h is the height of the drop (or distance dropped), and x is the total stretch of the chord
- The above can be rewritten as $mg(x_{max})= ½ k(x_{max}-x_L)^2$ for our system where:
    - $x_{max}$ is the distance from the top of the bungee cord and $x_L$ is the un-stretched length of the chord, meaning $x_{max}-x_L$ represents the total stretch the cord experiences during the drop.
- Solving for k: $½ k= (mg(x_{max}))/(x_{max}-x_L)^2$

Basis or brief theoretical background:

- These above equations are derived from the CWE theorem, which states that in a closed conservative system, the total (PE and KE) energy is constant. At the bottom of the jump, the energy from gravitational and bungee cord forces are equal, so we can equate the equations for the two as done above.
    - $x_{max}$ is used as the height (h) value because it represents the distance between the highest and lowest points of the jump, meaning it is the operative "height" of the drop.
- Since cord length is kept constant, the value of k should remain constant as well, and thus can be determined from the slope of line when $mg(x_{max})$ is modeled against $(x_{max}-x_L)^2$.

Hypothesis (or expectations):

- Though an elastic bungee cord does not represent an ideal Hooke's law spring, it's behavior should be able to be estimated as such, enabling us to find the value of k for a given chord length.

## *METHODS*:

Using the above equation derived from the CWE theorem as the basis of our experimental model, we varied the amount of mass placed on a hanging hook attached to set length of our elastic bungee cord to measure how displacement varies with force.

Fig. 1 – Experimental Variables

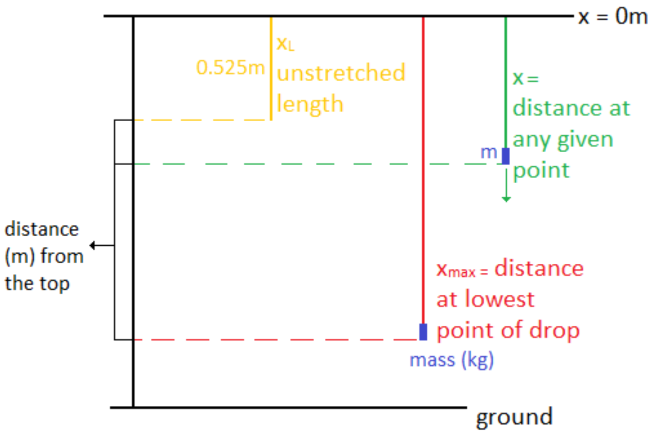


*Fig. 1 – For the sake of modeling our system, the above variables have been defined as shown.*

Fig. 2 – Diagram of Experimental Setup

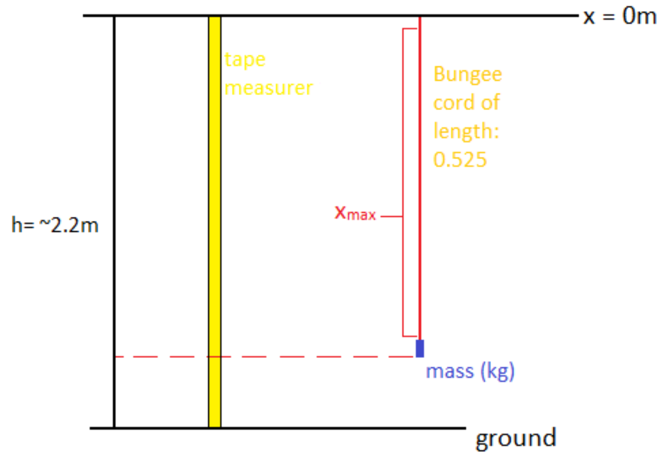


*Fig. 2 – Bungee cord was hung alongside tape measurer with variable mass attached to bottom end of cord. The mass was then released from the top bar and allowed to drop down to distance $x_{max}$, measured against the tape measure via slow-motion camera placed directly in front of setup.*

<u>Experimental Procedure</u>:

- The length of the elastic bungee cord was kept constant at 0.525m in length. Masses ranging from 0g-100g were added to the 50.0g hook attached to the end of the cord.
- The hanging mass was released from rest at the top of the cord (x = 0m), and the displacement of each of the masses were collected.
  - Displacement was measured against a tape measurer using slow-motion recording from *Couch My Video* app on iPad.

<u>Evaluation/Verification</u>:

- As a means of evaluating the integrity of our model, the same experiment was repeated with cord length 0.24m and with masses ranging from 100-225g.
  - This was to ensure the model was consistent with higher values for mass, as well as to confirm that changing the value of $x_L$ would change the value of k (which intuitively knew would).
  - This was a "test" of our model, not part of our main experiment.

***RESULTS***:

As mentioned above, data was collected from the displacement of the varied masses at the lowest point of their drops. This data was then used alongside the equation $mg(x_{max}) = \frac{1}{2}k(x_{max}-x_L)^2$ to model the functional relationship between force and displacement for our cord (specifically seeing if it was linear).

**Data:**

Fig 3. – Table of Raw Experimental Data

$x_L$ = 0.53m ± 0.02m

| **Total hanging mass (g) ±2%** | **$x_{max}$ (m) ±.02m** |
|---|---|
| 50 | 1.19 |
| 55 | 1.24 |
| 60 | 1.31 |
| 65 | 1.36 |
| 70 | 1.4 |

| | |
|---|---|
| 75 | 1.44 |
| 80 | 1.52 |
| 85 | 1.57 |
| 90 | 1.6 |
| 95 | 1.67 |
| 100 | 1.73 |
| 140 | 2.2 |
| 150 | 2.24 |

*Fig 3. – table of all experimentally relevant data, rounded based on the uncertainty of measurements.*

Fig. 4 – Table of Linearized Experimental Data

$x_L$ = 0.53m ± 0.02m

| **Total hanging mass (g)** | **W (N)** | **$x_{max}$ (m) ±.02m** | **W*$x_{max}$ (N*m)** | **$(x_{max}-x_L)^2$ ($m^2$)** |
|---|---|---|---|---|
| 50 | 0.491 | 1.19 | 0.584 | 0.442 |
| 55 | 0.540 | 1.24 | 0.669 | 0.511 |
| 60 | 0.589 | 1.31 | 0.771 | 0.616 |
| 65 | 0.638 | 1.36 | 0.867 | 0.697 |
| 70 | 0.687 | 1.4 | 0.961 | 0.766 |
| 75 | 0.736 | 1.44 | 1.06 | 0.837 |
| 80 | 0.785 | 1.52 | 1.19 | 0.990 |
| 85 | 0.834 | 1.57 | 1.31 | 1.08 |
| 90 | 0.883 | 1.6 | 1.41 | 1.16 |
| 95 | 0.932 | 1.67 | 1.55 | 1.30 |
| 100 | 0.981 | 1.73 | 1.70 | 1.45 |
| 140 | 1.37 | 2.2 | 3.01 | 2.79 |
| 150 | 1.47 | 2.24 | 3.30 | 2.94 |

*Fig. 4 – table of all the data relevant to the calculations needed to make linearize model, as well as the values of the components of our Hooke's law equation. All values rounded based on uncertainty of measurements and to appropriate number of significant figures from calculations.*

Fig. 5 – Table of linearized data for cord length 0.24m

| **Total hanging mass (g)** | **Total hanging mass (kg)** | **W (N)** | **xmax (m) ±.02m** | **W*$x_{max}$ (N*m)** | **$(x_{max}-x_L)^2$ ($m^2$)** |
|---|---|---|---|---|---|
| 100 | 0.1 | 0.981 | 0.88 | 0.863 | 0.410 |
| 125 | 0.125 | 1.23 | 1.03 | 1.26 | 0.624 |
| 150 | 0.15 | 1.47 | 1.14 | 1.68 | 0.810 |
| 175 | 0.175 | 1.72 | 1.24 | 2.13 | 1.00 |
| 200 | 0.2 | 1.96 | 1.36 | 2.66 | 1.24 |
| 225 | 0.225 | 2.21 | 1.53 | 3.37 | 1.65 |

*Fig. 5 – due to the small number of data points, this is not these are not the main experimental values, but instead an extra set of data points to verify the results of the main experiment.*

***Linearized* graph**:

Fig. 6 – Graph of $(x_{max}-x_L)^2$ vs $W*x_{max}$ for cords 0.53m and 0.24m

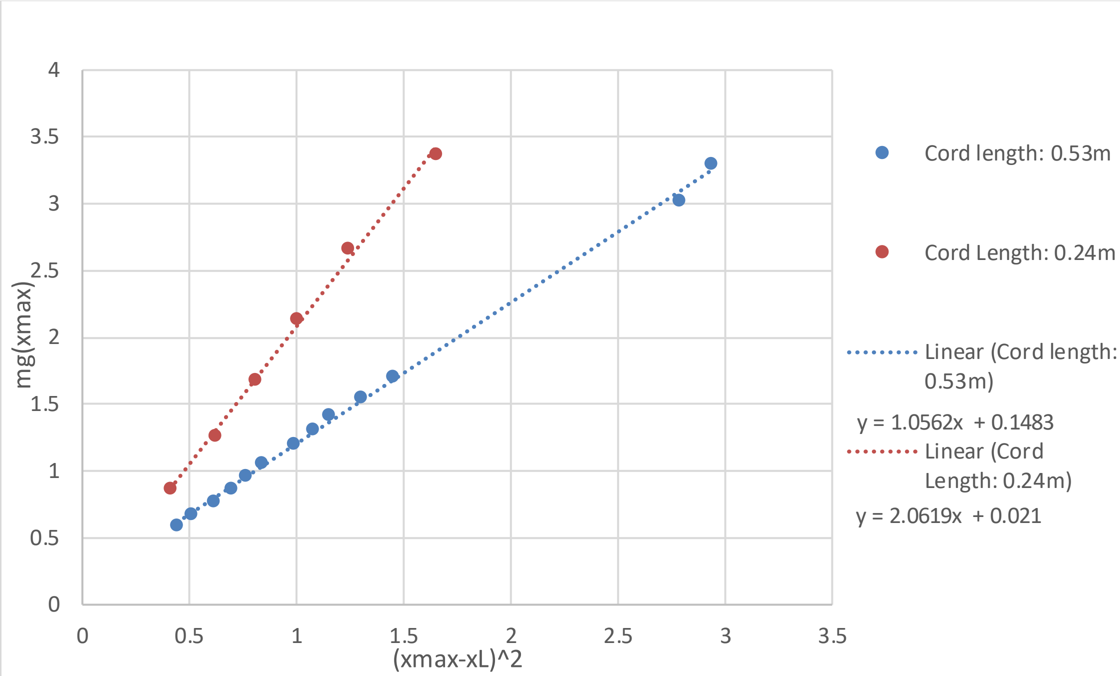


*Fig. 6 – The linearized graph of our data, the slope of which gives ½ of the k value of our cord when $x_L$=0.53m and 0.24m because* ½ k= $(mg(x_{max}))/(x_{max}-x_L)^2$.

**Linear equation**:

- Linear equation for cord length 0.53m: $mg(x_{max})= 1.056(x_{max}-x_L)^2 + 0.148$
- From $mg(x_{max})= ½ k(x_{max}-x_L)^2$, we know that the slope of this line is ½ the value of k, so k=2(1.0562)= 2.112

- For cord length 0.24m: $mg(x_{max})= 2.062(x_{max}-x_L)^2 + 0.021$
- k=2(2.0619) = 4.124

**Propagation of Uncertainty**:

Uncertainty for $(W*x_{max})$: 1% for masses, 2% for displacement -> 2%

Uncertainty for $(x_{max}-x_L)^2$: 2% for displacement->2%

Total uncertainty from measurements: 3%

**Regression analysis**:

(Only for cord length 0.53m because it is the main experiment)

uncertainty for slope= ±0.013N/m % uncert= 1%

uncertainty for y-intercept= ±0.019N*m % uncert= 13%

**Experimental value of interest**:

- Quantitively, the value of interest is the k value for our elastic bungee cord (2.112, when rounded for uncertainty).
  - This value, as mentioned above, is twice the slope of the line modeled by $(mg(x_{max}))/(x_{max}-x_L)^2$.
  - Linear regression analysis gave us an uncertainty of 1% for this slope value. It should be noted, however, that our measurements give the system a total uncertainty of 3%.
- Qualitatively, it is of interest that for our system, $(mg(x_{max}))$ and $(x_{max}-x_L)^2$ are linearly proportional. This supports the estimation of our bungee cord as an ideal Hooke's law spring.

For our model of a bungee cord with constant length (0.53m ± 0.02m), we were able to model the linear function $mg(x_{max})= 1.0562(x_{max}-x_L)^2 + 0.1483$. The slope of this equation is equivalent to ½ the value of k for our bungee cord at this specific length when estimated as an ideal Hooke's law spring.

## *DISCUSSION*:

**Error analysis**:

- In order to quantify the error in our system, we can plug experimental values for $x_{max}$, mass, and k in order to solve for $x_L$, and compare that value with the actual length of the cord.
  - For cord length 0.53m: $mg(x_{max}) = 1.056(x_{max}-x_L)^2 + 0.148$
  - For total hanging mass: 0.065kg, $mg*x_{max} = 0.867$, $x_{max} = 1.36$, and k = 1.056
  - Plugging in those values and solving for $x_L$ you get an $x_L$ of .535
    - This value is (.535m-.525m) only .01m off, which is a 2% error.
- This 2% error is less than the 3% uncertainty from all the measurements that went into this model, so our model can be considered an acceptable estimation.
- Our model should not have a y-intercept, which is almost definitely a resultant of uncertainties/error (the 13% uncertainty is much higher than the percent uncertainty of our measurements).

**Sources of uncertainty**:

- Measurements from video - the displacement (or $x_{max}$ values) was estimated by looking at slow-motion footage of the drop frame by frame and comparing the position of the mass with the hanging tape measurer beside it. This could be due to the angle of the camera relative to the mass its level with the tape measurer, or simple error in estimating the measurement.
- The bungee cord may have stretched as the experiment progressed, altering the $x_L$ throughout the experiment.
- The mass may have been dropped at slightly different heights for each drop, altering the height.
- Unaccounted forces such as air drag may have caused uncertainty (pertaining to $x_{max}$).
- The mass may have not been dropped perfectly perpendicular each time, which could alter $x_{max}$ values.

Our data indicated that the relationship between force of gravity and displacement is linear, which supports the claim that we can estimate our bungee cord as an ideal Hooke's law system for a given string length as well as find the cord's k value for that given string length.

## *CONCLUSION*:

- According to our experimental data, we can estimate the functional relationship of force and displacement for our bungee cord as an ideal Hooke's law system, provided string length is kept constant.
- For future calculations, we can use Hooke's law to model our bungee jump.
- The two cord lengths we examined were .53m and .24m, the first being approximately double the second. The k value for cord length .24m was almost double that of .53m, so it may be worthwhile to see how the values of k vary based on cord length. This would be extremely helpful to know when deciding what length of cord and static cord to use for our final jump.

**On my honor, I have neither given nor received any unacknowledged aid on this assignment.**

***Pledged: Harris Billings***