Relating Potential Energy of a Bungee Cord to its Displacement

Alex Herr Partner: Andy Cuthbert
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Abstract:
This report presents an empirically derived formula for the potential energy of a bungee cord. Hanging masses attached to a 0.05m bungee chord were dropped from a fixed height and the final length of the bungee cord (x_f) was measured using a slow-motion camera and a tape measure. Displacement was measured by subtracting the initial length from the final length of the cord ($\Delta X = x_f - x_i$). Potential energy of the bungee was assumed to be equal to the initial potential energy of the system ($P_{ef} = P_{ei} = m*g*x_f$). Plotting potential energy vs. stretch length and applying a power trend line resulted in the equation $P_e = 1.0445\Delta X^{1.977}$, where the slope is analogous to the term "½ k" in the potential energy formula for an ideal spring ($P_e = \frac{1}{2} kx^2$). Using the formula derived for the spring constant "k" from previous work, we determined that the coefficient in our experimentally derived formula was ß=.412. Applying this new coefficient to our bungee cord's potential energy formula we found $P_e = .412(k)\Delta X^{1.977}$. Given an initial height and mass, this equation can be used to calculate the length of bungee cord sufficient to get the mass proximal to the ground.

***Introduction*:**
The purpose of this investigation was to determine whether Hooke's Law expressed in potential energy ($P_e = \frac{1}{2} kx^2$), could be applied to a bungee cord. Since Hooke's Law models the potential energy of an ideal spring, we hypothesized our bungee would be modeled by an equation similar to Hooke's Law ($P_e = ßk(\Delta x^y)$) where "ß" and "y" represent constants. To find these constants we will first assume the initial potential energy of the system is equal to final potential energy of the bungee cord at max stretch. By dropping masses with different potential energies and measuring displacement we can graph the potential energy vs. displacement, and use our experimentally derived equation from last week ($k = 1.2254x_i^{-1.048}$) to determine the constant ß.

***Methods*:**
Figure 1 diagrams the experimental set-up. Increasing hanging masses were dropped and displacement of the bungee cord was measured to determine how potential energy of the bungee changes as a function of displacement.

Experiment 1: Determining the Potential energy of a bungee cord as a function of stretch length.

Step 1: Attach bungee chord hanging apparatus, and tie off loop to suspend the hanging mass.
Step 2: Hang tape measure proximal to the path of fall.
Step 3: One person drops mass from top of hanging apparatus on verbal command.
Step 4: Other person records video of the mass falling.
Step 5: Lowest point of descent is measured using video play back.

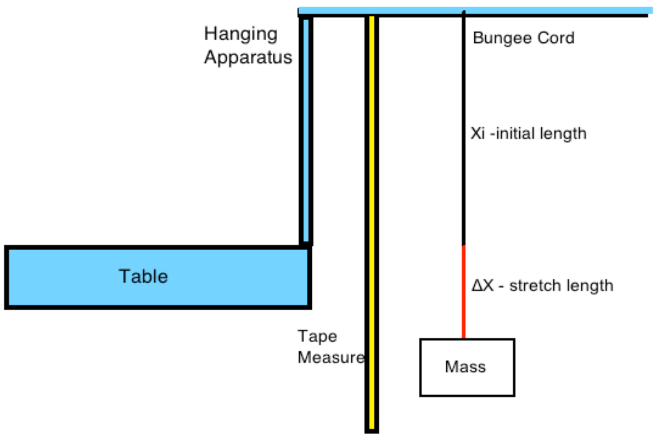

Figure 1: Diagram of the experimental design. Masses were dropped from the top of the hanging apparatus and displacement was measured using a slow-motion camera at the lowest point of the fall. Hanging mass in the diagram is at lowest point in fall where the P_e of the bungee is greatest.

RESULTS:

Data collected relates the potential energy of a bungee cord (P_e) to the displacement of the cord (ΔX**)**. Potential energy of the bungee cord at max stretch was assumed to equal the initial potential energy of the system. Data was collected by dropping the hanging masses and using a slow-motion camera with a tape measure to measure the final length of the cord. Raw data along with stretch lengths and potential energy values are reported in Table 1.

Hanging Mass (kg)	**Cord Length** x_i (m, ±0.01m)	**Final Length** x_f (m, ±0.01m)	**ΔX Stretch Length** (m, ±0.01m)	**Potential Energy** P_E **(m*g*x_f)** (j, ±0.01 j)
0.05	0.50	1.285	0.785	0.62965
0.06	0.50	1.37	0.87	0.80556
0.07	0.50	1.465	0.965	1.00499
0.08	0.50	1.6	1.1	1.2544
0.09	0.50	1.715	1.215	1.51263

Table 1. Displacement and potential energies of the bungee cord for all five falling masses at max stretch.

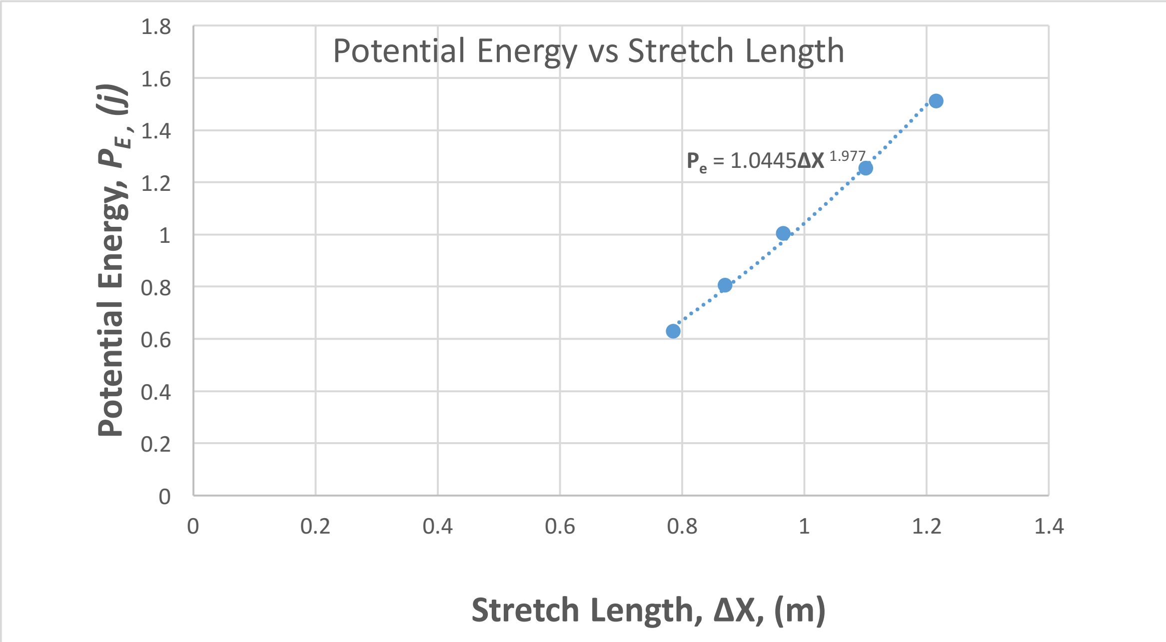

Figure 2: Potential Energy vs. Stretch Length. The potential energy of an ideal spring is modeled by $P_e=\frac{1}{2}(k)\boldsymbol{\Delta x^2}$, therefore the slope of this equation represents ß(k), where ß is a constant.

In previous work, a formula for the spring constant k was derived as a function of bungee cord length. This formula ($k= 1.2254x_o^{-1.048}$) can be used to determine the coefficient ß**.** Since $k=1.2254x_o^{-1.048}$, and x_o in our experiment was fixed at 0.05m we find k=2.53. From the potential energy equation, we find the slope (1.0445= ß *k) which means ß=0.412. Using this constant in our potential energy expression we find a general expression for potential energy as a function of k and ΔX where $P_e=0.412k(\Delta X^{1.977})$. Given any height and mass, this general expression can be used to calculate initial length of bungee cord to maximizes stretch and graze the ground. A linear regression analysis of the P_e vs ΔX determined the standard error of the slope to be ±.034.

Discussion
Experimental data showed that the potential energy for our bungee cord $P_e=0.412k(\Delta X^{1.977})$ is similar to the potential energy expression for an ideal spring $P_e= \frac{1}{2} k x^2$. Sources of error could come from our assumption that the potential energy of the mass is equal to potential energy of the extended bungee cord, as energy is likely lost in heat as the mass falls through the air. Another source of error could come from the limited number of trials performed for each mass drop (n=1). Our experiment validated our hypothesis that potential energy would change as a function of bungee stretch, and that this model would be similar to the potential energy expression for an ideal spring $P_e= \frac{1}{2} kx^2$, with a different coefficient and power value $P_e=$ ß kx^y.

Conclusion
Experiment showed that potential energy in a bungee cord is a function of displacement. Future studies should aim to discern how the length of static bungee effects the force of the mass during deceleration so that the bungee cord ride can be both safe and fun for its passenger