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TITLE: Finding the Spring Constant for a Given Elastic Cord by Varying Length

ABSTRACT:

We derived the spring constant for a given piece of elastic cord so that we may accurately predict the length of that cord needed for a successful bungee jump. Using Hooke's Law for an ideal system, we derived the spring constant, k, from the energy- related equation, $mgh = \frac{1}{2}kx^2$; this was done by measuring both the length of the cord in equilibrium with a mass of 0.160 kg and the maximum distance of elongation when the object was released at ten different cord lengths. Moreover, using a force detector, we measured the maximum force to make sure that the acceleration of the mass did not exceed three times the acceleration of gravity, as is in accordance with our lab procedure. By plotting the length of elongation at equilibrium and maximum distance, we found a linear relationship, $X_{max} = 4.7X_L$, where 4.7 represents the spring constant for the elastic cord. This spring constant will then be used to solve for the cord length needed to ensure safe passage to our egg of a given mass and at a given height.

INTRODUCTION:

Theoretical and Practical Background:

The overall purpose of the experiment is to develop an elastic cord bungee jump for an egg with a mass in the range of 100-170 g and for a jump height of 8-9 meters. Based upon previous experimentation, we had modeled a spring constant, *k*, for the elastic cord by measuring the displacement of elongation with various masses at one length. The spring constant was found using Hooke's law, $F_{spring} = -kx$, where *F* is the force of the spring, *k* is the spring constant, and *x* is the distance past the equilibrium point that the cord is stretched.

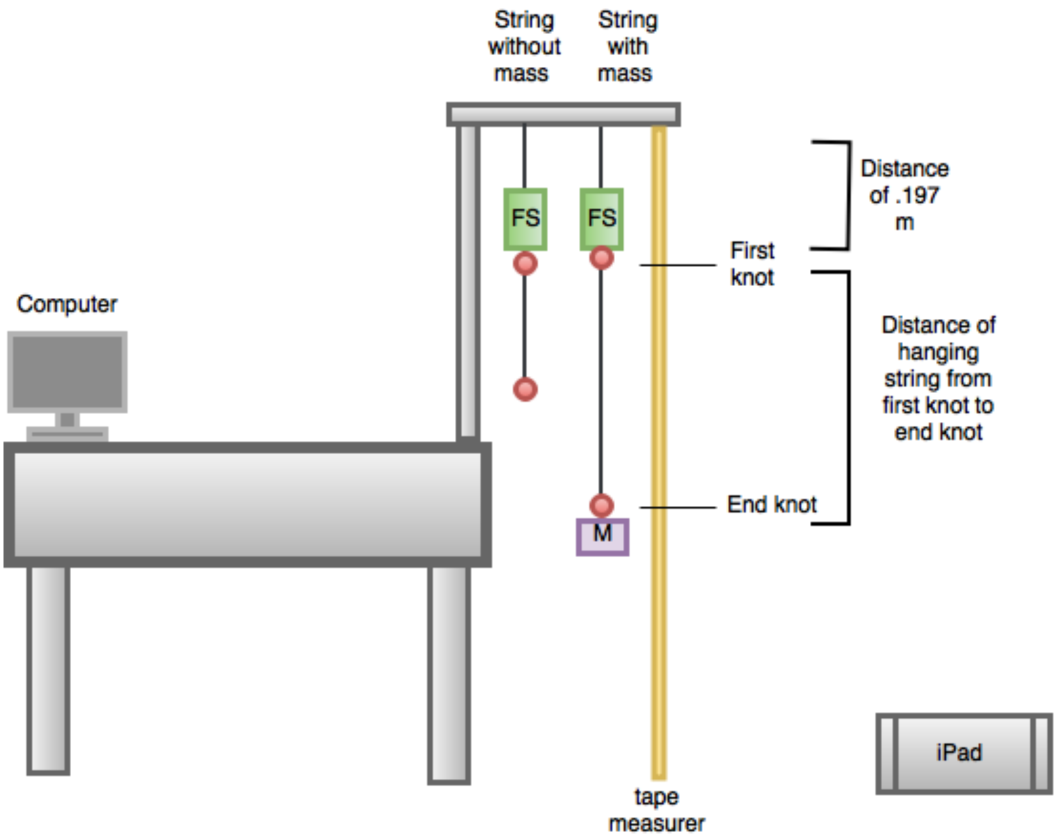
However, because our egg will be bungee jumping, rather than just being placed at the end of the cord, we need to account for the mass, m, gravity, *g*, and the height of the fall, *h*. Thus, this experiment is rooted within the conversation of work and energy theorem (CWE theory), which states that the sum of the potential (PE) and kinetic energies (KE) at the top with be equal to that at the bottom of the jump. Therefore, we use the equation, $mgh = \frac{1}{2}kx^2$, as mgh is representative of the energies at the top of the jump and $\frac{1}{2}kx^2$ is representative of the energies at the bottom, where *k* is the spring constant and *x* is the distance past the equilibrium point that the cord is stretched. Additionally, because we must stay within the force limit of no more than three times the acceleration of gravity, we need to calculate the force of the mass at max elongation.

We then predict that there will be a linear relationship between the distance of elongation at equilibrium and the max distance of elongation during the fall of the mass. Additionally, we predict there will be no difference between the force of the mass at equilibrium or the force of the mass at maximum displacement between cord lengths.

METHODS:

We measured the spring constant by measuring the length of the cord without the mass and then measuring the length of the cord with the hanging mass of 0.160 kg at both rest and at its max during the fall. This was done for ten different cord lengths.

Figure 1: **Diagram of experimental set-up**



Setup:
The force sensor was hung from the metal strip and connected to the Capstone computer program, which detected force. A small knot was tied in the middle of the elastic cord and was hung from the hook on the end of the force sensor. A knot was tied at the end of the string. A new knot in the middle of the cord was tied for each of the ten lengths.

Procedure (for each of the ten lengths):

- The cord without mass was measured from the first knot to the second knot using the tape measurer and recorded. The mass was then added and the length of the cord's displacement at equilibrium was measured from the first knot to the second knot.
- We were then able to determine the length of the cord at maximum displacement by releasing the end knot with the mass at the same height as the first knot. Using the slow motion video camera on the iPad, the motion was recorded and the video was then paused to ascertain the length of maximum displacement of the mass.
- The force sensor was zeroed and then used to measure the force at equilibrium of the mass. To find the force at maximum length, the end knot with the mass was released at the same height as the first knot.

RESULTS:

The aforementioned measurements were recorded in Excel; the length of the cord without mass and the length at maximum displacement during the release of the mass were plotted to create a linear model for the sum of the potential and kinetic energies at the bottom of the jump. This was found to be represented by the equation, $X_{max} = 4.7X_L$.

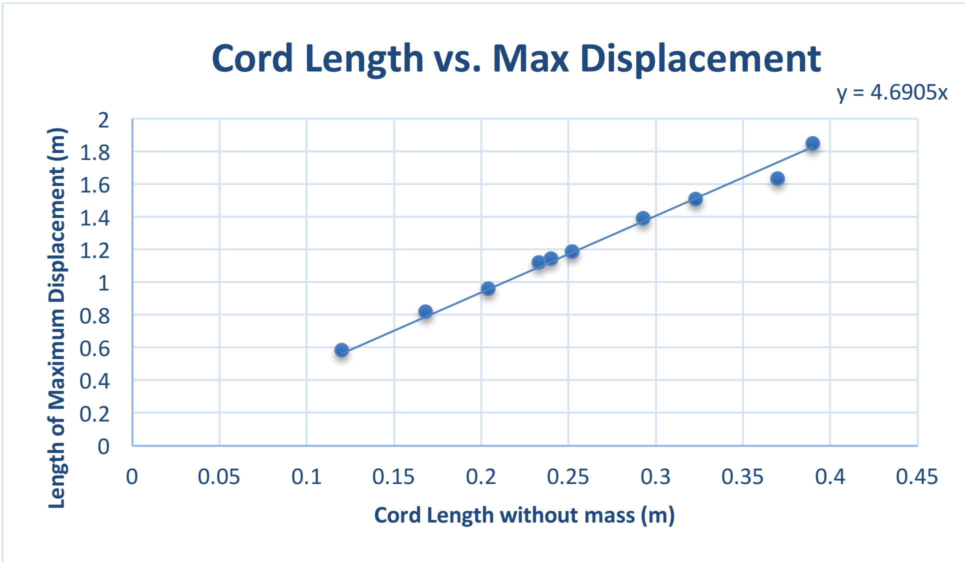
Table 1: Cord lengths without mass and with mass, at both equilibrium and maximum displacement. All lengths with mass were taken using a 0.160 kg mass.

Cord Length ± 0.001 m	Length at equilibrium (m) ± 0.001 m	Length at max displacement (m) ± 0.001 m
0.12	0.30	0.59
0.17	0.41	0.82
0.20	0.49	0.96
0.23	0.58	1.12
0.24	0.59	1.15
0.25	0.61	1.19
0.29	0.71	1.39
0.32	0.79	1.51
0.37	0.86	1.64
0.39	0.96	1.85

Table 2: Cord length without mass, force at equilibrium and the force at maximum displacement. All forces were recorded with a 0.160 kg mass. We observed there is little change between cord lengths for either force at equilibrium and force at max displacement.

Cord Length ± 0.001 m	Force at Equilibrium (N) ± 0.001 N	Force at Max Displacement (N) ± 0.001 N
0.12	-1.54	-3.78
0.17	-1.55	-3.73
0.20	-1.56	-3.72
0.23	-1.55	-3.74
0.24	-1.55	-3.73
0.25	-1.54	-3.72
0.29	-1.54	-3.81
0.32	-1.57	-3.82
0.37	-1.51	-3.83
0.39	-1.57	-3.74

Graph 1: Cord Length vs. Max Displacement The cord length without mass was graphed against the length of maximum displacement during the mass's fall, measured by the slow motion video app on the iPad.



The linearized equation is $X_{max} = 4.7X_L$, where X_{max} represents the cord's maximum displacement during the fall, $X_L$represents the length of the cord without the hanging mass, and 4.7 represents the spring constant, k.

Using Excel regression analysis, the slope was found to be 4.7 and the uncertainty for the slope was found to be 0.045. The percent uncertainty was found to be 0.96% for the slope. The intercept was found to be zero and therefore has no uncertainty.

The experimental value of interest is the spring constant, *k*, which is found in the latter end of the equation, $mgh = \frac{1}{2}kx^2$. Because $\frac{1}{2}kx^2$ represents both the spring constant and the distance past equilibrium in which the chord is stretched, we were able to find k by plotting the cord length without the mass with the maximum displacement of the cord during the release of the mass from the first knot.

The overall, modeled equation of $X_{max} = 4.7X_L$has a percent uncertainty of 1.26%, found using the propagation of uncertainty of a product.

Because at the time of the fall, we will have the mass, *m*, gravity, *g*, and the height of the fall, *h*, as well as the spring constant, *k*, for the elastic cord, we will have all the information needed to solve for the length of the cord using the equation, , $mgh = \frac{1}{2}kx^2$.

DISCUSSION:

Because there are no accepted values for the spring constant of our elastic cord, we are not able to say if our spring constant is accurate by comparing the percent error to the percent uncertainty for our slope. Therefore, we must look for an alternative measure to find a way to test the accuracy of our value. We can test the accuracy of our spring constant by choosing a given weight, *mg*, and height of fall, *h*, and calculating a theoretical elastic cord length with our spring constant to fit the fall. If we were able to successfully and barely miss hitting the floor with the object, we would be able to ascertain the accuracy of our spring constant. Potentially, we could use a similar methodology to this experiment: we could record the actual displacement of the object to the calculated theoretical one to ascertain accuracy even more aptly.

Given the experiment, greatest sources of error could be found in the size of the knots for each length. Because a new knot had to be tied for each length, there could be differences in the size of the knots and therefore inaccuracies in our calculations of the lengths of the cord, both without and with mass. The tightness of the knot could also affect whether the elastic cord stretched more or less and therefore caused inaccuracies in the length calculations. Additionally, due to time constraints, we did not repeat trials for each length. Without additional numbers of trials, we may lack reliability in our estimates. Having had at least two trials for each length would have shown whether we calculated anomalies for length or force.

One aspect we did not consider was the potential for change in the mass to affect the spring constant. Our previous experiment suggested that as the mass changes, the spring constant will change proportionally. However, because we didn't look at both the change in mass and the change in length simultaneously, we do not have a relationship between all three, the spring constant, the object's mass and the length of the cord. Therefore, using other studies, such as "Mass and Xmass of Elastic String" by Michael Shields, that explored the change in spring constant as related to change in weight and in cord length, we should be able to ascertain whether weight affects our final jump.

Our main results do support our hypothesis that there is a linear relationship between the distance of elongation at equilibrium and the maximum distance of elongation during the fall. We found the spring constant of 4.7 for our elastic cord, with a percent uncertainty of 0.95%. Since there was no intercept, we feel confident that our modeled spring constant is a good representation of the actual spring constant. Additionally, we found that there is no difference between the force of the maximum elongation between cord lengths. This is beneficial information as we can say that there will likely be no additional force in our final jump despite using a longer cord.

CONCLUSION:

In conclusion, we found an experimental equation, $X_{max} = 4.7X_L$, which informs us of our spring constant, 4.7. This information will prove useful in the final stages of our jump as we prepare to estimate the length of cord to use to ensure our egg safe passage down the balcony in the Science Center. Again, because at the time of the fall, we will have the mass, *m*, gravity, *g*, and the height of the fall, *h*, as well as the spring constant, *k*, for the elastic cord, we will have all the information needed to solve for the length of the cord using the equation, , $mgh = \frac{1}{2}kx^2$. Moreover, we observed little change in the force at equilibrium or at maximum displacement, and therefore we know that there will be little influence from the cord length in the overall force of the jump. As mentioned earlier, we still have not taken into account the mass of the object's possible effects on the spring constant. For future steps, we plan to use the information of our peers who have calculated a relationship between spring constant and mass to inform us of the appropriate length of cord to use for a given mass. Using all of the collected information and the knowledge of our classmates, we feel confident in the safety and thrilling nature of our bungee jump.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Elena Diller, November 28, 2016 at 10:34 pm