Lab Report Outline—the Bones of the Story

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TITLE: Characterizing the restorative force of a bungee in relation to its length using Hooke's Law

ABSTRACT:

The purpose of this experiment was to discover the relationship between the unstretched length of a bungee and its restorative force. The bungee was viewed as a spring, allowing us to quantify the restorative force using Hooke's Law, which states $F_{spring}=-kxi$. In this equation, the spring constant k was quantified for various bungee lengths. In order to quantify the spring constant, the displacement (x) of a given length of bungee due to changing force (F) was recorded, where force was the weight of the mass hanging from the bungee. Weight of the hanging mass was then plotted against displacement of the bungee for each length of bungee, yielding the spring constant of the bungee at the indicated length. These spring constants were then plotted against the length of bungee. A power regression was performed on these data and was then used to linearize the data with an uncertainty very similar to that of the experimental error. This showed that the spring constant of a bungee, and therefore its restorative force, is inversely related to its length.

INTRODUCTION:

In this experiment, we wanted to discern how the restorative force of a bungee changed as a function of its length. In order to do this, we assumed the bungee acted as a spring when stretched past its resting length, allowing us to use Hooke's Law (Equation 1). As a result, we were able to model the restorative force using spring constants, which could be measured at various lengths of bungee to discern the relationship between the length of our bungee and its restorative force. For a given length of bungee, we varied the force acting on the bungee by changing the mass hanging from it and measured the displacement of the bungee. According to the Second Law Equation for our system (Equation 3), these data could then be plotted to give the spring constant for the given length of bungee. We repeated this process for multiple lengths of bungee so that spring constants could be plotted against bungee length. A power regression analysis was performed to model the relationship between these data and was used to successfully linearize the data; this showed that the spring constant of the bungee, and therefore its restorative force, is inversely related to its length. This suggests that longer bungees will cause less deceleration in objects attached at the end.

METHODS:

The displacement of a given length of our latex bungee due to eight different masses was measured. Weight of the hanging mass was plotted against displacement of the bungee to yield the spring constant of the bungee at that length. This process was repeated for four different lengths of bungee, and spring constant was consequently plotted against the length of bungee. The relationship between spring constant and the length of bungee was then determined using regression analyses.

Relevant Equations:

(1) $F_{spring} = -kxi$ (Hooke's Law)

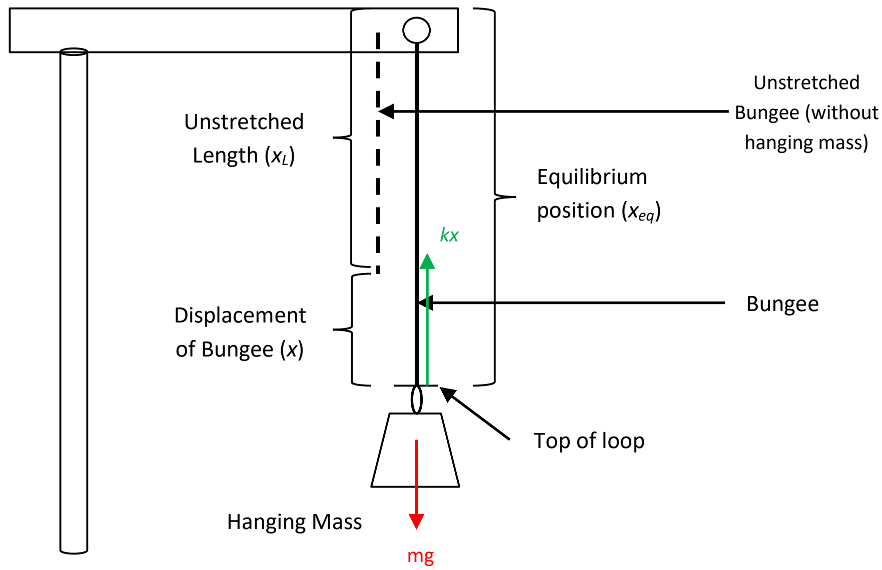
(2) $F_{tot} = ma$ (Newton's Second Law)

(3) $mg - kx = ma$ (Second Law Equation for our system)

(4) $mg = kx$ (Second Law Equation when a=0)

Figure 1: Diagram of experimental setup. Our bungee was hung from a stand secured to the edge of a table. A small loop was tied on the bungie so that a mass could be hung from it. The unstretched length of the bungie (x_L) - the length of the bungee when no mass was attached - was measured from the top of the stand to the top of the loop from which the mass was hung. The equilibrium position is the position at which the hanging mass attached to the bungee is at rest. The length of the bungee at this equilibrium position (x_{eq}) was measured in the same manner as the unstretched length. The displacement of the bungee (x) was calculated by subtracting the

unstretched length (x_L) from the length of the bungee at its equilibrium positon (x_{eq}). The restorative force of the bungee (kx) and the weight of the hanging mass (mg) counteract one another.



- We hung a bungee from a horizontal stand which was secured to the edge of a table.
- We then tied a small loop at a random location on the bungee so that a mass could be hung from it.
- The unstretched length of the bungee (x_L) was measured from the top of the stand to the top of the loop.
- Next, we hung eight different masses on the loop and measured the length of the bungie from the top of the stand to the top of the loop at its equilibrium position (x_{eq}) – the position at which the hanging mass was at rest.
- The displacement of the bungee (x) is the difference between x_{eq} and x_L. Given that the system is not accelerating in this setup, the Second Law Equation for this system is $mg = kx$.
- Therefore, the spring constant (k) can be interpreted as the slope of the line plotting weight of the hanging mass (mg) vs. displacement of the bungee (x).
- The spring constant was calculated for multiple lengths of bungee so that spring constant could be plotted as a function of bungee length.
- We then performed a power regression analysis and linearized our data according to this regression in order to determine the relationship between the spring constant of the bungee and its length.

RESULTS:

We measured the displacement of a constant length of bungee due to various hanging masses in this experiment. We then plotted weight of the hanging mass against the displacement of the bungee and performed a linear regression analysis. The slope of this regression was interpreted as the spring constant of the bungee at a given length. We found the slope constants for various lengths of bungee and plotted these data. A power regression analysis was performed and the data were linearized according to this regression.

Table 1: Displacement data for unstretched length of bungee (x_L) = 0.380 m. The length of the bungee at its equilibrium position (x_{eq}) was measured for each mass and used to determine the displacement of the bungee (x). Weight was determined by multiplying the mass of the hanging mass by its acceleration due to gravity, 9.81 m/s^2.

Hanging Mass (kg) ± 0.001kg	x_L (m) ± 0.001m	x_{eq} (m) ± 0.001m	Displacement x (m) ± 0.002m	Weight of Hanging Mass (N) ± 0.001N
0.050	0.380	0.455	0.075	0.491
0.060	0.380	0.489	0.109	0.589
0.070	0.380	0.512	0.132	0.687
0.080	0.380	0.536	0.156	0.785
0.100	0.380	0.592	0.212	0.981
0.110	0.380	0.628	0.248	1.079
0.120	0.380	0.663	0.283	1.177
0.130	0.380	0.704	0.324	1.275
0.150	0.380	0.782	0.402	1.472
0.200	0.380	0.990	0.610	1.962

Table 2: Displacement data for unstretched length of bungee (x_L) = 0.553 m. The length of the bungee at its equilibrium position (x_{eq}) was measured for each mass and used to determine the displacement of the bungee (x). Weight was determined by multiplying the mass of the hanging mass by its acceleration due to gravity, 9.81 m/s^2.

Hanging Mass (kg) ± 0.001kg	x_L (m) ± 0.001m	x_{eq} (m) ± 0.001m	Displacement x (m) ± 0.002m	Weight of Hanging Mass (N) ± 0.001N
0.050	0.553	0.668	0.115	0.491
0.060	0.553	0.702	0.149	0.589
0.070	0.553	0.738	0.185	0.687
0.080	0.553	0.775	0.222	0.785
0.100	0.553	0.866	0.313	0.981
0.110	0.553	0.926	0.373	1.079
0.120	0.553	0.983	0.430	1.177
0.130	0.553	1.047	0.494	1.275
0.150	0.553	1.175	0.622	1.472
0.200	0.553	1.515	0.962	1.962

Table 3: Displacement data for unstretched length of bungee (x_L) = 0.651 m. The length of the bungee at its equilibrium position (x_{eq}) was measured for each mass and used to determine the displacement of the bungee (x). Weight was determined by multiplying the mass of the hanging mass by its acceleration due to gravity, 9.81 m/s^2.

Hanging Mass (kg) ± 0.001kg	x_L (m) ± 0.001m	x_{eq} (m) ± 0.001m	Displacement x (m) ± 0.002m	Weight of Hanging Mass (N) ± 0.001N
0.050	0.651	0.793	0.142	0.491
0.060	0.651	0.827	0.176	0.589
0.070	0.651	0.873	0.222	0.687
0.080	0.651	0.922	0.271	0.785
0.100	0.651	1.035	0.384	0.981
0.110	0.651	1.093	0.442	1.079
0.120	0.651	1.164	0.513	1.177

0.130	0.651	1.247	0.596	1.275

Table 4: Displacement data for unstretched length of bungee (x_L) = 0.199 m. The length of the bungee at its equilibrium position (x_{eq}) was measured for each mass and used to determine the displacement of the bungee (x). Weight was determined by multiplying the mass of the hanging mass by its acceleration due to gravity, 9.81 m/s^2.

Hanging Mass (kg) ± 0.001kg	x_L (m) ± 0.001m	x_{eq} (m) ± 0.001m	Displacement x (m) ± 0.002m	Weight of Hanging Mass (N) ± 0.001N
0.050	0.199	0.239	0.040	0.491
0.060	0.199	0.251	0.052	0.589
0.070	0.199	0.264	0.065	0.687
0.080	0.199	0.277	0.078	0.785
0.100	0.199	0.313	0.114	0.981
0.110	0.199	0.333	0.134	1.079
0.120	0.199	0.352	0.153	1.177
0.130	0.199	0.376	0.177	1.275

Table 5: Displacement data for unstretched length of bungee (x_L) = 0.478 m. The length of the bungee at its equilibrium position (x_{eq}) was measured for each mass and used to determine the displacement of the bungee (x). Weight was determined by multiplying the mass of the hanging mass by its acceleration due to gravity, 9.81 m/s^2.

Hanging Mass (kg) ± 0.001kg	x_L (m) ± 0.001m	x_{eq} (m) ± 0.001m	Displacement x (m) ± 0.002m	Weight of Hanging Mass (N) ± 0.001N
0.050	0.478	0.581	0.103	0.491
0.060	0.478	0.608	0.130	0.589
0.070	0.478	0.643	0.165	0.687
0.080	0.478	0.676	0.198	0.785
0.100	0.478	0.754	0.276	0.981
0.110	0.478	0.804	0.326	1.079
0.120	0.478	0.854	0.376	1.177
0.130	0.478	0.912	0.434	1.275

Figure 2: An example scatter plot showing weight of the hanging mass (mg) vs. displacement of the bungee (x) at unstretched length x_L = 0.380 m (see *Table 1*). A linear regression analysis was performed and yielded a linear equation, which was modified to incorporate mg and x as our variables.

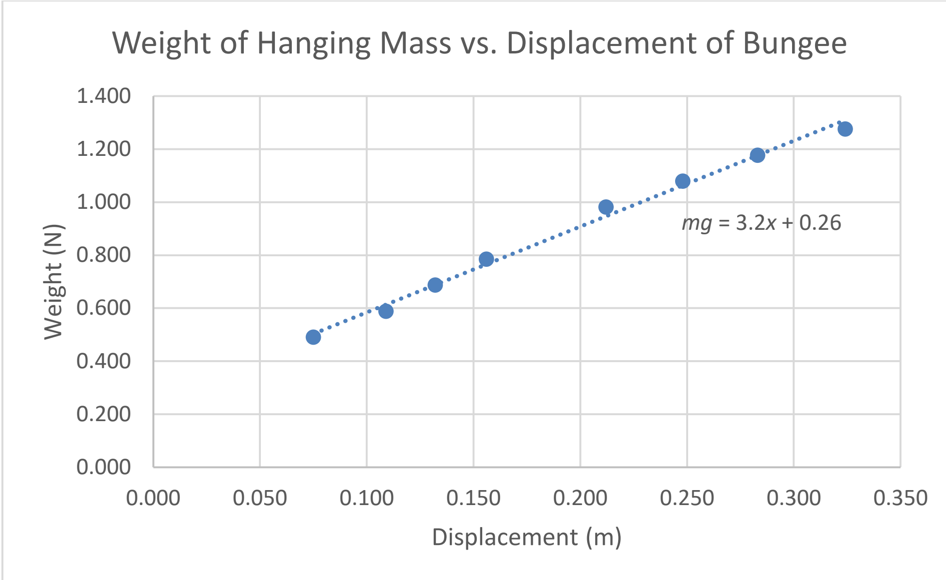


Table 6: Spring constants compiled from the linear regression analyses of the weight vs. displacement plots. The final column lists the linearized data, which are the inverse values of the spring constants.

x_L (m)	Spring Constant k (N/m)	Linearized Data: $1/k$ (m/N)
0.199	5.7	0.18
0.380	3.2	0.31
0.478	2.4	0.42
0.553	2.06	0.486
0.651	1.72	0.581

Figure 3: A scatter plot showing the spring constant of the bungee (k) plotted as a function of its unstretched length (x_L). A power regression analysis was performed and yielded the following equation, which was modified to incorporate k and x_L as our variables.

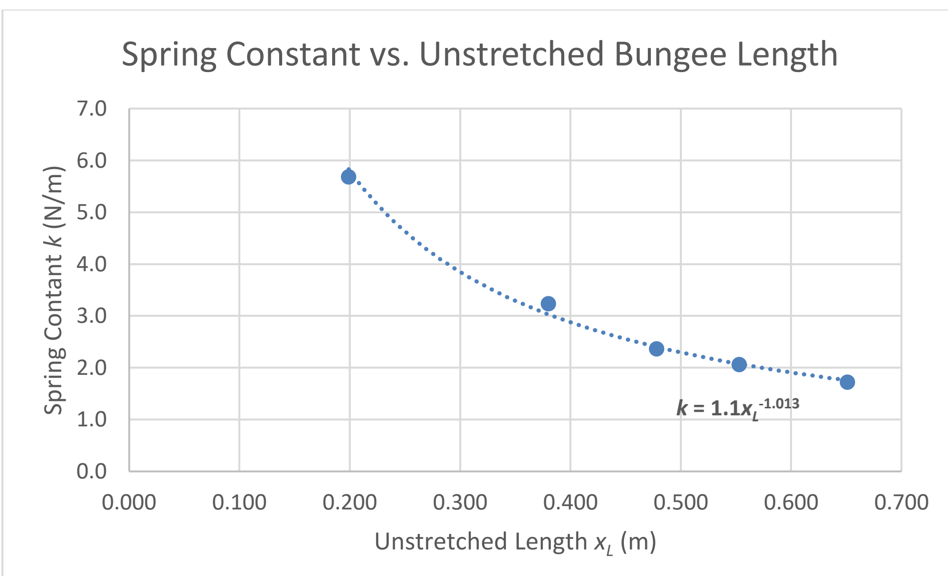


Figure 4: A linearized scatter plot showing the inverse of the spring constant of the bungee ($1/k$) as a function of its unstretched length (x_L). A linear regression analysis was performed and yielded the following equation, which was modified to incorporate $1/k$ and x_L as our variables.

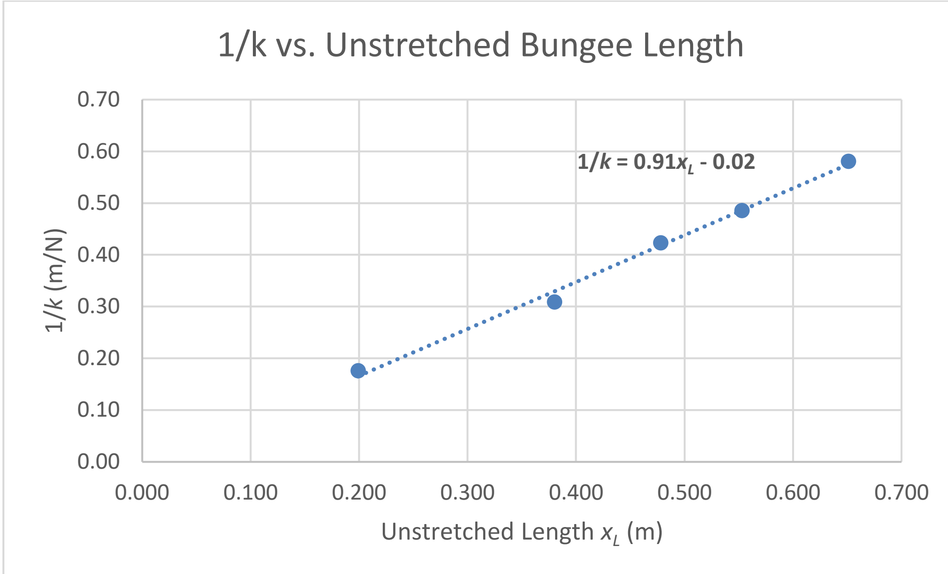


Table 7: The results of the regression analysis of the linearized data performed in Excel. The coefficient of the x-variable is the slope of the linear regression, and the standard error was interpreted as the uncertainty associated with this slope. The value of the y-intercept and its standard error have also been included.

	Coefficients	***Standard Error***
Intercept	-0.01507	0.019306
X Variable	0.906207	0.040396

Equation of linearized graph: $1/k = 0.91x_L - 0.02$
Uncertainty for slope = ± 0.04
% Uncertainty for slope = ± 4%
Uncertainty for y-intercept = ± 0.02
% Uncertainty for y-intercept = ± 100%

The power regression analysis of the spring constant (k) vs. unstretched bungee length (x_L) plot suggested an inverse relationship due to the negative exponent in the equation from *Figure 3*: $k = 1.1x_L^{-1.103}$. In order to linearize our data according to this regression, we plotted the inverse of the spring constant ($1/k$) against the unstretched bungee length (x_L). These data showed a linear relationship, modeled by the equation from *Figure 4*: $1/k = 0.91x_L - 0.02$. Successful linearization of our data according to our power regression analysis confirms that there is an inverse relationship between spring constant and unstretched bungee length.

DISCUSSION:

The percent uncertainty of the slope of the linearized data, which was obtained using Excel regression analysis, was reasonably small at 4%. The percent uncertainty associated with the mass of our hanging masses alone was 2%, so the percent uncertainty of our results (the slope of the linearized data) is not significantly larger than the percent uncertainty associated with our experiment overall, suggesting the linearization is reliable. The percent uncertainty associated with this slope could likely be improved by increasing the number of data points. Since there is not an accepted value for slope that this can be compared to, the degree of accuracy of our results cannot be discerned.

The most significant source of error in this experiment was the variability in the measurements of the unstretched bungee length and the length of the bungee at its equilibrium position. We had to make sure not to disturb the bungee when measuring its length, so the large distance between the bungee and the tape measure used to make measurements decreased the accuracy of these measurements. An additional source of error was the uncertainty in the mass of the hanging masses. We did not mass each hanging mass used in our experiment on a balance, which significantly reduced the certainty with each mass, which consequently affected the precision of our spring constant values.

Overall, our results show that the restorative force of a bungee, when modeled using Hooke's Law, decreases as the length of the bungee increases. The percent uncertainty associated with the plot linearizing data according to this relationship is very similar to the percent uncertainty associated with our experimental procedure, suggesting our results are reliable.

CONCLUSION:

When modeled using Hooke's Law and spring constants, the restorative force of a bungee shows an inverse relationship to its length. That is, a bungee's restorative force will decrease as its length increases. This becomes important when attempting to maximize the deceleration of the egg which will be attached to the bungee; a very long bungee will not cause the egg to decelerate as quickly.

As a point of future study, one might try to discover if the restorative capacity of a bungee can be compromised by extensive stretching, just as it can be for traditional springs.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Joseph R. Zoeller