Elizabeth McDonald

Experimentally determining the extent of spring like behavior in a bungee cord

ABSTRACT

This report aimed to continue properly designing guidelines for a quality bungee jump to be taken by an egg on a thin cord. Although all necessary components needed to solve for unstretched cord length (L) given a known mass of the egg (m) and height of bungee release (h) were calculated last class, our team acknowledged that the assumed behavior of the cord (predicted to be spring-like) may be inaccurate. Keeping constant unstretched cord length (L), disks of varying masses were added onto the end of the string and cord displacement was recorded. Potential energy was calculated for each mass and plotted against cord displacement to derive the resulting behavior of the cord. We observed that the cord did indeed transfer its gravitational potential energy into spring potential energy through our resulting equation $y=0.576k\Delta x^{2.0977}$, comparable to the spring equation for energy, $0.5k\Delta x^2$. Using our more precise description of our elastic cord, we can rework our equation for L to yield more accurate optimal L lengths for our bungee when testing it from a given height and mass.

INTRODUCTION

The goal of bungee makers is to provide a thrilling and safe fall. Although one could add more thrill to the fall using some inelastic length of cord, one must also make sure that this added thrill does not result in a dangerous acceleration past 3g. We therefore controlled for this condition by only using elastic cord. To ensure safety, our goal for the past two labs has been to accurately derive an equation that predicts an appropriate unstretched cord length that does not stretch to exceed a given height from which the egg (of mass m) is launched. In our last lab, we recognized that the spring constant (k) differs based on length of the unstretched cord (L); therefore, we developed a function to predict k given L: $k(L)= 1.0913x^{-0.936}$. Recognizing that the entire behavior of the cord from initial release to rest can be modeled using the CWE equation, we manipulated the CWE equation using our known value for k to give us an equation for L given a known drop height and mass of the egg:

$$mgh = \frac{1}{2}k\Delta x^2$$

$$mgh = \frac{1}{2}k(L)\Delta x^2$$

$$\sqrt{\frac{2mgh}{k(L)}} = \Delta x$$

$$h = L + \Delta x$$

$$h = L + \sqrt{\frac{2mgh}{k(L)}}$$

Thus, we can solve for L given h and m. However, this assumes that our bungee will behave like an ideal spring (gravitational potential energy will completely convert to spring potential energy). Therefore, for this lab, we sought to determine if $mgh = \frac{1}{2}k\Delta x^2$ is accurate. We hypothesize that our bungee cord will mostly follow the behavior of an ideal spring.

METHODS:

In order to determine whether our bungee cord behaves like an ideal spring, we can determine displacement of the cord when different masses are attached. Plotting mgh over the displacement will give us an equation that varies with displacement given a mgh, which we can then compare to $mgh = \frac{1}{2}k\Delta x^2$.

Figure 1 on the last page shows disks of variable masses (0.17 kg, 0.15kg, 0.13kg, 0.11kg and 0.09kg) dropped from an unstretched cord length of 0.22m. Maximum displacement was determined using a slow motion camera.

- A cord was tied off such that the mass would hang from an unstretched cord length of 0.22m
- The mass was then lifted above the drop height by one partner who subsequently released the mass while the other partner took a slow motion video of the released mass
- Using the ruler as a reference in the video, both partners determined maximum displacement of the cord before the mass began oscillating back up
- The trial was repeated for mass 0. .17 kg, 0.15kg, 0.13kg, 0.11kg and 0.09kg

RESULTS:

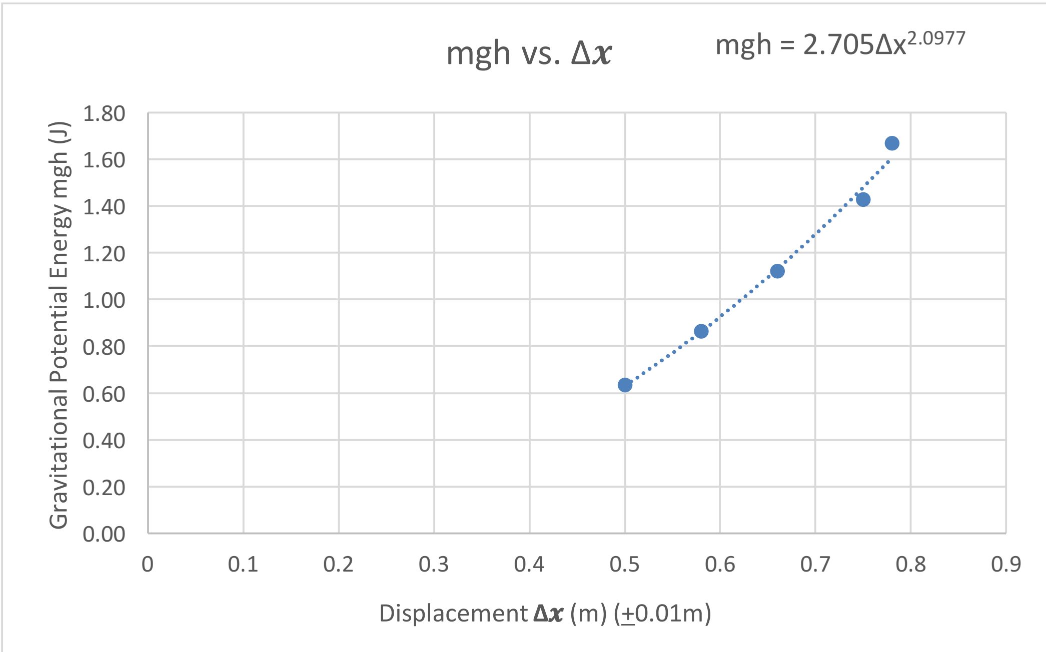
In order to compare our results to the assumption $mgh = \frac{1}{2}k\Delta x^2$, we derived an equation that varies with displacement given mgh. To do so we first multiplied our tested masses by gravitational acceleration (g) and height (L+Δx), giving mgh. Then, we plotted mgh over displacement (Δx) for each mass.

Table 1 Recorded displacement of a cord of unstretched length 0.22m with the addition of varying masses.

Mass ***m*** (kg) (±0.01kg)	Unstretched Length ***L*** (m) (±0.01m)	Displacement $\Delta \boldsymbol{x}$ (m) (±0.01m)	Height ***h*** (m) (±0.01m)
0.17	0.22	0.78	1.00
0.15	0.22	0.75	0.97
0.13	0.22	0.66	0.88
0.11	0.22	0.58	0.80
0.09	0.22	0.50	0.72

Table 2 Gravitational potential energy computed by multiplying h, g and m at certain mass. Displacement of the cord at each mass is also recorded.

Displacement $\Delta \boldsymbol{x}$ (m) (±0.01m)	Gravitational Potential Energy ***mgh*** (J)
0.78	1.67
0.75	1.43
0.66	1.12
0.58	0.86
0.5	0.64



Graph 1 Displacement vs. Gravitational Potential Energy of an Unstretched Cord (0.22m). The slope, 2.705 Nm, varies with displacement and is comparable to 0.5k.

Using our experimental data,

$$mgh = 2.705\Delta\text{x}^{2.0977}$$

Comparable to

$$mgh = \mu k(L)\Delta x^2$$

Plug in k(0.22m) =4.693, so μ =0.576; therefore,

$$mgh = 0.576k(L)\Delta x^{2.0977}$$

Manipulating the derived equation from the intro, use this more precise behavior for the cord to solve for L

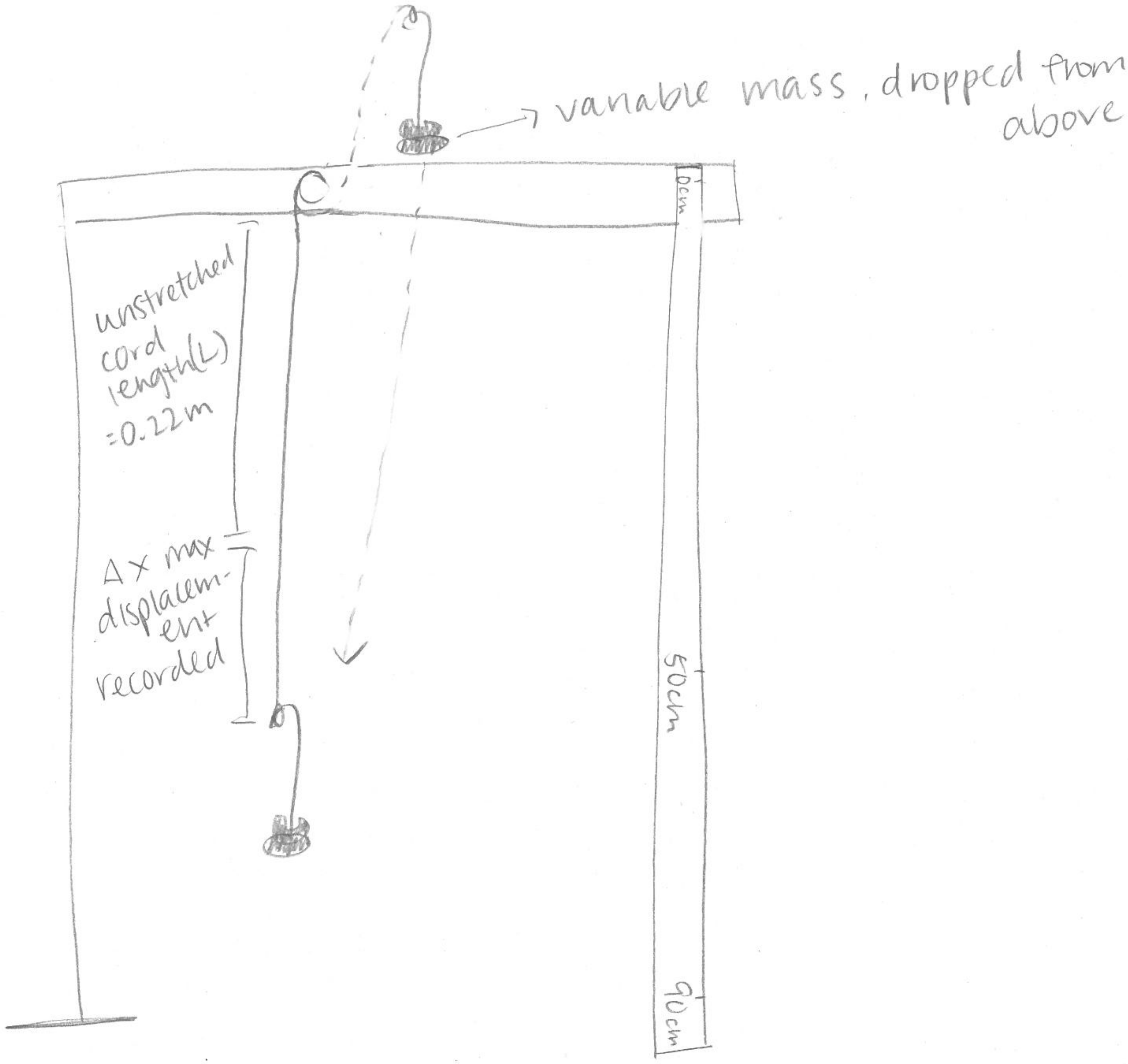
$$L^{2.0977} = h^{2.0977} + \frac{mghL^{0.936}}{0.6286}$$

DISCUSSION:

Because none of our results from releasing the egg have been quantified, uncertainty and error analysis cannot be calculated. However, qualitatively, we can compare our derived equation that models the behavior of our cord, $mgh = 0.576k(L)\Delta x^{2.0977}$, to the accepted behavior of springs, $mgh = 0.5k\Delta x^2$. We are unable to tell whether the slight differences in our model's behavior, including a larger constant accompanying k and a larger exponential value, will significantly affect our predicted L values. However, because the goal of the bungee lab is to give as accurate a prediction for L given h and m, we will continue to use $0.576k(L)\Delta x^{2.0977}$ in our derivation of an equation for L. One possible reason why our cord acts slightly different from an ideal spring include the fact that we dropped the masses from high elevation rather than releasing from unstretched height. Another reason may be that our cord has some weight of its own. Our cord also swayed a bit from side to side after it was released.

CONCLUSION:

Combining two reports, we aimed to derive an equation that would accurately predict a safe length for an egg's bungee jump given drop height and egg mass. This was done by characterizing the behavior of the cord. Noticing that the spring constant of the cord varies with given unstretched cord length, we first derived an equation to represent k at different cord lengths. Then, we noticed that the cord may not behave like an ideal spring and determined its actual behavior by plotting mgh against displacement. Finally, combining these two characteristics together, we were able to manipulate the CWE equation that describes our bungee system from start to finish to derive an equation for L given h and m.



variable mass, dropped from above
0cm
unstretched cord length(L) =0.22m
Δx max displacement recorded
50cm
90cm