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TITLE: Experimental Characterization of Bungee Cord Displacement – Testing the Assumptions of Hooke's Law

ABSTRACT: This report presents an experimental characterization of the relationship between bungee cord initial potential energy and final kinetic energy in terms of cord displacement for a dynamic system. The primary purpose was to determine whether the bungee cord behaved sufficiently close to an ideal spring when greatly elongated, and to determine any necessary adjustments to make in predicting bungee displacement. Cord length (L) and spring constant (k) were held constant and hanging mass (m) of the system was varied for two trials each at five mass values. Slow-motion video recording was used with measuring tape to determine height (h) – full length of stretched cord in the dynamic system – and bungee cord displacement (dx). According to CWE theorem formulations and Hooke's Law, total energy of the system is conserved during bungee fall such that mgh = $1/2k(dx)^2$ if the bungee acts as an ideal spring under constant acceleration by gravity (g). The experimental relationship of mgh versus dx for the bungee cord was plotted and a power fit was conducted. The experimentally derived power of dx was 1.643 ± 0.003. The cord k-value divided by the constant of the power fit, 2.044 ± 0.007, provided the experimentally derived denominator for the fraction of k, 1.918 ± 0.012. These values represented the adjustments made for Hooke's Law assumptions in the final experimentally derived model for free fall of a bungee system, mgh = 1/1.918*(1.403L^(-0.945))*(h-L)^1.643, which predicted height from known mass and cord length with 4.501% error.

INTRODUCTION:

In a bungee jump, damage can be incurred to the jumper by impact with the ground or deceleration that is too large. The project goals of the Bungee Challenge include designing a quality bungee cord experience in which a jumper (the egg) avoids damage while experiencing thrill from the maximum speed achieved and resulting deceleration.

According to the CWE theorem and conservation of energy, the total energy of the system at the top prior to bungee jump should be equal to the total energy of the system at the bottom after fall (see Equation 1). Potential energy at the bottom is negligible and there is no kinetic energy at the top prior to bungee jump. For an ideal spring, Hooke's law is assumed (see Equation 2) and potential energy can be determined as a function of weight and height (see Equation 3). Height can be equated to the sum of cord length and dynamic displacement (see Equation 4). Therefore, the theoretical formulation for the relationship between potential energy and kinetic energy in terms of bungee cord displacement in a dynamic system is shown in Equation 5.

In the final Bungee Challenge, gravity will be known (9.81 m/s^2), height will be provided by the instructors, and mass will be measured for the egg provided. Because dx is equal to the difference of height and cord length, the only unknown will be cord length. Therefore, the purpose of this experiment in furthering the project goals was to determine an equation for the length of cord needed to bring the egg as close to the ground as possible without causing impact and damage. It was expected that the bungee cord would not meet assumptions of an ideal spring and that adjustments to the theoretical formulation for the relationship between initial potential energy and bungee cord displacement would be necessary.

EQUATION 1: $(PE + KE)\ top = (PE + KE)\ bottom$
EQUATION 2: $KE = ½k(dx)^2$
EQUATION 3: $PE = mgh$
EQUATION 4: $h = L + dx$
EQUATION 5: $mgh = ½k(h-L)^2$

PE = potential energy (J)
KE = kinetic energy (J)
k = spring constant
dx = bungee cord dynamic displacement (m)
m = mass (kg)
h = height of drop (m), determined as maximum height – minimum height during dynamic fall
L = bungee cord length (m)

METHODS:

This report presents the experimental determination of the relationship between conserved energy and bungee displacement in a dynamic system. A determination of potential energy of the system prior to bungee drop was needed for comparison to bungee displacement. The power curve relating mgh and dx was used to determine the extent to which Hooke's Law assumptions were held for the bungee cord in comparison to an ideal spring. Cord length (L) and spring constant (k) were held constant and hanging mass (m) of the system was varied for two trials each at five mass values.

Figure 1: *Diagram Set-Up*. Experimental set-up for bungee system attached to stationary fixture. Knotted loops connected bungee cord to stationary fixture and hanging mass to bungee cord. **h** (m) was the height of the full drop (maximum height – minimum height), **L** was the bungee cord length (m), **m** was the mass of the hanging mass (kg), and **dx** was the cord displacement during the drop (m). *A* shows determination of cord length (without hanging mass), *B* shows initial set-up before drop and *C* shows bungee position at maximum height of drop.

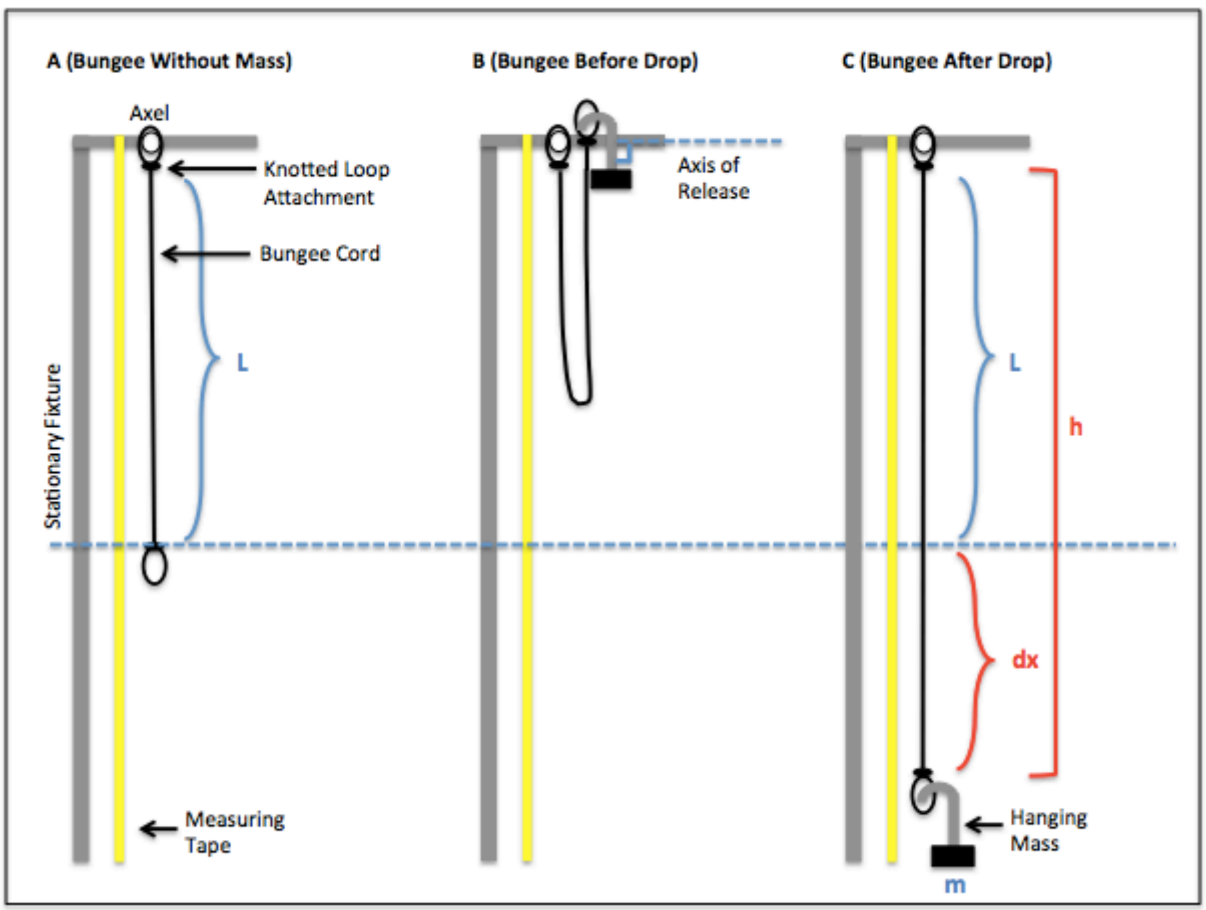


Figure 1 illustrates the general experimental setup for the bungee system with an attached hanging mass. The bungee cord was attached to a stationary fixture by a knotted loop free to move around an axel. A measuring tape was secured with its origin at this location (and increasing length measurement moving downward towards the ground). Cord length was measured by measuring tape from bottom of knot attachment with stationary fixture to top of knot attachment with hanging mass. Constant k-value was determined for this length using a previous experimental derivation for the relationship between k and cord length, k = 1.4026L ^(-0.945). Height (h) value was the full length of stretched cord in free fall and was determined by slow-motion video camera and measuring tape. Cord displacement (dx) was determined as the difference of height and cord length. Constant acceleration (9.81 m/s^2) by gravitational force (g) was assumed. For each experimental trial, the following steps were taken:

- Partner 1 ensured proper attachment of the bungee cord to the axel of the stationary fixture.
- Partner 1 attached the hanging mass to the bungee cord via a knotted loop. Additional mass added to the hanging mass were secured with tape (to avoid separation during the dynamic drop).
- Partner 1 held the knot (attaching the bottom of the bungee cord to the hanging mass) at the leveled release point, with the hanging mass perpendicular to the axis of release.
- Partner 2 prepared the slow-motion video camera (of an iPad), at a height even with the projected maximum height of the bungee cord with hanging mass upon drop.
- Partner 2 alerted partner 1 when the video recording start button was selected, and partner 1 dropped the bungee cord with attached hanging mass.
- When not in use, the hanging mass was removed from the bungee cord to avoid unnecessary cord stretching.

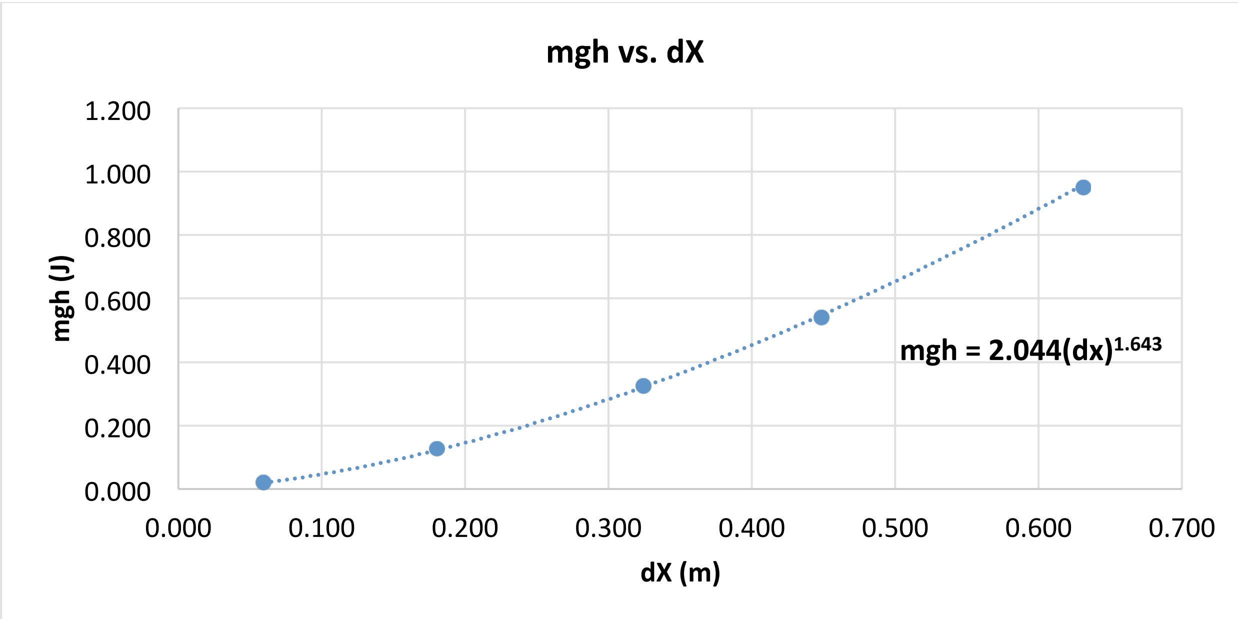
RESULTS:

In general, the data for this report relates initial potential energy at the top (prior to bungee drop) to final kinetic energy at the bottom (after bungee drop) of a dynamic bungee system in terms of cord displacement (dx) as displayed in Figure 1. The product of mass, gravity, and the average of measured height for each mass was used to determine initial potential energy values (mgh) as listed in Table 1. Average cord displacement in the dynamic drop for each mass is also listed in Table 1.

Table 1: Raw data and averages for height and cord displacement from two trials per mass added to a bungee system, used to determine relationship between starting potential energy and ending kinetic energy in terms of displacement.

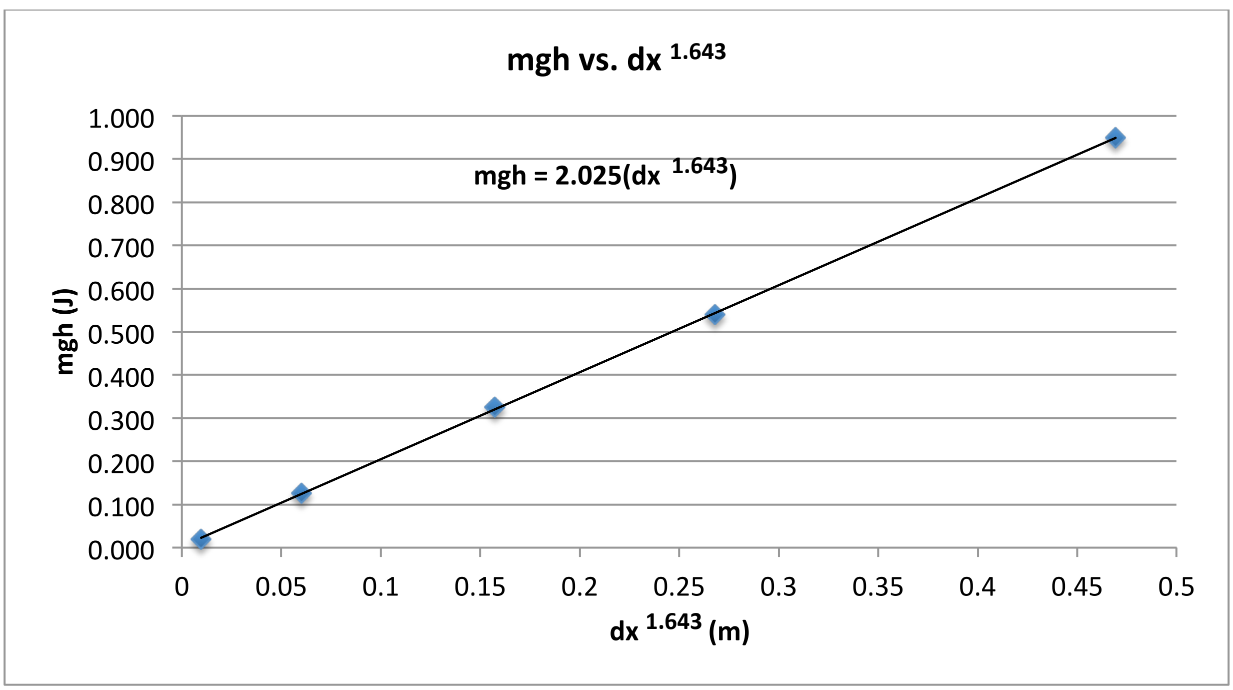
Cord Length: 0.337m			k: 3.920				
Mass (kg)	**Trial 1: h (m)**	**dX (m)**	**Trial 2: h (m)**	**dX (m)**	**Avg. h (m)**	**Avg. dX (m)**	**mgh (J)**
0.005	0.400	0.063	0.393	0.056	0.397	0.060	0.019
0.025	0.518	0.181	0.517	0.180	0.518	0.181	0.127
0.050	0.660	0.323	0.663	0.326	0.662	0.325	0.324
0.070	0.785	0.448	0.786	0.449	0.786	0.449	0.539
0.100	0.962	0.625	0.974	0.637	0.968	0.631	0.950

Figure 2: ***Relationship of Initial Potential Energy of Bungee System Versus Cord Displacement***. The power curve describes the relationship between cord displacement (dx) and potential energy of the bungee system for conserved energy (initial potential energy, mgh, equal to final kinetic energy).



Power Curve Equation: $mgh = 2.044(dx)^{1.643}$

Figure 3: ***Linearized Relationship of Initial Potential Energy of Bungee System Versus Cord Displacement***. The slope describes the linearized relationship between cord displacement (dx) and potential energy of the bungee system for conserved energy (initial potential energy, mgh, equal to final kinetic energy).



Linear Equation: $mgh = 2.025(dx^{1.643})$

Excel Regression Analysis:

uncertainty for slope = 0.007m   % uncert = 0.346%

Experimental Values of Interest:

Experimental denominator for fraction of k: The value obtained was 1.918 ± 0.012. The k-value of the cord, 3.920 ± 0.025, was determined from the previous experimentally derived formulation comparing cord length and k-value. The k-value divided by the constant of the power curve equation, 2.044 ± 0.007, provided the experimentally derived denominator for the fraction of k. The uncertainty was determined from the propagation of uncertainty for uncertainty in k divided by uncertainty in the power fit constant.

Experimental power of dx: The value obtained was 1.643 ± 0.003. This was determined from the graph of mgh versus dx. The uncertainty was determined from the average standard error across two trials for each mass value.

The experimental denominator for fraction of k and the experimental power of dx were both utilized in determining the experimental formulation for the relationship between bungee cord initial potential energy and final kinetic energy in terms of cord displacement for a dynamic system,
mgh = 1/1.918*(1.403L^(-0.945))*(h-L)^1.643. Because the sum of cord length and cord displacement is equal to the height of drop, dx was replaced in the experimental formulation with (h-L).

DISCUSSION

Experimental and theoretical formulations of the relationship between initial potential energy and final kinetic energy in terms of cord displacement based on Hooke's Law assumptions for a bungee system were compared. The purpose of this experiment was to explore the degree to which Hooke's Law assumptions could be accepted; therefore, the "actual" bungee cord behavior in terms of these formulations was unknown.

In this experiment, potential sources of uncertainty included stretching of the tied bungee loops (for attachment of cord to stationary fixture and hanging mass to cord) and limitations in the quality of the slow-motion video recording. The attachment loops were created using the bungee cord itself; therefore, these loops likely stretched in the same manner as the bungee cord during the dynamic drop. This addition in stretching cord length may have resulted in estimations of cord displacement and height of drop that were lower than actual values. Measuring tape readings were made difficult by blurred video recordings. There may have been increased uncertainty in estimations of cord displacement and height of drop due to this systematic error in blurred measuring tape readings.

A test for acceptability was created to determine whether the experimental model for the bungee jump or the theoretical model more accurately predicted the height of drop from known bungee cord length and mass of the system. One trial was completed in which the same procedures were utilized (as described in the Method section) to obtain a slow-motion video recording for the drop of the bungee with hanging mass. For a bungee cord of length 0.698m, actual height of drop (obtained experimentally) was 1.455m.

The experimental model predicted h = 1.522m, which was 4.61% error from the actual height of drop. The theoretical model based on Hooke's Law assumptions, predicted h = 1.800m, which was 23.7% error from the actual height of drop. Because percent error for the experimental model was over five times smaller than percent error for the theoretical model based on the actual height of drop, the experimental model should be considered acceptable for use in the Bungee Challenge. These results support the hypothesis that the bungee cord would not meet assumptions of an ideal spring and that adjustments to the theoretical formulation for the relationship between initial potential energy and bungee cord displacement would be necessary. Furthermore, the experimental model resulted in a conservative prediction for height of drop; the predicted height value for known cord length and mass was above the actual height of drop. Therefore, use of this model during the final Bungee Challenge may further ensure that the egg is not damaged during the bungee drop due to floor impact by predicting conservatively.

CONCLUSION:

The proposed purpose of this report was to determine an equation for the length of cord needed to bring the egg as close to the ground as possible without impact for the final Bungee challenge when height of drop, mass, and gravity will be known. The experimentally derived formulation relating initial bungee cord potential energy (before drop) to final kinetic energy (at maximum height of drop) in terms of dx may be used to determine cord length for the Bungee Challenge.

The experimentally derived formulation does not consider the impact of multiple bungee cords together or of stationary string in the bungee system on cord displacement or length required to meet drop requirements. Therefore, future studies may aim to determine the benefit of multiple cords or stationary string on creating the "thrill" factor of the bungee jump – maximum possible speed and deceleration. Furthermore, prior to the final Bungee Challenge, this model should be tested to ensure that total force of the bungee system does not exceed 3mg to ensure safety of the egg jumper.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Emily Perszyk