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TITLE: *Evaluating the effects of hanging mass and unstretched bungee cord length on the maximum stretching distance of a bungee*

ABSTRACT:

Bungee cords behave elastically, allowing them to stretch beyond their original length then ultimately return to that length. In order to predict the maximum stretching distance of a bungee cord, we have to evaluate the respective roles of the hanging object's mass and the unstretched bungee cord's length. Given the mass of an attached object, we intend to gain the ability to predict the maximum displacement from equilibrium (**Δx)** of a bungee cord of a given unstretched length ($\mathbf{x_0}$). To experimentally evaluate this relationship, we dropped a hanging mass attached to the end of a bungee cord and used a slow-motion video recording app to find **Δx**. Applying this procedure for three different unstretched bungee lengths with a fixed hanging mass, we were able to find the relationship between **Δx** and $\mathbf{x_0}$ at that given mass, characterized by the slope of the graph's trendline. Changing the hanging object's mass twice more and repeating this process, we used the slopes from these **Δx** versus $\mathbf{x_0}$ graphs for a given mass to plot $\mathbf{\Delta x/x_0}$ versus mass. This provided us with the ability to predict the relationship between maximum displacement from equilibrium and unstretched bungee length as a function of the hanging object's mass. The linear equation we found for this relationship was $\mathbf{\Delta x/x_0}$ = 4.1969(**mass**) + 0.3733. Thus, if we are told the mass of the hanging object and we desire to stretch the bungee a certain distance from the equilibrium, we have the necessary tools to choose the appropriate unstretched bungee length to accomplish this goal.

INTRODUCTION:

In order to ensure that a bungee cord does not stretch beyond a given distance when a hanging object of mass ***m*** is attached, we need to develop a better understanding of our bungee cord's properties. In particular, we must evaluate the respective roles of the mass's size and the original length of the bungee cord that we choose. In the bungee experiment at the end of the semester, accurately predicting the maximum stretching distance from equilibrium (**Δx**) of our bungee system will allow us to closely approach the ground without breaking the egg due to impact. The equilibrium position of the bungee system ($\mathbf{x_{eq}}$) is the distance from the origin where the attached mass, when released from that position, will be in steady state and thus not oscillate upward or downward. In order to theoretically predict **Δx**, we need to know the mass of the hanging object and the spring constant (***k***) of the bungee (experimentally determined in the last bungee lab as $\boldsymbol{k = 1.1692(unstretched\ length)^{-1.048}}$ (eq. 1)). Using equation 1 allows us to deal with an easily measured variable in our measurements such as unstretched length ($\mathbf{x_0}$), while **k** can be experimentally found (as we did during the last bungee lab) but not directly measured with the instruments available to us. It is important to find **k** because it is a piece of the story that informs us of the stretching characteristics of or bungee in a way that unstretched bungee length cannot. By manipulating the mass and unstretched bungee length, we will observe different **Δx** values. The experimentally observed **Δx** can be used to derive a corresponding model to predict **Δx**, which will inform us of the respective roles of changes to mass and unstretched bungee length on a real-world bungee's stretching behavior. The uncertainty in our experimental model can help us evaluate whether there are other factors we need to consider in calculating the unstretched length of bungee cord to use during the bungee challenge.

We hypothesize that increasing the original bungee cord length or the mass of the hanging object will result in a proportional increase in the maximum stretch length of the bungee cord. If the uncertainty of our experimental model is sufficiently low, we will use this model as part of our bungee experiment to predict how much unstretched bungee cord should be used to stretch the cord with an attached hanging mass, ***m***, a specific distance that will approach but not impact the ground.

METHODS:

In order to ensure the bungee system can reach maximum stretching distance without impacting the floor, we used an L-shaped hanging apparatus that suspended the bungee system above the ground. The bungee system was tied in a knot to the top of the hanging apparatus, while the hanging mass was attached to a knot tied at the distal end of the bungee cord. To ensure consistent data, lengths (i.e. $\mathbf{x_0}$, $\mathbf{x_{eq}}$, and $\mathbf{x_{max}}$) were measured from the top knot to the bottom knot.

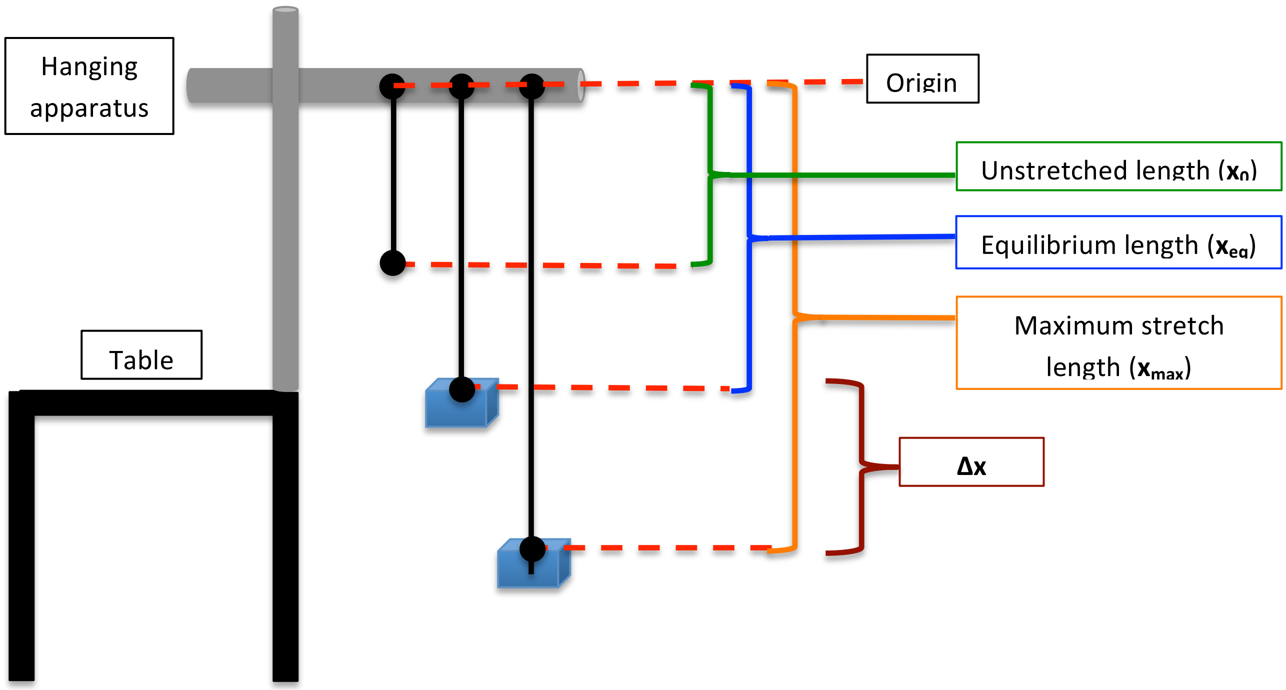


Figure 1: Diagram of the bungee cord experimental setup, where $x_{max} - x_{eq} = \Delta x$ (eq. 2).

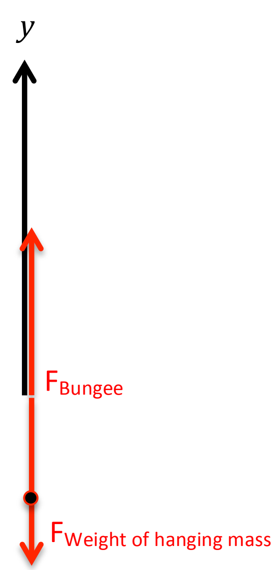


Figure 2: Free body diagram of the bungee system

Prior to dropping the bungee system from the origin, we measured unstretched bungee cord length ($\mathbf{x_0}$), the mass of the hanging object, the height of the top knot relative to the ground, and the equilibrium position of the bungee system with hanging mass attached ($\mathbf{x_{eq}}$). Upon dropping the mass, we used a slow-motion video recording app to capture the maximum stretch distance from the origin ($\mathbf{x_{max}}$). Subtracting $\mathbf{x_{eq}}$ from $\mathbf{x_{max}}$ provided us with **Δx** (equation 2), the maximum stretch distance from equilibrium. We repeated this process using the same mass of the hanging object, changing only the unstretched bungee length. The purpose of doing so was to identify the relationship between the unstretched bungee length and **Δx** at a specified mass. We repeated the same procedure, using three different unstretched bungee lengths for at least two other hanging masses.

With the collected data, we graphed maximum bungee displacement from equilibrium (**Δx**) against unstretched bungee length (**x_0**) for each of our three masses. We fit a linear trendline through the points on each graph and used the equation of the trendline as a means to quantify the relationship between **Δx** and **x_0** for each of the three masses. Plotting the slope of each graph's trendline against its corresponding mass allowed us to model how the relationship between **Δx** and **x_0** changes as a function of mass. This experimental model could prove useful in the bungee experiment, in which we will be provided the mass of the hanging object and expected to select the unstretched bungee length (**x_0**) required to drop the attached hanging mass as close as possible to the ground from a certain drop height. Since we are able to predict **Δx** from **x_0** at a given mass, and since we indirectly explored the relationship between **x_0** and **x_{eq}** for a given mass during the first bungee lab, we can therefore synthesize the two bungee labs using equation 2 to find the maximum stretch length of the bungee from the origin (**x_{max}**) for any given mass. This will allow us to drop the egg as close as possible to the ground during the bungee experiment while avoiding impact.

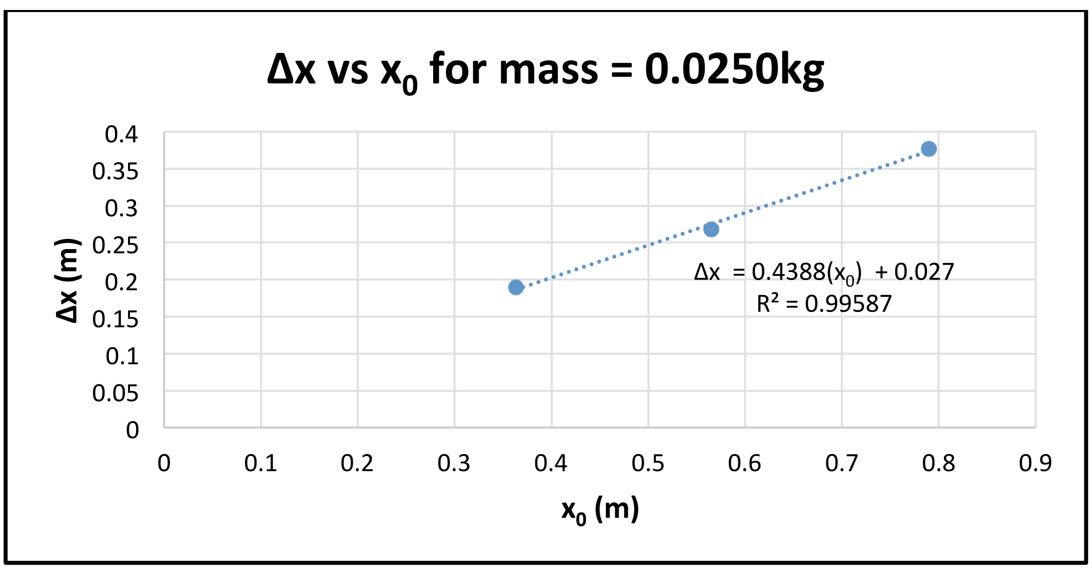
RESULTS:

Maximum displacement from equilibrium vs. unstretched bungee length

We experimentally measured the maximum displacement of the bungee system under conditions of a changing unstretched bungee length while keeping hanging mass constant. We repeated this for three different masses.

mass (kg) ±0.0001	**x_{eq}** (m) ±0.001	**height** (m) ±0.001	**X_0** (m) ±0.001	**k** (N/m)	**x_{max}** (m) ±0.001	**Δx** (m) ±0.0014
0.0250	0.873	2.16	0.790	1.497	1.25	0.377
0.0250	0.632	2.15	0.565	2.127	0.90	0.268
0.0250	0.410	1.92	0.363	3.381	0.60	0.190

Table 1: Raw data for a hanging object with a constant mass of 0.0250kg.



Graph 1: Relationship between x_0 and Δx at a constant hanging object mass of 0.0250kg.

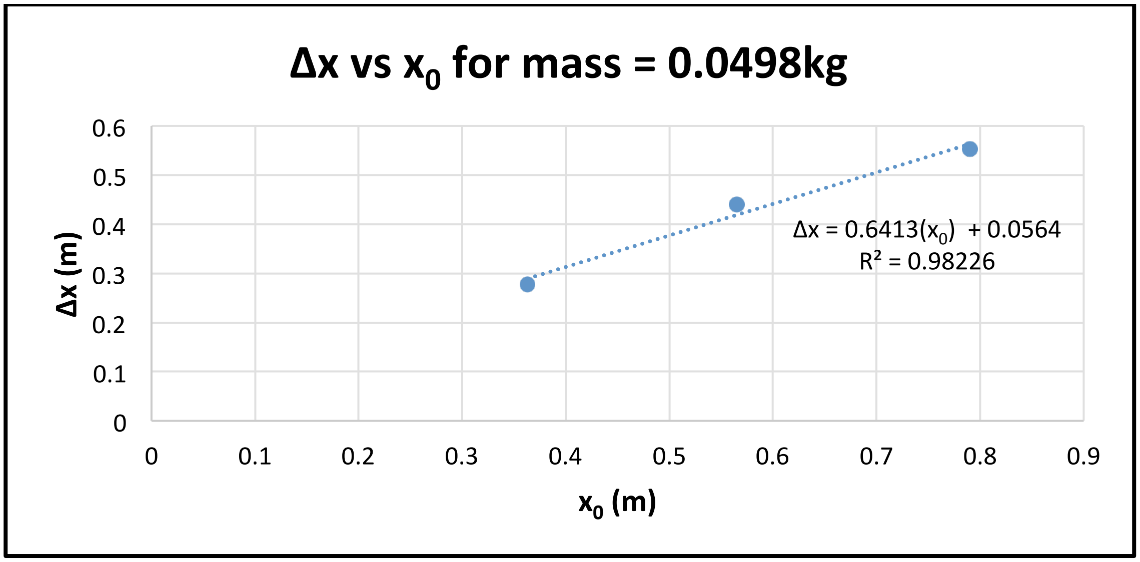
Equation of line: **Δx** = 0.4388(**X_0**) + 0.027

Uncertainty of slope: 0.028 % uncertainty: 6.4%

We are interested in the coefficient of x_0, 0.4388, because this is the slope of the graph and it relates the unstretched bungee length to the maximum length of bungee stretch from equilibrium. The uncertainty for this variable of interest was found using regression analysis in excel.

Mass (kg) ±0.0001	x_{eq} (m) ±0.001	**height** (m) ±0.001	X_0 (m) ±0.001	**k** (N/m)	X_{max} (m) ±0.001	**Δx** (m) ±0.0014
0.0498	0.945	2.11	0.790	1.497	1.498	0.553
0.0498	0.690	2.15	0.565	2.127	1.130	0.440
0.0498	0.445	1.92	0.363	3.381	0.723	0.278

Table 2: Raw data for a hanging object with a constant mass of 0.0498kg.



Graph 2: Relationship between x_0 and **Δx** at a constant hanging object mass of 0.0498kg.

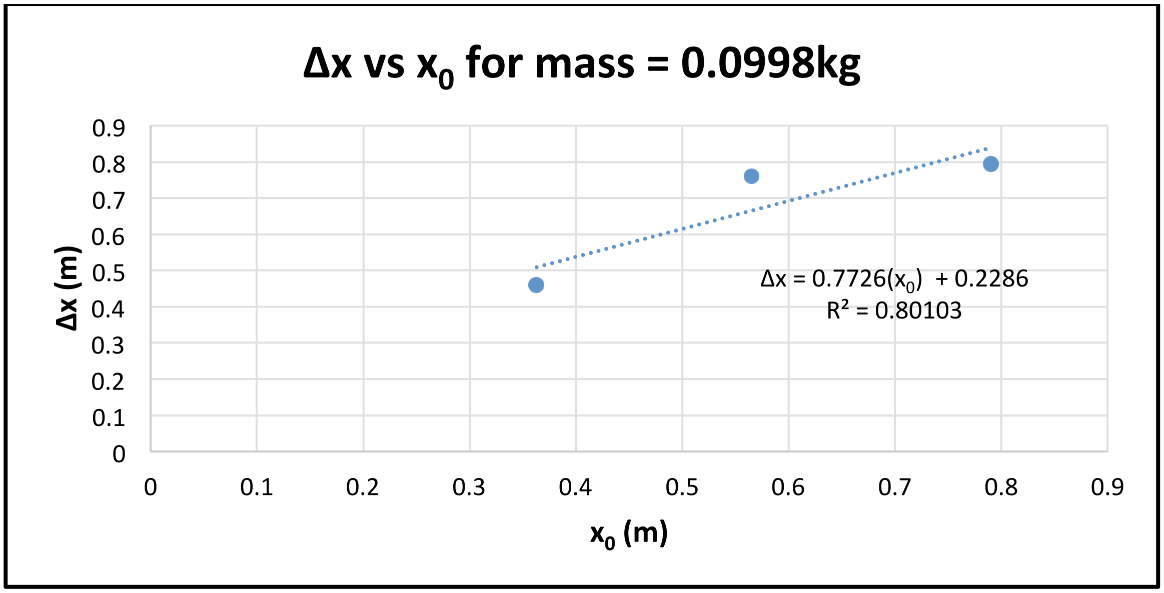
Equation of line: **Δx** = 0.6413(X_0) + 0.0564

Uncertainty of slope: 0.086 % uncertainty: 13.4%

We are interested in the coefficient of x_0, 0.6413, because this is the slope of the graph and it relates the unstretched bungee length to the maximum length of bungee stretch from equilibrium. The uncertainty for this variable of interest was found using regression analysis in excel.

mass (kg) ±0.0001	x_{eq} (m) ±0.001	**height** (m) ±0.001	X_0 (m) ±0.001	**k** (N/m)	x_{max} (m) ±0.001	**Δx** (m) ±0.0014
0.0998	1.046	2.16	0.790	1.497	1.840	0.794
0.0998	0.885	2.15	0.565	2.127	1.645	0.760
0.0998	0.564	1.92	0.363	3.381	1.023	0.459

Table 3: Raw data for a hanging object with a constant mass of 0.0998kg.



Graph 3: Relationship between $\mathbf{x_0}$ and $\mathbf{\Delta x}$ at a constant hanging object mass of 0.0998kg.

Equation of line: $\mathbf{\Delta x} = 0.7726(\mathbf{X_0}) + 0.2286$

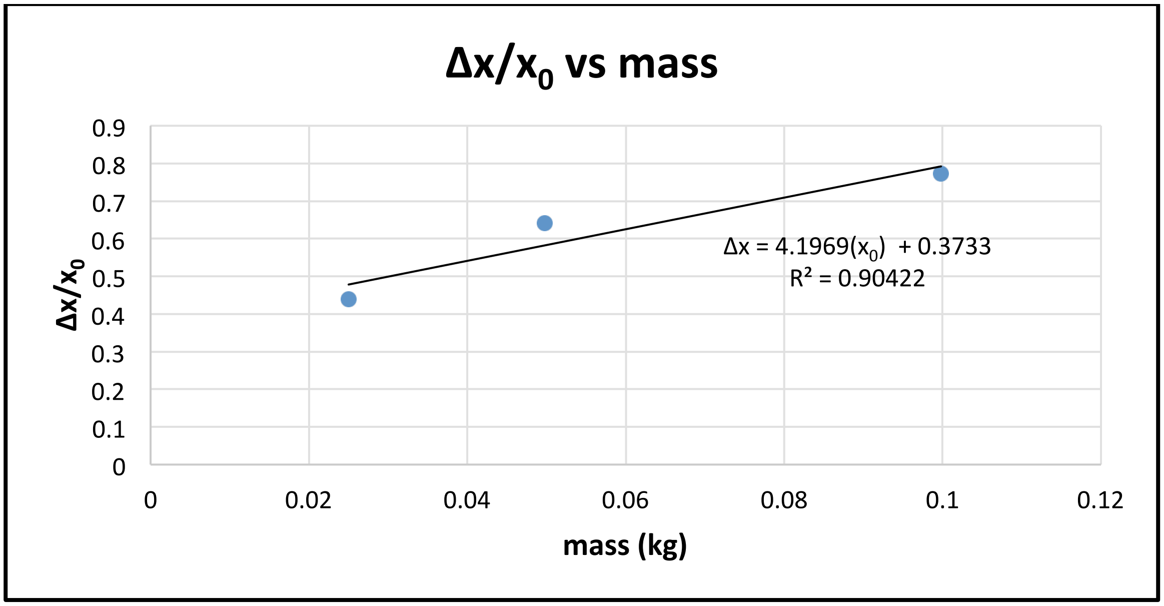
Uncertainty of slope: 0.385 % uncertainty: 49.8%

We are interested in the coefficient of $\mathbf{x_0}$, 0.7726 because this is the slope of the graph and it relates the unstretched bungee length to the maximum length of bungee stretch from equilibrium. The uncertainty for this variable of interest was found using regression analysis in excel.

The trendline equation slopes from graphs 1, 2, and 3 inform us of the relationships between $\mathbf{x_0}$ and $\mathbf{\Delta x}$ for each hanging mass. For example, with a hanging mass of 0.998kg, we can predict that an unstretched bungee length of 0.5m will result in a $\mathbf{\Delta x}$ equal to 0.7726*(0.5m) + 0.2286, or 0.6149m. While this is a powerful tool for each particular hanging masses, in order to gain the ability to predict the relationship between $\mathbf{x_0}$ and $\mathbf{\Delta x}$ for any mass, we must make mass the independent variable and the relationship between $\mathbf{x_0}$ and $\mathbf{\Delta x}$ the dependent variable. To do so, we plotted the slopes from graphs 1, 2, and 3 against their corresponding masses (Graph 4).

mass (kg) ±0.0001	$\mathbf{\Delta x / x_0}$
0.0250	0.4388
0.0498	0.6413
0.0998	0.7726

Table 4: Compiled data of $\mathbf{\Delta x / x_0}$ relationship and corresponding mass from graphs 1, 2, and 3.



Graph 4: Plotted relationship between $\Delta x/x_0$ at three different masses of interest.

Equation of line: $\Delta x/x_0$ = 4.1969(**mass**) + 0.3733

Uncertainty of slope: 1.37 % uncertainty: 32.5%

Uncertainty of y-intercept: 0.09 % uncertainty: 24.1%

We are interested in the coefficient of the mass, 4.1969, because this is the slope of the graph and it relates the hanging object's mass to the predicted relationship between **Δx** and x_0. The uncertainty for this variable of interest was found using regression analysis in excel.

DISCUSSION:

This experiment revealed that there is a directly proportional relationship between maximum bungee displacement from equilibrium and the unstretched bungee length, and that the ratio of **Δx** to x_0 (the slope of the relationship between these two variables) increases with increasing hanging mass. We attribute this increase in $\Delta x/x_0$ to an increased downward force on the bungee system due to the force of weight as mass increases.

Although a polynomial, logarithmic, or power equation would have improved correlation of our data points of $\Delta x/x_0$ versus mass, we chose to fit a linear trendline to these data. The reason we did so was because of the relatively large uncertainty of the slope—32.5%--in addition to the range of uncertainties of the slopes relating **Δx** and x_0 (6.4% for 0.0250kg, 13.4% for 0.0498kg, and 49.8% for 0.998kg). What this shows us is that more than three data points are necessary to relate **Δx** and x_0 at each of the three masses. We expect that more data would likely decrease uncertainty and provide us with more confidence in our equations.

There were several potential sources of uncertainty within our experiment that may have accounted for our large uncertainty. The slow motion video recording app could only be slowed down to a certain extent when reviewing video footage, making precise measurements of x_{max} quite difficult. With these imprecise measurements, the relationship between **Δx** and x_0 at any given mass or unstretched bungee cord length may have been recorded incorrectly. Another possible source of uncertainty could be the initial velocity of the dropped hanging mass. Ideally, the initial velocity of the hanging mass would be zero at the time of drop. However, if there was an initial velocity (downward or upward), this could have affected the maximum stretching distance.

Verifying the effectiveness of our final equation $\Delta x/x_0$ = 4.1969(**mass**) + 0.3733, would require selecting a mass and unstretched bungee length different from those measured in this lab, substituting those values into said equation to find a theoretical **Δx**, and then carrying out this setup to find the experimental **Δx**. Using the accepted and actual **Δx** values, if the % error was smaller than the % uncertainty then we could say our result was accurate. All things considered, the uncertainty of our equation was very large, so we might be hesitant to accept the legitimacy of our experimental model even if it was, by definition, "accurate."

The results of this experiment support our original hypothesis that increases in hanging mass or unstretched bungee cord length result in proportional increases in the maximum stretching distance measured from equilibrium. However, our expectation that the uncertainty of this relationship would be sufficiently small to validate using the equation in our bungee experiment did not hold up. Although we have no more lab periods to add data or explore the properties of our bungee in greater depth, we will still predict how much unstretched bungee length to use in the bungee experiment using the experimental model described in this paper. Additionally, we will use the Bungee Journal as a resource to evaluate and incorporate into our bungee challenge any bungee properties that have widespread support from the papers written by our peers.

CONCLUSION:

Based on the data that we collected, interpreted, then analyzed, we believe the experimental uncertainty of our model to predict **Δx** at a given $\mathbf{x_0}$ and mass causes room for concern but also holds some merit. The largest hanging mass tested (0.0998kg) qualitatively appeared to have a **Δx** to $\mathbf{x_0}$ ratio very similar to the theoretical ratio (slope of the trendline), potentially implicating improved prediction power of our model with larger masses (the egg is expected to be somewhere between 0.10 and 0.17kg, larger than any mass we tested). This does not, however, indicate that applying the results of this experiment will make for a successful bungee challenge. With no lab time left to replicate this experiment in the hopes of minimizing uncertainty, we are left with more questions than answers regarding our bungee choices the day of the bungee challenge. We maintain a high confidence in using the results of our first bungee lab as an indicator of our bungee's elastic properties. Still, identifying the appropriate length of unstretched bungee cord that will stretch a desired distance remains inconclusive, forcing us to either take our chances with the large uncertainty or to adopt a different methodology based on a literature review of papers from the bungee journal. As it relates to our understanding of bungee cord behavior, this experiment shed light on the complexity of a bungee system, including the forces acting on the system and the variables that must be considered in formulating an experimental model that effectively captures the factors impacting our bungee.

On my honor, I have neither given nor received any unacknowledged aid on this lab report.

Pledge: Shlomo Honig