Brett Becker and Brantley Lawson **Section: Physics 113-02** **Date: 11/14/2016**

Dynamics: Designing a Bungee Jump

ABSTRACT: The purpose of this Bungee II experiment was to use the Conservation of Work and Energy (CWE) Theorem to model the behavior of a mass on a bungee cord. As CWE Theorem indicates, we assumed that the energy of a mass at the top of our bungee cord experiment is the same as the energy at the bottom of our bungee cord experiment. Therefore, the total energy of the system can be modeled by the following equation:

$$E_{top} = E_{bottom}$$
$$(PE + KE)_{top} = (PE + KE)_{bottom}$$
(1) $$\mathbf{mgh = \frac{1}{2}k(\Delta x)^2}$$

**note: m = mass in kg; g = 9.81 m/s²; h = height in m; k = spring constant; Δx = elongation distance of bungee in m*

We experimentally varied mass in order to determine an experimental model of equation (1). This experimental model was determined to be:

(2) $$\mathbf{mgh = (constant)k(\Delta x)^2 = 1.4612\ (\Delta x)^2}$$

Our previously calculated value of k **[k = 1.2372(1/L) – 0.6803]** from our Hooke's Law style static bungee experiment was combined with equation (2) in order to determine an equation that could be used to find the length (L) of a bungee cord necessary for a given mass and height of drop. This is equation (3):

(3) $$\boldsymbol{h = L + \left(\frac{mgh}{\frac{1.075}{L} - 0.5912}\right)^{\frac{1}{1.5789}}}$$

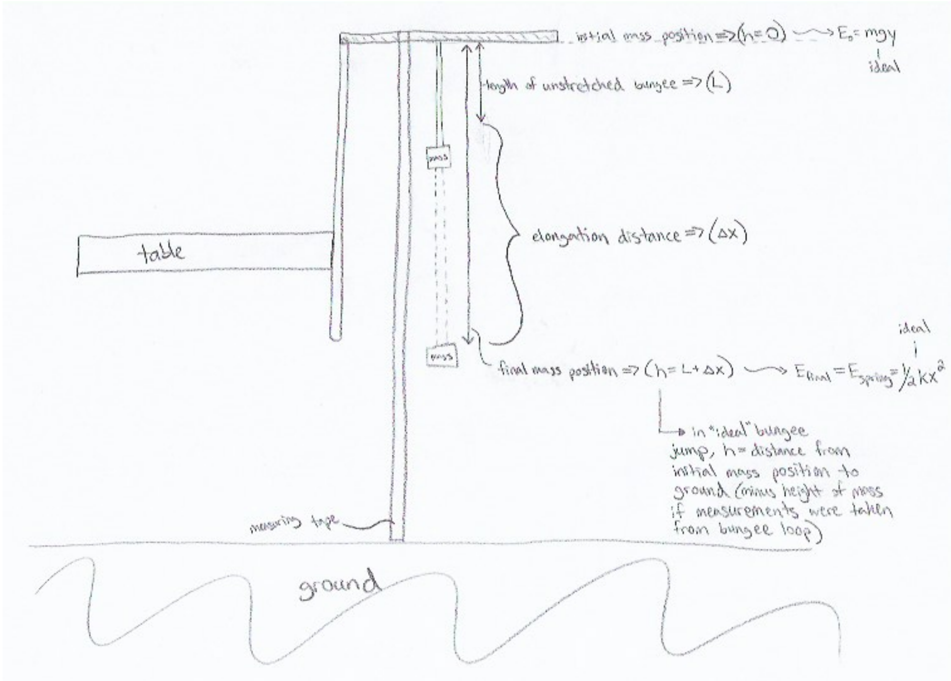
**note: h = height in m; L = length of unstretched bungee cord in m; m = mass in kg; g = 9.81 m/s²*

Equation (3) can be used for our final bungee drop to determine the necessary length of bungee cord (L) for the provided height and mass.

INTRODUCTION:
Purpose/Question: The purpose of this experiment is to use CWE Theorem and our previous calculation of k from Bungee 1 to determine an equation that relates length of unstretched bungee cord to mass and height of drop.
Relevant equations: See equation (1), (2), and (3) above.
Basis: The basis behind this experiment is founded upon CWE Theorem and the assumption that total energy is conserved. The CWE Theorem leads to the algebraically/experimentally determined equation (3) that can then be used to determine length of bungee cord necessary for a given mass and drop height.
Hypothesis: The elongation distance of the bungee cord at unstretched length of 0.524 m will result in a power trendline equation similar to that of equation (1) for CWE Theorem.

METHODS: The overall method of this experiment involved assessing the elongation distance (Δx) for varying masses (0.050 kg to 0.10 kg in increments of 0.01 kg) while maintaining an unstretched bungee length of 0.524 m. This data was then used to model the CWE Theorem equation (equation 1) and determine an experimental equation that relates length (L) with mass (m) and height (h) for our particular system (equation 3).

Figure 1: Diagram of Bungee Cord Drop



- **Procedure:**
    - Bungee cord was set up as indicated in Figure 1 with length of unstretched bungee cord = 0.524 m
    - A set mass of 0.05 kg was added to the bungee cord and dropped from the top-most position (i.e. height = 0 m).
    - The total height (h) of the bungee drop was measured using the measuring tape and a slow motion camera; this total height was recorded.
    - The above two steps were repeated for five total masses (i.e. 0.050 kg, 0.060 kg, 0.070 kg, 0.080 kg, 0.090 kg, 0.10 kg)
    - Elongation distances (Δx) were calculated by subtracting L from h for each trial
    - mgh vs. Δx was plotted to determine a CWE Theorem-like equation
    - This CWE-like equation was combined algebraically with our k value from the static state Bungee 1 experiment to determine the constant for our experimental CWE equation
    - This constant was then used to algebraically determine the final equation relating L with a given mass and height of drop

RESULTS/CALCULATIONS: The most important result of this experiment was equation (3), which has been rewritten below. This was found via the calculations described above in the procedure.

Equation 3:

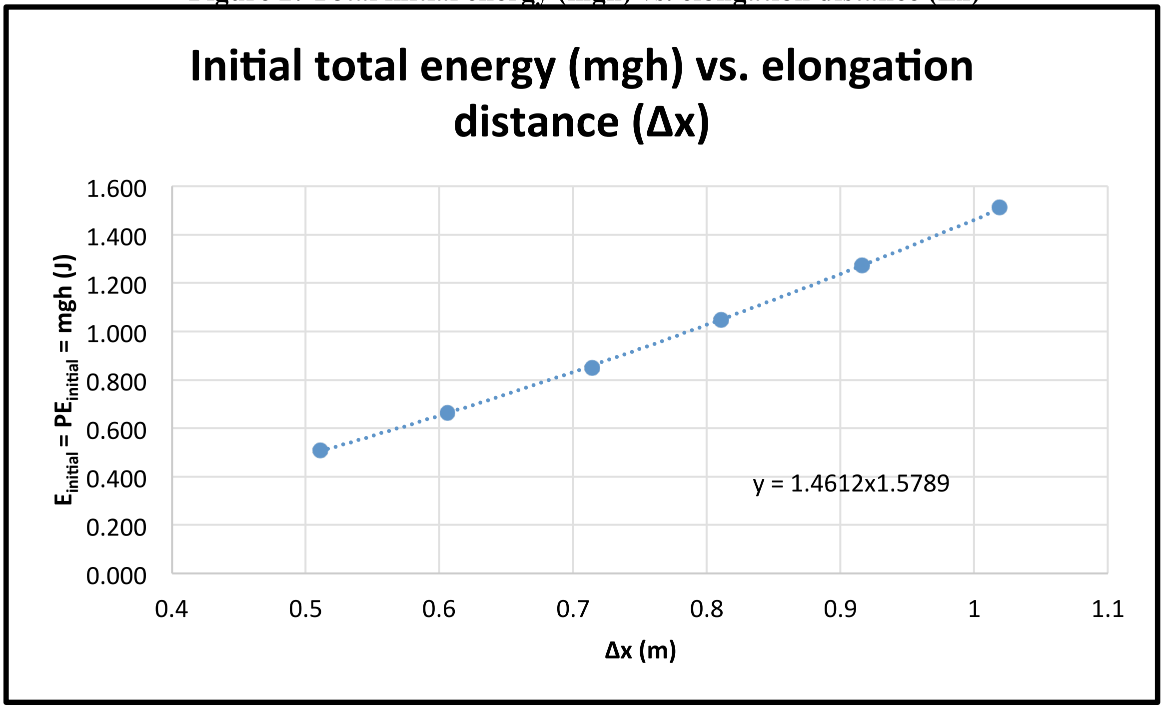
$$\boldsymbol{h = L + \left(\frac{mgh}{\frac{1.075}{L} - 0.5912}\right)^{\frac{1}{1.5789}}}$$

Table 1: Dynamic Bungee II Data

mass (kg)	$E_{initial}$=mgh (J)	L (m)	h (m)	Δx (m)
0.050	0.508	0.524	1.035	0.511
0.060	0.665	0.524	1.130	0.606
0.070	0.850	0.524	1.238	0.714
0.080	1.048	0.524	1.335	0.811
0.090	1.271	0.524	1.440	0.916
0.10	1.514	0.524	1.543	1.019

**Note: This table shows the raw data including: m = mass in kg; E_{final} = mgh = initial energy in J; L = length of unstrecthed bungee cord in m; h = total height of bungee drop in m;* Δx = elongation distance in m

Figure 2: Total initial energy (mgh) vs. elongation distance (Δx)



<u>Experimental equations of interest</u>: Overall, the experimental results of interest are the two related equations of equation (2) and equation (3). Equation 2 ($y = 1.4612x^{1.5789}$) is the experimental representation of CWE Theorem and was used with our previously determined value of k to determine equation 3 $\left(h = L + \left(\frac{mgh}{\frac{1.075}{L} - 0.5912}\right)^{\frac{1}{1.5789}}\right)$, which allows for the determination of unstretched bungee cord length provided a mass and drop height for a bungee drop.

The major result of this experiment was equation (3), which relates the height and mass of a bungee drop to the length of necessary unstretched bungee cord. This equation, developed through a combination of Bungee 1's k value and experimentally determined CWE theorem for varying masses, is vital for the final egg bungee drop during the final week of lab.

DISCUSSION:

The results obtained during this experiment show the relationship between length of unstretched bungee cord (L) and the mass/height provided for a given bungee drop.

Error analysis — The expected theoretical equation was resultant of the CWE Theorem, namely, $mgh = \frac{1}{2} kx^2$. The experimentally determined CWE equation was $mgh = 1.4612x^{1.5789}$, which was rearranged using our static determination of k value to become equation 3. Although there is no way to quantify the percent error of our equation, we can determine the acceptability of equation 3. This acceptability determination will be completed during the last week of lab when we drop the egg for our final bungee jump. If our calculations and equation are correct, the egg will not be harmed and will come sufficiently close to the ground. Additionally, the experimental CWE equation can be compared to that of the theoretical CWE equation. These two equations are not similar, which indicates that energy is not conserved perfectly in our system. We did not have an "ideal" scenario and thus determined our final equation based on our non-ideal data.

Uncertainty could have arisen in this experiment due to rounding errors in algebraic calculations and/or the measurements/measurement devices. Additionally, air resistance, tension, elevation, imprecise drops or measurements, and other factors that are difficult to predict in a lab setting could have affected the results.

The primary results of this experiment were as follows: a) our dynamic system does not exhibit conservation of energy and is not similar to that of the theoretical model of E_{spring} and b) we developed an equation that relates length of unstretched bungee cord with provided values for mass and height of a bungee drop. These results do support our hypothesis that the data would suggest a CWE-like equation and final result; however, we are unsure at this point with regards to the accuracy of our results. This will be determined in the last week of lab through our egg bungee drop.

CONCLUSION: The experiment revealed that our system does not exhibit ideal CWE behavior (i.e. complete conservation of energy in a spring-like manner – see equations 1 and 2). Additionally, this experiment revealed an equation relating mass and height of a bungee drop with the unstretched length of needed bungee cord for that drop – see equation 3. The purpose of this experiment to determine this equation to predict a bungee drop parameters was achieved. The overall implication of this experiment is that the amount of bungee cord necessary for a bungee drop can be determined using equation (3). This is particularly important for the final week of lab when we do the final egg bungee jump.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Brett T. Becker