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Determining k and an Equation that Represents Its Value with Respect to Varying Strand Numbers

## *ABSTRACT:*

The spring constant, k, for a given spring is often provided when it is needed for theoretical calculations. However, we often want to use spring-like cords with unknown spring constants to perform experiments. For example, in the Bungee Challenge, we will need to know the spring constant for the bungee cord material we will employ. In this experiment specifically, we wanted to know how the spring constant, k, changes with respect to the number of strands used. To do so, distances travelled were recorded for a hanger that was allowed to fall from equilibrium position. The distances travelled were measured at three different masses for four different strand numbers. By knowing the distances travelled of a given mass, we were able to calculate the k value for each run; the runs done at a given strand number were averaged, giving four $k_{ave}$ values: 3.36 for 1 strand (with a 5.06% uncertainty), 7.53 for 2 strands (with a 7.53% uncertainty), 10.86 for 3 strands (with an 8.99% uncertainty), and 12.99 for 4 strands (with a 9.88% uncertainty). These values were graphed with respect to the strand number, n, and the line of best fit was found to be $k_{ave} = -0.51\ n^2 + 5.77\ n - 1.92$ with an $R^2$ value of 0.99, showing an extremely high degree of correlation between our experimental data and the trendline. Additionally, data was linearized through graphing the negative distances travelled (-x) versus weight (2mg); the slope of the line of best fit of a graph for each strand number was found to be equal to k. These values were 3.17 for 1 strand (5.35% uncertainty), 7.36 for 2 strands (4.81% uncertainty), 10.86 for 3 strands (0.06% uncertainty), and 12.75 for 4 strands (2.07% uncertainty). Though the $R^2$ value shows uncertainty of the equation of our model and uncertainties of other values were propagated throughout, we are unable to determine the % error of our experimental results because there are no known or expected values from theory. However according to theory, the spring constant is multiplicative; using this, we assumed accuracy for our $k_{ave}$ value for 1 strand and used it to calculate theoretical values and % errors for all other strand numbers to be able to estimate the accuracy of our values. Our calculated $k_{ave}$ values were found to be within the range of accepted values, but our k values calculated through linearization were not. To determine the accuracy of our equation for $k_{ave}$, a test was derived to predict the distances travelled with a known mass and strand number and a calculated $k_{ave}$. In total, we were able to model the k value as it varies with strand number in multiple different ways, and data supported the hypothesis that k would be larger for higher strand numbers. Potential sources of uncertainty include air resistance and the measured equilibrium value.

## *INTRODUCTION:*

Purpose: In this experiment, we want to know if k varies with the number of strands of bungee cord used. We used potential energy theory to manipulate the values of the distances travelled to find $k_{ave}$, the average spring constant. By calculating average $k_{ave}$ values using a different number of strands of cord, we wanted to use these values to find an equation for k that can be evaluated to give the $k_{ave}$ for any value of strands. We also sought to linearize our data, providing an alternative method of calculating the spring constant. After comparison and results, we will use these values in the future when creating our bungee for the Bungee Challenge.

Equations:

$W_{nonconservative} = \Delta KE + \Delta PE = \Delta ME_{total}$

$W_{conservative} = -\Delta PE_{total}$

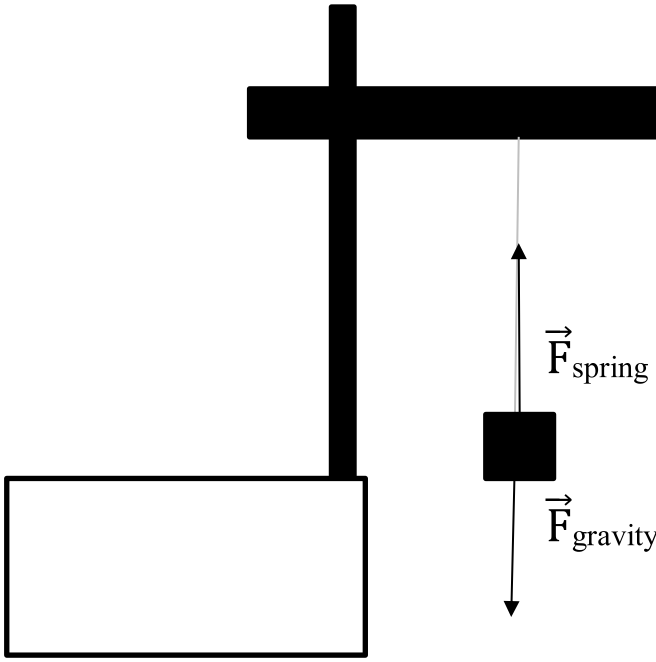



**Diagram 1: Forces on the System:** This diagram shows the forces on the system, the spring force and the force of gravity.

$\Delta PE_{\text{simple harmonic oscillator}} = ½\ k\ (x_f - x_i)^2$
where k = spring constant
where x is the distance travelled (m)
$\Delta PE_{gravity} = mg\ (x_f – x_i)$
where m = mass of hanging object and
where y is distance from the ground, initial and final (m)

$W_{nonconservative} = \Delta KE + \Delta PE = \Delta ME_{total} = 0$ because there are no nonconservative forces acting on the system (both the spring force and gravity are conservative).
$\Delta KE = 0$ because the system starts and stops at rest. Therefore, $\Delta PE = 0$.

Because we know that $\Delta PE_{total} = 0$, we can manipulate the above equations to deduce an equation that gives an experimental value for k consistent with Diagram 1:

$W_{conservative} = -\Delta PE_{total} = - \Delta PE_{spring} - \Delta PE_{gravity}$
$0 = - ½\ k\ (x_f – x_i)^2 - mg\ (x_f – x_i)$
$½\ k\ (x_f – x_i)^2 = -mg\ (x_f – x_i)$
$k = - 2\ mg\ (x_f – x_i)\ /\ (x_f – x_i)^2$
$k = - 2\ mg\ /\ (x_f – x_i)$
$k = -2\ mg\ /\ x$ where x is the distance travelled.

Brief Theoretical Background: Because there is no way to quantitatively measure k, we measured distances during the drop of a mass on a hanger and used them to calculate an experimental value for k. It is known that $\Delta PE_{total} = 0$ J and that by the definition of $W_{conservative}$, $W_{conservative} = 0$ J as well. We can break this down into the $PE_{spring}$ and the $PE_{gravity}$ because these are the only conservative forces on the system. Through manipulation of the known equations, we found the equation k = - 2 mg / x. We determined the distance travelled, x, through experimentation, and the masses are known. We then evaluated to find k.

Hypothesis: I hypothesized that k will vary with strand number used. I thought that a higher strand number would give a higher k because the distance travelled will be smaller.

***METHODS***:

Overall Method: In this experiment, an iPad was used to measure the fall distance of a hanger; this distance was used to find the distance travelled of the hanger. The strand number n, the number of strands of identical bungee cord on which the hanger was hung, was varied. For each strand number, 3 runs were performed, each of a different mass; each of these distances travelled were used to calculate a separate k value using the equation k = -2 mg / x. The three calculated k values per strand number were averaged to give four separate $k_{ave}$ values, one for each strand number used. Strand number was graphed versus $k_{ave}$, and a line of best fit through these points was found. Uncertainty was propagated. Data was also linearized by graphing –x versus W (2mg), as the slope of the graph was found to be equal to k. *Excel* linear regression was used to find the % uncertainty for these values.

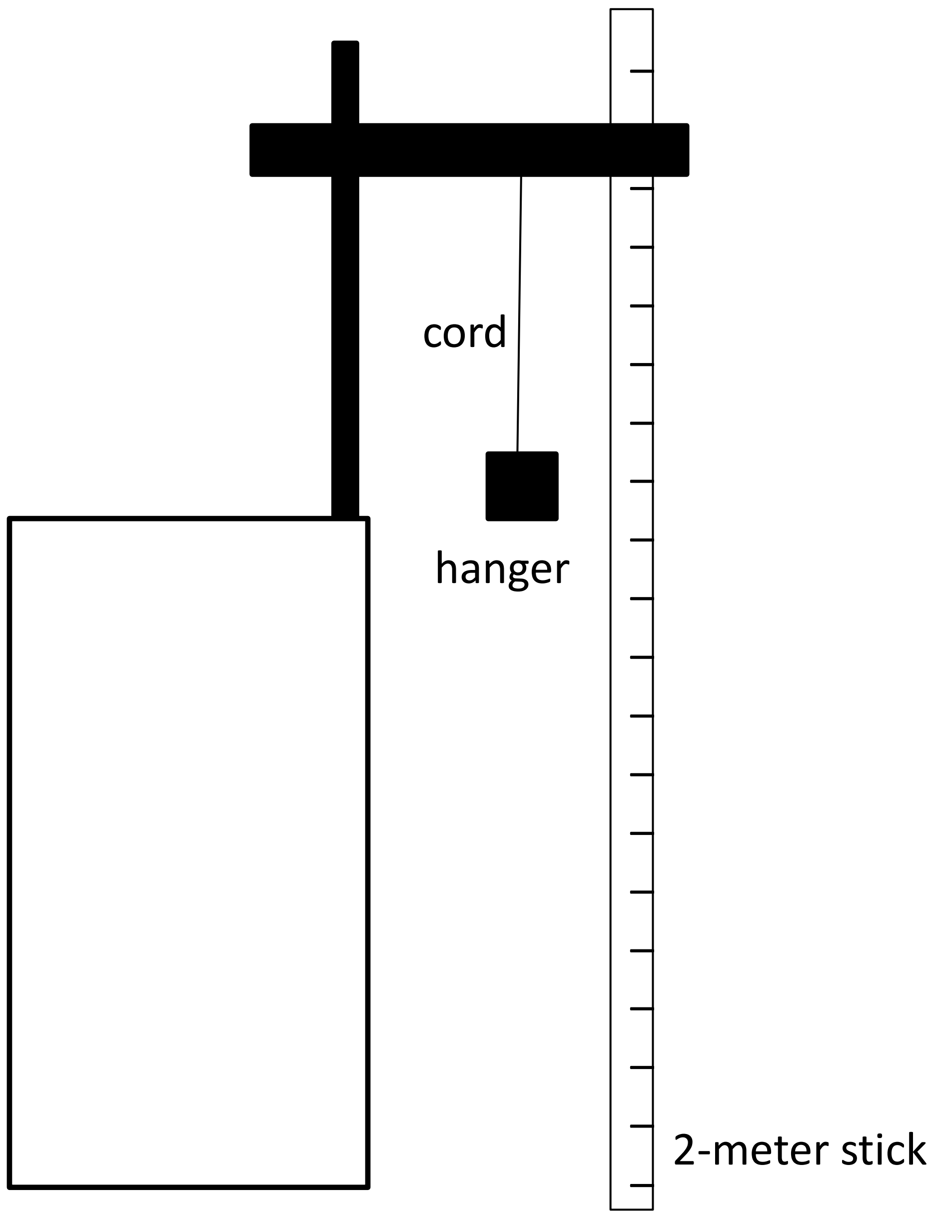



**Diagram 2: Experimental Setup**

Description of Setup: Varying masses were hung from a hanger attached to a bungee cord of a varied strand number. Equilibrium values were kept constant by using a 2-meter stick affixed to the clamp.

Procedure:

- The equilibrium distance, or the cord length, was kept constant at 0.38 m.
- In each test, the mass was released from the equilibrium distance.

- Using an iPad to take slow motion video, the distance travelled was calculated.
- The number of strand was varied to calculate different k values per strand number.
- Additionally, 3 masses were used per each strand number to give a wide range of values for the distance travelled to be used to calculate $k_{ave}$. The masses that were used were 0.105 kg, 0.155 kg, and 0.205 kg.

***RESULTS***:

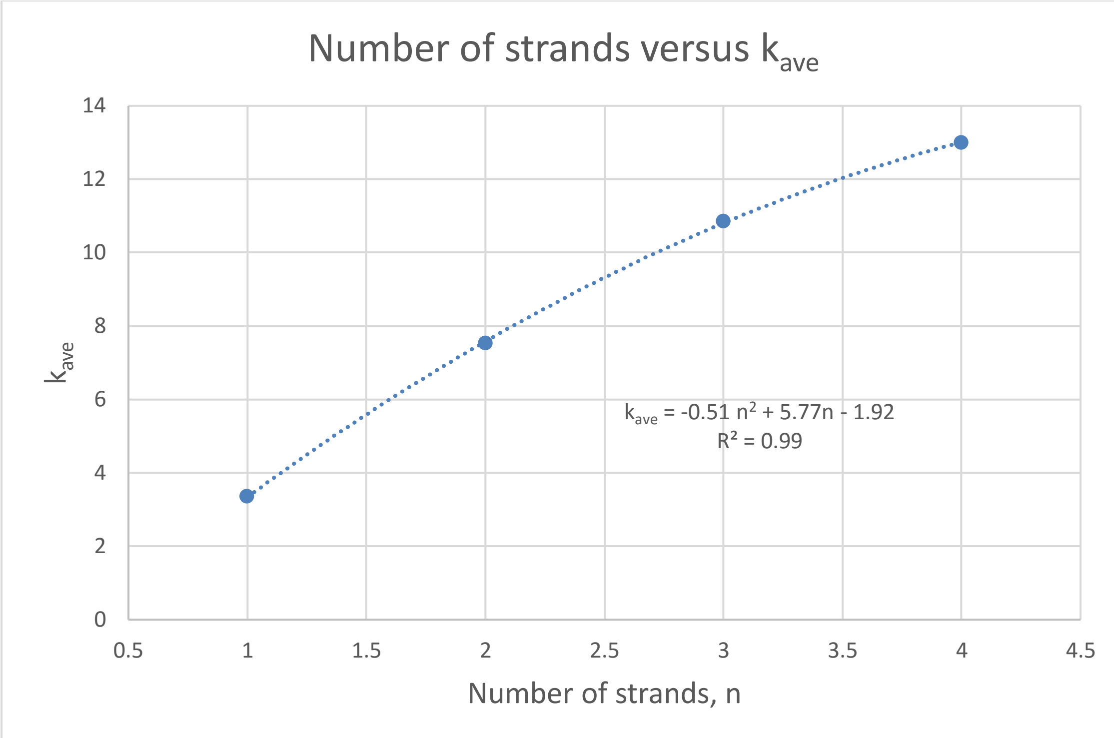
Four strand numbers were tested: 1, 2, 3, and 4 strands. The equilibrium position for the differing strand numbers were found to be equal. Three runs, one at each mass, were taken per strand number; the masses tested were 0.105 kg, 0.155 kg, and 0.205 kg. The mass was dropped from equilibrium position, and the fall was recorded with an iPad camera using the slow-motion video function. The fall distance was calculated and used to determine the distance travelled (x). These values were used to calculate three values of k for each strand number, and these values were averaged to determine the $k_{ave}$ for each strand number. These values were graphed to show the relationship between the number of strands and the $k_{ave}$. A curve of best fit was found, and its $R^2$ value was found to calculate uncertainty. Uncertainty was propagated. Additionally, data was linearized by graphing (–x), the negative distance travelled, as we set the ground as the origin, versus weight (2mg); equations of best fit were found for each strand number. The slope of these equations is equal to the value of k for that strand number. Uncertainty for these values was found through *Excel* regression analysis.

| Strands | Mass (±0.01kg) | $x_{initial}$ (±0.005m) | $x_{final}$ (m) (±0.005m) | x (±0.007m) | k | % uncertainty |
|---|---|---|---|---|---|---|
| 1 | 0.105 | 1.34 | 0.80 | -0.54 | 3.82 | 9.61% |
| 1 | 0.155 | 1.34 | 0.40 | -0.94 | 3.24 | 6.49% |
| 1 | 0.205 | 1.43 | 0.10 | -1.33 | 3.02 | 4.90% |
| 2 | 0.105 | 1.34 | 1.09 | -0.25 | 8.24 | 9.93% |
| 2 | 0.155 | 1.34 | 0.89 | -0.45 | 6.76 | 6.63% |
| 2 | 0.205 | 1.34 | 0.81 | -0.53 | 7.59 | 5.05% |
| 3 | 0.105 | 1.34 | 1.15 | -0.19 | 10.84 | 10.21% |
| 3 | 0.155 | 1.34 | 1.06 | -0.28 | 10.86 | 6.92% |
| 3 | 0.205 | 1.34 | 0.97 | -0.37 | 10.87 | 5.23% |
| 4 | 0.105 | 1.34 | 1.19 | -0.15 | 13.73 | 10.61% |
| 4 | 0.155 | 1.34 | 1.10 | -0.24 | 12.67 | 7.08% |
| 4 | 0.205 | 1.34 | 1.02 | -0.32 | 12.57 | 5.35% |

**Table 1: Raw Data:** This table shows the raw data collected as well as the k values per each run calculated using the theoretically-manipulated equation (k = -2mg / x). Slow-motion video was recorded using an iPad to measure $x_f$ values; ±0.005m uncertainty was assumed for these values. % uncertainties were propagated using product and division rules for uncertainty.

| Strands | $k_{ave}$ | % Uncertainty |
|---|---|---|
| 1 | 3.36 | 12.59% |
| 2 | 7.53 | 12.96% |
| 3 | 10.86 | 13.40% |
| 4 | 12.99 | 13.83% |

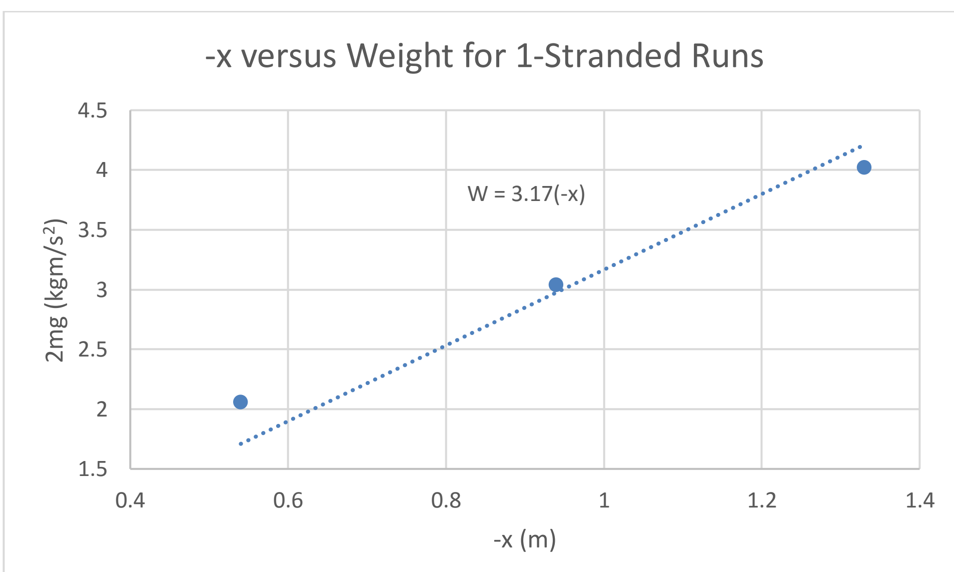
**Table 2: $k_{ave}$ values per each strand:** The average k value, $k_{ave}$, for each strand number was calculated. Uncertainty was propagated using the simple sum or difference of individual quantities rules.




**Graph 1: Number of strands versus $k_{ave}$:** This graph shows the $k_{ave}$ value depending on the number of strands of cord used. $k_{ave}$ was calculated by averaging the calculated k values. The line of best fit for this data was determined to be $k_{ave}$ = -0.51 $n^2$ + 5.77 n - 1.92, where n is the number of strands. The $R^2$ value was 0.99, showing that this line is highly representative of the data, giving a low uncertainty.

Equation of the curve-fit from the graph:
$k_{ave}$ = -0.51 $n^2$ + 5.77 n- 1.92

Uncertainty: Because this graph shows a non-linear relationship between the number of strands and the $k_{ave}$ value, we cannot use *Excel* regression analysis. Because of this, an $R^2$ value was found using *Excel*. This $R^2$ value shows how well the non-linear trend line fits our data. Our $R^2$ value is 0.99, which shows that this trend line fits our data very well (as $R^2$ = 1 signifies a perfect fit and $R^2$ = 0 signifies no correspondence at all).




**Graph 2: -x versus 2mg for 1-Stranded Runs:** This graph shows –x (negative distance travelled) versus the weight, 2mg, for runs taken with 1 strand of the bungee cord. The line of best fit was found to be W = 3.17(-x). This graph is

linearized as the slope of this is equal to k, giving us another value for the experimental value of interest, $k_{1strand}$ = 3.17. Because we expect the k value to go to zero for small distances travelled and small masses, we found the line of best fit through the y-intercept set to zero.

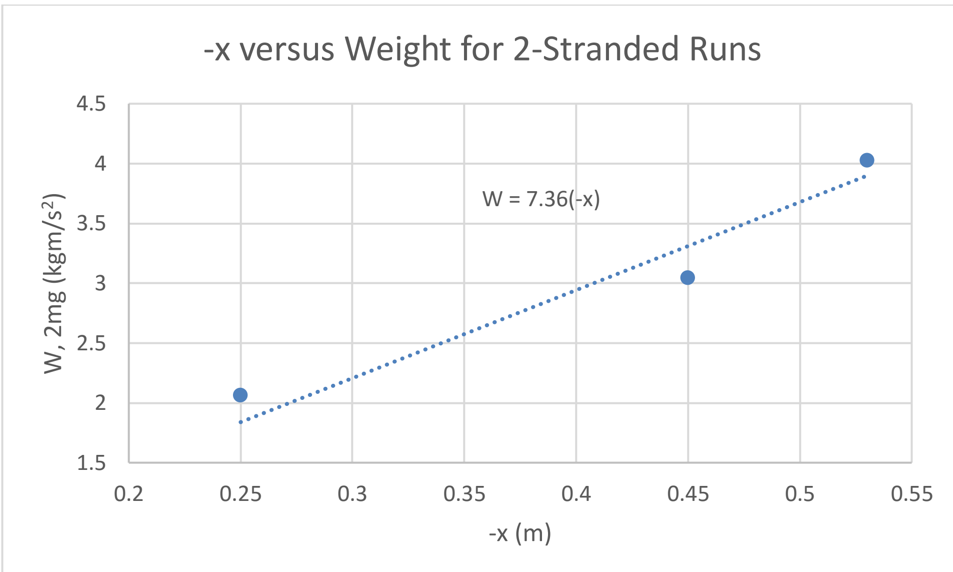
Equation: W = 3.17(-x)

Using *Excel* regression analysis,

uncertainty for slope= 0.166 % uncert= 5.25%

uncertainty for y-intercept= 0 % uncert= 0

The uncertainty for the y-intercept for this graph and for all others is zero or not applicable as the line of best fit was set to go through y=0. This is the case for Graph 2, 3, 4, and 5.



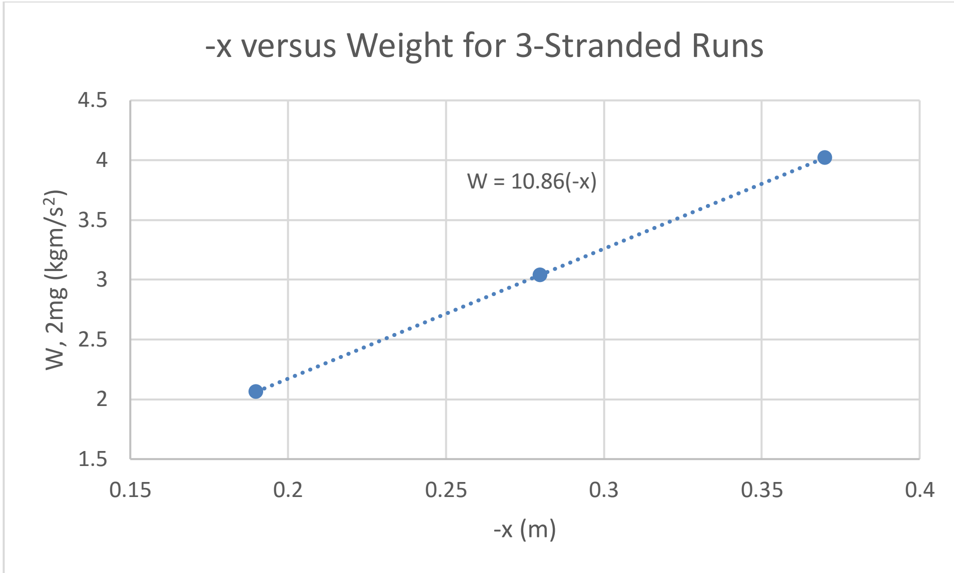

**Graph 3: -x versus 2mg for 2-Stranded Runs:** This graph shows –x (negative distance travelled) versus the weight, 2mg, for runs taken with 2 strands of the bungee cord. The equation of best fit for this linearized data was found to be W = 7.36(-x). The slope of this is equal to k, giving us another value for the experimental value of interest, $k_{2strands}$ = 7.36. Because we expect the k value to go to zero for small distances travelled and small masses, we found the line of best fit through the y-intercept set to zero.

Equation: W = 7.36(-x)

Using *Excel* Regression Analysis,

uncertainty for slope= 0.354 % uncert= 4.81%

uncertainty for y-intercept= 0 % uncert= 0%



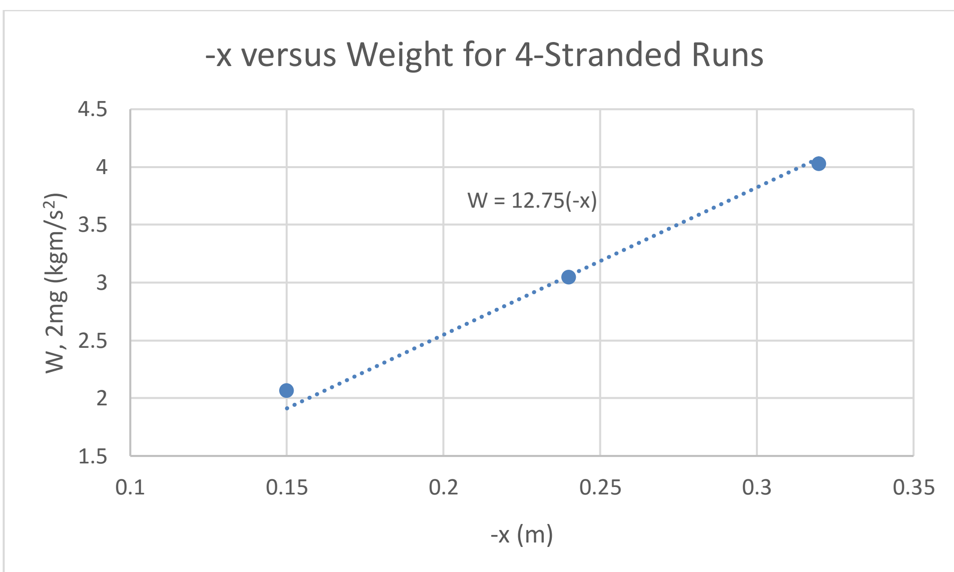

**Graph 4: -x versus 2mg for 3-Stranded Runs:** This graph shows –x (negative distance travelled) versus the weight, 2mg, for runs taken with 3 strands of the bungee cord. The line of best fit for this linearized data was found to be W = 10.86(-x). The slope of this is equal to k, giving us another value for the experimental value of interest, $k_{3strands}$ = 10.86. Because we expect the k value to go to zero for small distances travelled and small masses, we found the line of best fit through the y-intercept set to zero.

Equation: W = 10.86(-x)

Using *Excel* Regression Analysis,

uncertainty for slope= 0.007 % uncert= 0.06%

uncertainty for y-intercept= 0 % uncert= 0%




**Graph 5: -x versus 2mg for 4-Stranded Runs:** This graph shows –x (negative distance travelled) versus the weight, 2mg, for runs taken with 4 strands of the bungee cord. The line of best fit for this linearized data was found to be

W = 12.75(-x). The slope of this is equal to k, giving us another value for the experimental value of interest, $k_{4strands}$ = 12.75. Because we expect the k value to go to zero for small distances travelled and small masses, we found the line of best fit through the y-intercept set to zero.

Equation: W = 12.75(-x)

Using *Excel* Regression Analysis,

uncertainty for slope= 0.264 % uncert= 2.07%

uncertainty for y-intercept= 0 % uncert= 0%

Experimental values of interest:
The experimental values of interest in this experiment are the k values per strand number. They were found 2 ways: averaging k values for the specific runs at varying strand count ($k_{ave}$) and graphing (-x) versus W to give a linearized graph where the slope of the equation of best fit is equal to $k_{\#strand(s)}$.

With regard to the four $k_{ave}$ values calculated through averaging experimental k values,

$k_{ave}$: 3.36 (1 strand), 7.53 (2 strands), 10.86 (3 strands), 12.99 (4 strands)

% uncertainties of values of interest: 12.59% (1 strand), 12.96% (2 strands), 13.40% (3 strands), 13.83% (4 strands).

Technique used for propagation: In accordance with the equation used, k = - 2 mg / x, uncertainties for the distance travelled ($x = x_f - x_i$) was propagated using the simple sum or difference formula. This was found to be equal to 0.007m. Uncertainty of the mass, m, was estimated to be 0.01 kg. The % uncertainty for each $k_{ave}$ value was calculated using the product or division without higher powers rule. Because $k_{ave}$ was taken, the % uncertainty for $k_{ave}$ was found through propagation of the individual % uncertainties of each k following the simple sum or difference rule.

With regard to the $k_{\#strand(s)}$ values calculated through linearization of the data,

$k_{\#strand(s)}$: 3.17 (1 strand), 7.36 (2 strands), 10.86 (3 strands), 12.75 (4 strands)

% uncertainties: 5.25% (1 strand), 4.81% (2 strands), 0.06% (3 strands), 2.07% (4 strands)

Technique used for propagation: Because this data was linearized to give our value of interest as the slope of the line of best fit, *Excel* regression analysis was used to determine % uncertainties.

Summary of Results:
Raw distances travelled were calculated using slow-motion video with an iPad. Using these values, 4 $k_{ave}$ values (corresponding to 4 strand number values) were calculated, and their uncertainties were propagated. These values were 3.36 for 1 strand (with a 5.06% uncertainty), 7.53 for 2 strands (with a 7.53% uncertainty), 10.86 for 3 strands (with an 8.99% uncertainty), and 12.99 for 4 strands (with a 9.88% uncertainty). These values were used to graph a relationship between the number of strands and the $k_{ave}$ value. The relationship was found to be nonlinear, following the equation $k_{ave} = -0.51 n^2 + 5.77 n - 1.92$. The uncertainty of this equation was found using an $R^2$ value; this value was found to be equal to 0.99, which shows an extremely high degree of relation between the data and this equation.
Additionally, data was linearized to give a second value for k, $k_{\#strand(s)}$, per each strand number through graphing (-x) (the negative distance travelled) versus weight (2mg). The slope of the line of best fit through each of these graphs is equal to $k_{\#strand(s)}$. These values were found to be 3.17 for 1 strand (5.35% uncertainty), 7.36 for 2 strands (4.81% uncertainty), 10.86 for 3 strands (0.06% uncertainty), and 12.75 for 4 strands (2.07% uncertainty).

***DISCUSSION***:

Error analysis:
There are no accepted or expected values from theory or other sources. Because of this, we cannot compare % uncertainty versus % error because we cannot calculate the % error.

From theory, we do know that the spring coefficient is proportional to the number of springs. In other words, the k for of 2 identical springs should be twice the k for 1 spring identical to the 2; in our case, 2 the k for 2 strands should be twice the value of k for 1 strand, and so on. Although we do not know its error, we can use our $k_{ave}$ value for 1 strand to calculate the theoretical value for k of the other strand numbers. We can then compare this value to our experimental $k_{ave}$ values and our experimental $k_{\#strand(s)}$ values to calculate % error.

| Theoretical: | $k_{ave}$ Values: | % Uncertainty: | % Error: | $k_{\#strand(s)}$ Values: | % Uncertainty | % Error: |
|---|---|---|---|---|---|---|
| $k_1 = 3.36$ | $k_{ave1} = 3.36$ | - | - | $k_{1strand} = 3.17$ | 5.25% | 5.65% |
| $2(k_1) = k_2 = 6.72$ | $k_{ave2} = 7.53$ | 12.96% | 12.05% | $k_{2strands} = 7.36$ | 4.81% | 9.52% |
| $3(k_1) = k_3 = 10.08$ | $k_{ave3} = 10.86$ | 13.40% | 7.73% | $k_{3strands} = 10.86$ | 0.06% | 7.73% |
| $4(k_1) = k_4 = 13.44$ | $k_{ave4} = 13.83$ | 13.83% | 2.90% | $k_{4strands} = 12.75$ | 2.07% | 5.28% |

Because the % errors of all values for the experimental values are less than the % uncertainties, our $k_{ave}$ values are within the range of acceptable values. The $k_{\#strand(s)}$ values found as the slope of the lines of best fit for Graphs 2-5 all have a higher % error than % uncertainty, however; this means that these values are not within the range of accepted values as calculated using our $k_{ave1}$ to calculate theoretical k values. It is important to remember, though, that these % error values and analysis are based on an assumption of our $k_1 = k_{ave1}$ value as acceptable.

Test of Acceptability: We can also test the acceptability of our $k_{ave}$ equation by using our equation of best fit giving $k_{ave}$ to predict the distance travelled of a fall using a known strand number and known mass, and a calculated known $k_{ave}$.

For example, we could test our model using 5 strands, n = 5.

$k_{ave} = -0.51\ n^2 + 5.77\ n - 1.92 \rightarrow k_{ave} = -.51\ (5)^2 + 5.77\ (5) - 1.92 = 14.18$

$k = -2\ mg / (x_f - x_i) \rightarrow 14.18 = -2\ (0.105\ kg)\ (9.81\ m/s^2) / (\Delta x) \rightarrow \Delta x = 0.15\ m$

To test this calculated distance travelled, x, we could drop a mass of 0.105 kg from the equilibrium position and measure the x with iPad slow-motion video. We could then compare our experimental distance travelled to our calculated distance travelled determine the acceptability of our model.

Sources of uncertainty: One source of uncertainty is that we did not factor in air resistance. Because this another force upward, against the force of gravity, this could potentially decrease the distance travelled (as the mass would fall a shorter distance due to air resistance), and thus our $k_{ave}$ values would be larger than they truly are. This could present an issue if our equation for $k_{ave}$ was used in an area where the air resistance was significantly more or less than it was under experimental conditions. Another point of error is that there is not a known value for the equilibrium of the cord. For this reason, we had to estimate the equilibrium. If we underestimated or overestimated, this would change our $k_{ave}$ values. It is believed that this is a more serious point of error in terms of the Bungee Challenge because while we expect a largely consistent amount of air resistance from room to room in an enclosed space, if our equilibrium value was incorrect, consequences in the fall distance are of more importance. Lastly, it is possible that after testing the bungee with various masses, the cords were more stretched out toward the end of the experiment than at the beginning, leading to higher distances travelled. However, we attempted to alleviate this area of uncertainty by testing small masses (which did not stretch the cord to undue lengths) and by testing the strand value of 1 last, as it was expected to stretch the most.

Main Result: The results do support my original hypothesis that $k_{ave}$ would vary with strand number and that the larger strand numbers would have a larger $k_{ave}$ because their distance travelled during the fall would be smaller than smaller strand numbers. Although we do not know the theoretical or expected k value of any of the strand numbers nor can we calculate it with theory, we used our $k_{ave}$ value for 1 strand to calculate the k values for other numbers of strands. Doing so, we found that our $k_{ave}$ values are within the range of acceptable values but our $k_{\#strand(s)}$ values calculated through linearization are not within the range of acceptable values.
However, although we can calculate the % error with respect to our $k_{ave}$ value for 1 strand, we cannot conclusively find these values acceptable or unacceptable because our $k_{ave}$ value for 1 strand does have its own uncertainty. Additionally, a test was conceived to determine the accuracy of our model.

***CONCLUSION***:

Experimental Outcomes: In this experiment, we did find that $k_{ave}$ changes with strand number used.
We also found the equation $k_{ave} = -0.51\ n^2 + 5.77\ n - 1.92$ that can be used to determine $k_{ave}$ for any number of strands, n. $k_{ave}$ values as well as $k_{\#strand(s)}$ values for each strand number tested were also calculated.

Implications of these conclusions and proposed subsequent work: When the spring coefficient is not given, it is not straightforward to calculate. However, we showed one way that it can be done and tested for acceptability. Although we found an equation which allows us to predict $k_{ave}$ using the line of best fit through our $k_{ave}$ values, we are still unsure of the accuracy of this value. However, an experimental test of accuracy was devised to ascertain the accuracy of this equation. This experimental test of accuracy could be that which was proposed earlier through comparing the experimental distance travelled of a run of a set mass to the calculated distance travelled.

**Report Outlines are *individual assignments*. Cite any work not your own, acknowledge any aid, and pledge the report:**

**On my honor, I have neither given nor received any unacknowledged aid on this assignment.**

***Pledged: Emily Limmer***