Modelling a cord with Hooke's Law

Date: 10/26

ABSTRACT:

In *Modelling a cord with Hooke's law,* we worked to relate an elastic cord's physical properties to Hooke's law. By measuring the displacement of this cord at varying masses, we were able to calculate the associated K values at those lengths of the cord. These specific K values were then used to build a graph showing how K changes at any given length of the cord. In this way, we were able to quantitatively understand how this elastic cord behaves when stretched and when mass is added. This ends up serving as tremendous help because it can potentially provide key insight into the acting forces on the cord for our future bungee free fall experiment.

INTRODUCTION:

This research was intended to find the Hooke's law relationship (if any) inherent to our cord, which will be used in an actual bungee free fall. The goals of this lab can thus be divided into two main parts: the first was to find K values of the cord at different lengths. This was done by varying mass and plotting weight v.s. Δx. The second part of the research involved using these data to build a model of the cord's K values at any given length, where we can visually see how K varies at any length. From these data we will be able to understand how our cord exactly behaves according (pun intended) to Hooke's law. Additionally, since the graph of k v.s. length was not linear, we used other Hooke's law relationships (1/k v.s. length) to linearize our data in order to compute error analysis.

Relevant equations: $F_{spring} = K \cdot \Delta x$, F=ma

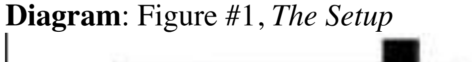
The theoretical basis of this experiment comes from these two equations above. Hooke's law states that the force of a spring is equal to the change in x (displacement) multiplied by the K constant. Therefore, in order to know K, we must find Δx and F_{spring}. Additionally, F=ma allows us to model the forces acting on the hanging mass attached to the cord, where weight is antiparallel to the F_{spring}.

Hypothesis: I suspect that the cord's behavior can be accurately modeled by Hooke's law.

METHODS:

Initially, we hung masses from the cord at varying lengths. This allowed us to measure the displacement of the cord. By taking the slope of a weight v.s. Δx graph, we received K values at those given lengths. We performed these trials exactly seven times, yielding seven K values. With these data, we were then able to see how K changes with cord length. Since this is not a linear relationship, rather, a decay, we linearized the data using 1/k v.s. length in order to perform linear regression analysis, or simply error analysis.

Diagram: Figure #1, *The Setup*



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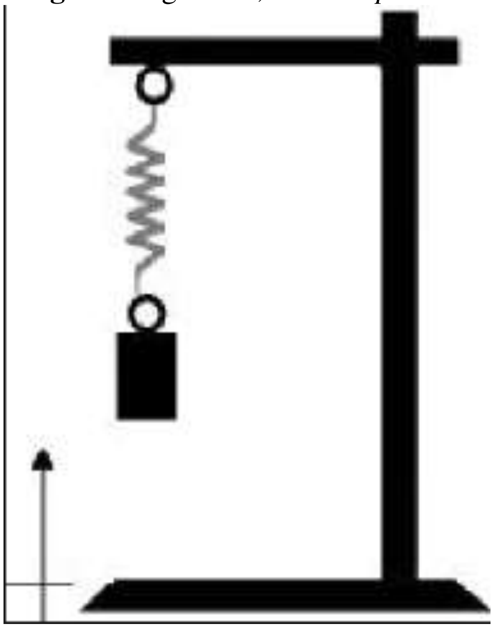


Hanging mass where the "spring" is our elastic cord we predict will model Hooke's law.

Setup:
The setup was simple. We had a steel structure, such as the one above, clamped to the desk that allowed us to hang the cord down vertically. We varied the cord length and hung a 50 gram implement that allowed us to add weight onto the cord. We then carried out our procedure to yield data.

Procedure:

- Measured cord length as it hung from the top of the steel beam.
- Added a mass (20g, 50g, 100g, etc.)
- Recorded displacement values of the length at varying masses.
- Repeated measuring displacement values for about seven different masses at that length.
- Computed K value based on weight v.s. Δx (at that given length).
- Then repeated the steps above to calculate seven different K values.
- Plotted K v.s. length in excel.
- Performed linearization by using Hooke's Law relationships (1/k v.s. length).
- Linear regression analysis.

RESULTS:

We received excellent results that seems to show how our cord follows Hooke's law behavior. Each graph also had an R^2 value of .99 or more.

Table 1.) Raw Data of Trial 1

added (kg)	Mass (kg)	Weight	X(equil) (m) +/- .00001m	Spring Strech (Xl =.64m)
	0.05	0.4905	0.775	0.135
0.02	0.07	0.6867	0.845	0.205
0.05	0.1	0.981	0.987	0.347
0.07	0.12	1.1772	1.103	0.463
0.1	0.15	1.4715	1.295	0.655
0.12	0.17	1.6677	1.438	0.798
0.15	0.2	1.962	1.649	1.009

Now, from the previous procedure section, it is clear that we measured change in x of different masses. We did this with seven different masses and the raw data of such a trial is listed above. However, another seven raw data tables like this were created at different lengths in order to find those seven different K values. Since we calculated seven different values of K, it is not necessary to display all seven trials.

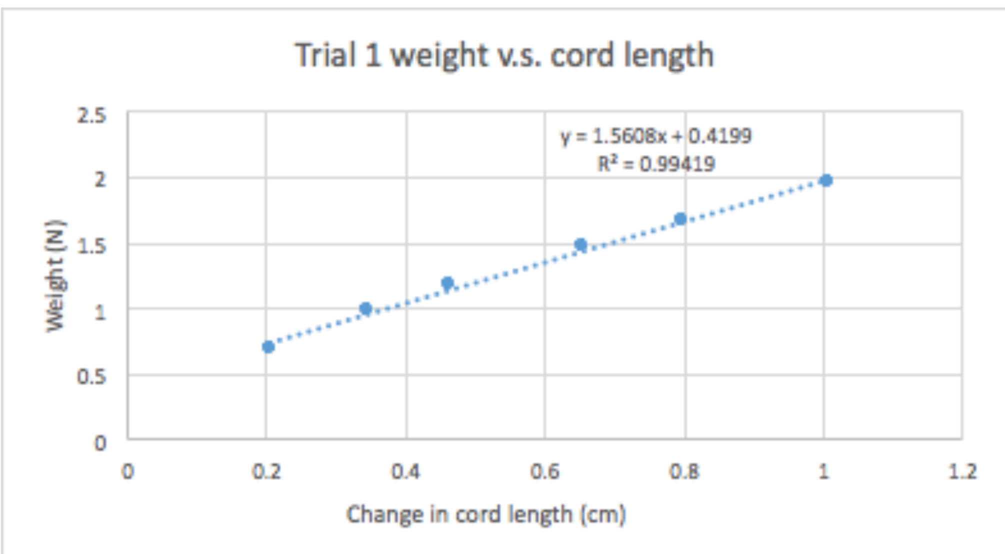
We did this with seven different masses and the raw data of such a trial is listed above. However, another seven raw data tables like this were created at different lengths in order to find those seven different K values. Since we calculated seven different values of K, it is not necessary to display all seven trials. Rather, it is more important to see how we calculated one K value from a weight v.s. Δx graph (Graph 1), while bearing in mind that we did this same trial method at different lengths in order to find seven other K values as well.

Table 2.) Specific raw data taken to build a weight v.s. Δx graph (Graph 1).

stretch	weight
0.205	0.6867
0.347	0.981
0.463	1.1772
0.655	1.4715
0.798	1.6677
1.009	1.962

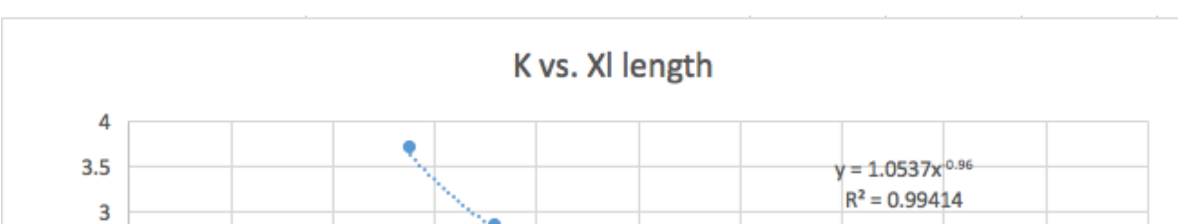
The stretch is the Δx in cm and the weight is in Newtons. We plotted weight v.s. Δx in order to calculate K from the slope. This is an example of a separate table we built in Excel (from the raw data) in order to create the graph listed below.

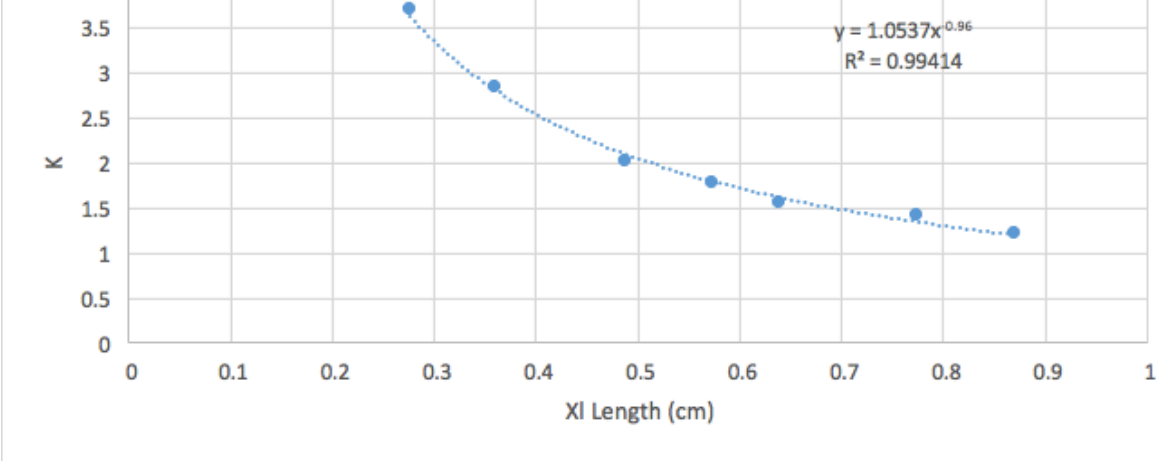
Graph 1.) Weight v.s. Δx



This graph is essentially a form of Hooke's law, where weight (force) is plotted against Δx and the slope is K. This models $F_{spring} = K \cdot \Delta x$. Notice the R^2 value of close to .994 tells us the data is sound.

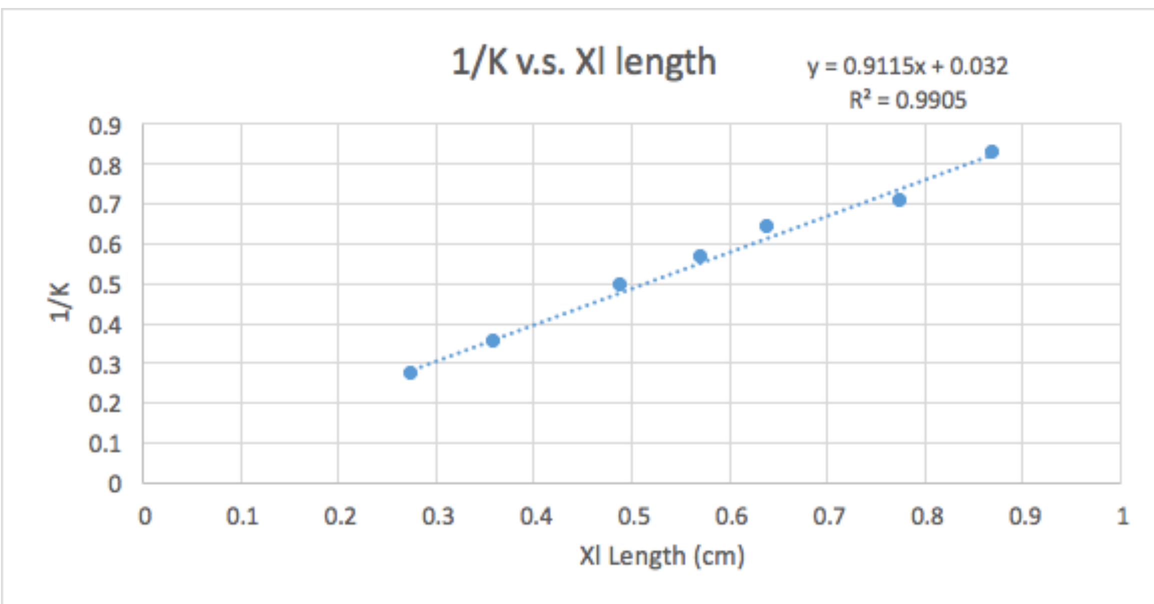
Graph 2.) Graph displaying K v.s. Cord lengths





Here we have K plotted against a given cord length. As noted in the procedure section, we performed seven trials in order to find seven different K values at varying lengths. This allowed us to build this exact graph above where we can now see how K behaves when length increases or decreases. As length increases, k decreases and vice versa (although not linearly). R^2 value very close to .994 and the exponent in the equation is -.96.

Graph 3.) Linearized graph of Graph 2.



The Hooke's law relationship 1/k v.s. length can be taken advantage of here in linearizing our data in Graph 2. Linearizing our data makes this far easier to perform regression analysis and find the error associated with slope and y-inercept. R^2 value of .9905.

Linear equation: y= .9115x+.032

Linear Regression Data taken from Graph 3:

SUMMARY OUTPUT	
Regression Statistics	
Multiple R	0.99523909
R Square	0.99050084
Adjusted R Sq	0.98860101
Standard Erro	0.02089737
Observations	7

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ANOVA

	df	SS	MS	F	Significance F
Regression	1	0.2276791	0.2276791	521.362494	2.9962E-06
Residual	5	0.0021835	0.0004367		
Total	6	0.2298626			

	Coefficients	Standard Error	t Stat	P-value	Lower 95%	Upper 95%	Lower 95.0%	Upper 95.0%
Intercept	0.0319579	0.0240489	1.32887144	0.24130258	-0.0298618	0.09377757	-0.0298618	0.09377757
X Variable 1	0.91152227	0.03992063	22.8333636	2.9962E-06	0.80890303	1.01414152	0.80890303	1.01414152

***Excel* regression analysis**:

uncertainty for slope= .0399206
uncertainty for y-intercept= .024089

Pertinent/Summarized Results: Tables show each trial which was used to calculate graphs of weight v.s. Δx. From a graph of weight v.s. Δx, K values were yielded from the slope. By then plotting seven different values of K against each cord length, the relationship between these two variables, length of the cord and its K constant, could be perceived as a decay. Linearizing this data simply allowed us to perform error analysis, which associated errors in slope and y.

DISCUSSION:

Our results show a sound relationship between the cord and Hooke's law. The K values decay as length is increased which well models our interpretation. The error analysis is also rather strong, as we have .99 R^2 values and low associated error values in slope and y-intercept. Overall, the cord is quantitatively a happy fit with Hooke's law. Of course, as in any experiment, there are always variables that do affect the integrity of the data. One source of error I noticed is simply measuring Δx. For the most part, measuring the change in x involved a tape measure and "eyeing" where the mass was hanging from relative to it. It was almost impossible to have the hanging mass completely still while directly measuring behind it, so some relative guessing was definitely involved. The limit of this error is, however, not seen in our very strong R^2 values, which suggest not a lot of error was at play. Furthermore, our -.96 value in the exponent of the equation in Graph 2 is quite close to 1. Although it is not exactly 1, this may have been a result of error carried through from "eyeing" the measurements. As stated earlier, I do believe that our lab results were very strong and that error was not significantly propagated throughout the experiment. These results therefore support our hypothesis that the cord can be modeled by Hooke's law.

CONCLUSION:

Since our data supports our hypothesis, the definite conclusion is that we can move forward with our future bungee experiment by being confident that the cord is modeled through Hooke's law. Moreover, we can now use the K v.s. cord length relationship to find force and displacement at any given length of the cord. The larger question that persists here now is whether or not these are the only things we should consider for a bungee free fall experiment. More research should take into account, for example, using multiple elastic cords or adding inelastic cord lengths.

Pledged