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TITLE: Does Force Vary with the Ratio X_s/X_b Like It Varies with X in F=-kx?

ABSTRACT:

We want to create a model analogous to the equation of a spring, F=-kx, that predicts the magnitude of the maximum force, F_{max}, that acts on a constant oscillating mass based on the ratio between static string length, x_s, and the stretchy string length, x_b. This will help us easily pick the lengths of static and stretchy string when we drop an egg from 8 meters high. We measure F_{max} and vary the ratio: We keep the stretchy string length and the mass constant but change the static string length. We found that the ratio x_s/x_b relates directly with the magnitude F_{max}; as the ratio increased, the greater the magnitude of F_{max}, according to the equation $F_{max}= \alpha (x_s/x_b) + \beta$. However, when we tested our model with a given ratio x_s/x_b and measured the F_{max}, we did not find the calculated k to be consistent with the experimental k. Therefore, our model cannot be analogous to the equation of a spring. For further steps, instead of finding the relationship between x_s/x_b and F_{max}, we would focus on finding the k based on the distance stretched of a bungee with varing x_s and F_{max}.

INTRODUCTION:

We want to make a model that predicts the magnitude of maximum force, F_{max}, on a constant mass when given the ratio of the static string, x_s , and the stretchy string, x_b. Therefore, we want to graph F_{max} versus x_s/x_b, so the equation takes the following form:

$F_{max}= \alpha (x_s/x_b) + \beta$

- F_{max} is the magnitude of the force when the mass is at the bottom of its oscillation
- α and β are constants that find when we create our model, and α is what is analogous to the k in the equation of a spring, F=-kx, where the force is directly related to the spring constant, k, and the distanced stretched from equilibrium, x.
- x_s is the length of the static string
- x_b is the length of the stretchy string

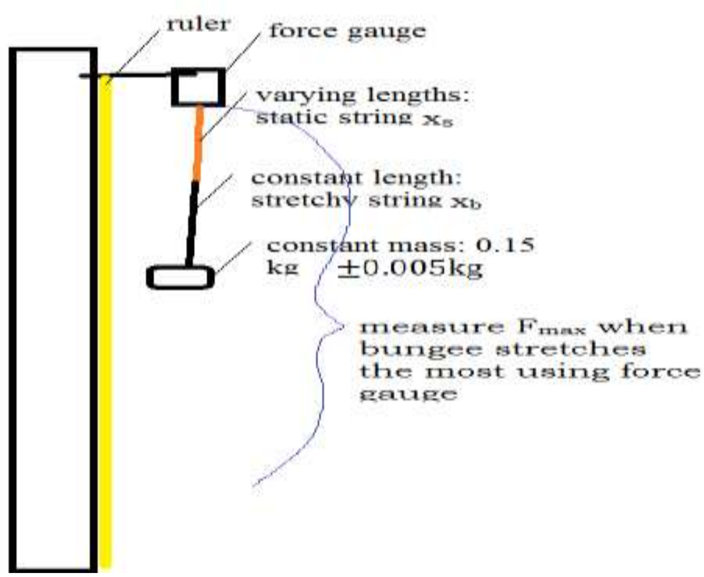
We assume that the ratio x_s/x_b has a linear relationship with F_{max} because our model behaves like a spring. In the equation of a spring, $F_{spring}=-kx$, where k is the spring constant and x is the distance stretched, the distance is linear in relation to the magnitude of force. Because x_b is constant, the units of the ratio are still meters, so it is analogous to x in the equation of a spring. So, the coefficient α is equivalent to the k of our bungee.

Hypothesis:
Our bungee can be treated as spring with the equation F=-kx, so the bigger the ratio between static string and stretchy cord is, the higher F_{max} will be.

METHODS:

We test different ratios of static string and stretchy cord to see how F_{max} responds. The longer the static string is, the bigger the ratio between static string and stretchy cord, because we hold the stretchy cord constant in the creation of our model. Varying the stretchy string length will not only create more uncertainty but also does not have any effect on F_{max}. This is because in the spring equation F=-kx, x represents the distance the spring stretches, not how long the spring is.

Figure 1: Setup and Measurements: To measure $F_{max,}$ we drop the mass when the top knot of the static string lines up with the knot connecting the mass to the bungee cord, and measure F_{max} at the bottom of the first oscillation.



The static string is tied by a knot to a force gauge that will measure F_{max}. The stretchy string is tied to the bottom of the static string, and the mass is tied to the stretchy string. A ruler will be beside our bungee system to help measure the length of the varying static string.

-Measure the length of the static string knot to knot each time we change the length.
-Drop the mass when the knot on the bottom of the stretchy string lines up with the knot tied onto the force gauge.
-Measure F_{max} the first time the bungee stretches the most, or the first time in its oscillation.
-Measure F_{max} twice for each length of static string to ensure consistency.

RESULTS:

We recorded F_{max} for varying lengths of static string. We found the ratio of static string to the constant stretchy string, and found that the bigger the ratio, the greater the magnitude of the force, $F_{max.}$ The ratio has a positive linear relationship with the force.

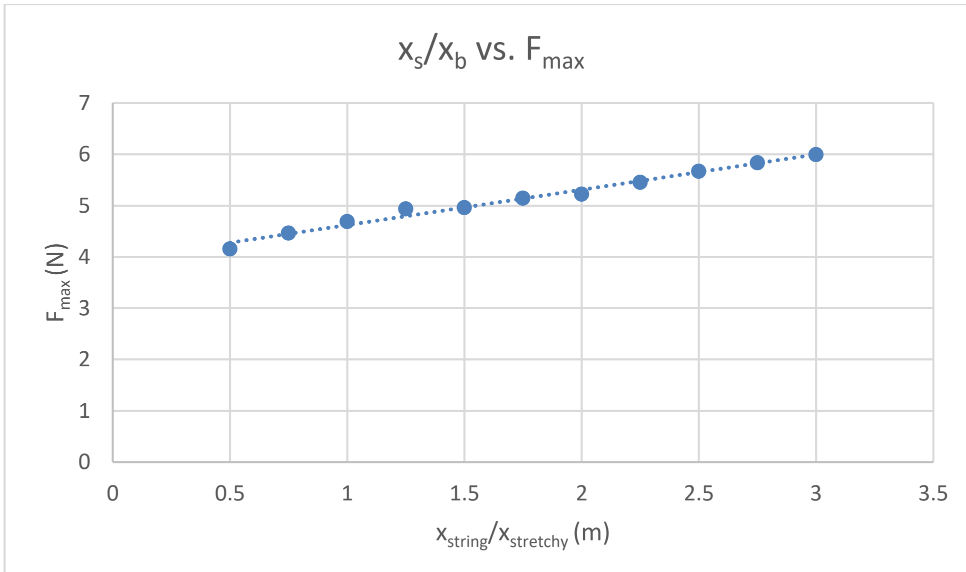
Table 1: Table of F_{max} in Response to Varying Ratios of x_s/x_b —Two trials of F_{max} for each x_s were averaged. X_b is constant, 0.2±0.01 meters. Because this is a constant, the units of the ratio are still meters.

X_s, length of static string (m) ±0.01 m	*$F_{max\ avg}$ (N)*	*X_s/X_b (m) ±0.01 m*
0.10	4.2	0.50
0.15	4.5	0.75
0.20	4.7	1.00
0.25	4.9	1.25
0.30	5.0	1.50
0.35	5.1	1.75
0.40	5.3	2.00
0.45	5.5	2.25
0.50	5.7	2.50

0.55	5.8	2.75
0.60	6.0	3.00

Figure 2: Linearized Graph of the Magnitude of x_s/x_b vs F_{max}, where x_s/x_b is the ratio of the static string and the stretchy string, and x_b is the 0.2 meters, the length of the stretchy part of the bungee.

$F_{max}= 0.69 (x_s/x_b) + 3.9$



Equation of graph:

$F_{max}= 0.69 (x_s/x_b) + 3.9$

We keep the y intercept because our model has the condition that the mass is constant, around 0.15±0.005 kg, and our model may not behave exactly like an ideal spring.

uncertainty for slope= ±0.03 N/m % uncert= 4.1%

uncertainty for y-intercept= ±0.05 N % uncert= 1.4%

The value of interest is the coefficient of the term x_s/x_b, because this is the equivalent to spring constant of a spring of our bungee.

value obtained = 0.69±0.03 N/m % uncert=4.1%

We obtained these values through regression analysis of F_{max} vs. x_s/x_b in Excel.

DISCUSSION:

From our results, we see that is a direct relationship between F_{max} and x_s/x_b, giving us the equation $F_{max} = 0.69\ (x_s/x_b) + 3.9$. So when given a ratio, we can predict the F_{max} on the constant mass, because we assume that our model is analogous to the equation of a spring.

When given a maximum F_{max}, we know the appropriate ratio of bungee to give the egg a successful bungee jump experience, that is, maximize thrill without it hitting the ground. In our case the maximum force, or thrill of the egg, is 3g, three times the acceleration of gravity. We can use our equation, now that we know the correction term and the spring constant of our bungee, to make a bungee cord with the right ratio of static string and stretchy string that will maximize F_{max} but not exceed 3g.

We tested our model by finding the F_{max} of a ratio of 0.2 meters of static string and 0.3 meters of stretchy string. The F_{max} we measured for this circumstance was 4.5 N. Using our equation, the spring constant value we get is 0.9 N/m. We use this as our accepted value.

Experimental Value from graph: 0.69±0.03 N/m %uncertainty: 4.1%

Accepted Value: 0.9 N/m

% Error=|Accepted Value-Experimental Value|/Accepted Value = 23%

The uncertainty of our experimental value, 4.1% is smaller than the percent error 23%; therefore, the equation of a spring, $F=-kx$, cannot be analogous to our model.

The discrepancy between the accepted value and the experimental value may be due to a mathematical error: Because we held the length of the stretchy string constant, we treated it as a scaler with no units in the ratio, when, in fact, it should be treated as a length with units.

However, we did find that the greater the length of the static string, x_s, the greater the magnitude of F_{max}. So instead of relating the ratio of static string and stretchy string to F_{max}, we can relate the length of x_s to F_{max}. This way, we can better use the equation of a spring, F=-kx.

Sources of uncertainty include (most important first):
-each time we changed the length of the static string, the knots skewed its length, because we did not account for the length of the loop.
-the static string may not be perfectly inelastic
-the loop of the knot on the stretchy string also stretches, but we did not account for the length of the loop
-it was hard to be sure if the knots line up when we dropped the mass, because we held the mass above eye level
-when we tested our model to obtain an accepted value, we had to use another mass, because our original mass broke
-there was a lot of extra cord that hung from our bungee. Although we avoided the mass catching on to the extra cords, some contact was inevitable.

Our main results do not support our hypothesis that the equation of a spring can be applied to the relationship between F_{max} and the ratio of the length of the static string and the stretchy string. This makes sense, because the equation of a spring defines x as the distance stretched, not the ratio of different lengths of bungee strings.

CONCLUSION:

Even though we found that the ratio of the static string and the stretchy string varies directly with F_{max}, we cannot apply the equation of a spring, F=-kx, to our equation F_{max}= 0.69 (x_s/x_b) + 3.9. Therefore, instead of trying to relate the ratio between static and stretchy string to force, we should relate the distance stretched based on varying lengths of static string to force. This model would be more analogous to the equation of a spring.