

**TITLE:** Determination of the Force of a Mass on a Bungee Cord in Relation to its Length

**ABSTRACT:**

An elastic cord, when stretched, will provide a certain restoring force to return to equilibrium. In a bungee jump, this restoring force prohibits the person (or egg) from colliding with the ground. This force needs to be strong enough to arrest the object in free fall, but not be so strong as to cause significant injury to the object on its journey back to the equilibrium position of the cord (for the case of our egg, it cannot exceed three times the gravitational force). In this experiment, we measured the effect of length on this restoring force by varying the length of cord used to drop an object of constant mass. We found that the restoring force at a given mass is constant despite different lengths of cord. This information will allow us to model a bungee jump for an egg so that the restoring force does not cause irreparable harm to the egg on its return trip.

**INTRODUCTION:**

A mass dropped from a certain height attached to a bungee cord experiences a restoring force due to the bungee cord. This restoring force can often be a limiting factor to a bungee jump – forces as much or greater than three times the weight of an object can be achieved. We want to be able to experimentally determine the conditions best suited to give an egg a bungee jump without breaking the egg – either by the egg hitting the ground or breaking from the restoring force applied by the bungee cord. Thus, knowledge of the force exerted by the cord on the return journey is crucial to keep the egg intact during a bungee jump.

Relevant equation(s) specific to this experimental purpose or setup, identifying variables:

$$(PE + KE)_f = (PE + KE)_i$$

$$mgh = \frac{1}{2} kx^2$$

$$F = -kx$$

Where:

F = spring force (N)

k = spring constant

x = spring displacement from the equilibrium position (m)

m = mass (kg)

g = gravitational constant (m/s<sup>2</sup>)

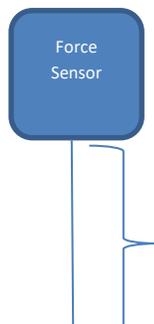
h = height (m)

Theoretical Background: The theoretical background for this experiment is twofold: the CWE theorem provides a basis for the potential and kinetic energies of the mass at the top and bottom of the bungee cord, and Hooke's Law provides an assessment of the force the object experiences as a function of displacement from the equilibrium position of the bungee cord.

Hypothesis: We hypothesize that with a longer cord length, the restoring force on the mass will increase similarly.

**METHODS:**

We used a force sensor to measure the force on a constant mass given a certain length of bungee. This is important, as a force greater than three times the weight of the egg used in our egg drop would put too much strain on the egg and cause it to break.



$L$ **Setup:**

Varying lengths,  $L$ , of bungee cord were attached by a knot to a force sensor. Total height of the system was held constant. To the opposite end of the bungee cord, a constant mass  $m$  was placed. This allowed us to measure the force on the cord solely as a function of cord length.

**Procedure:**

- Force sensor was connected to the Capstone system and hung from a rod at a specific height
- Varying lengths of bungee cord were knotted to the force sensor
- A knot was made in the opposite end of the bungee cord, and a hanging mass was placed on this knot
- Both knots were tightened as to minimize slipping during free fall and to keep measurements consistent
- The mass was dropped five times for each length of cord from a marked starting spot (the same experimenter dropped the mass each time to minimize uncertainty)
- The average force for each length was used to construct a graph

**RESULTS:**

**Introduce the Results:** Data was collected from the force graph in the Capstone computer system. The maximum force achieved (at the lowest point in the fall) was used, as this gave the greatest force on the system throughout the entire drop, and this force is the most relevant to our experimental questions. Length versus average force was graphed in Excel.

**Significant Constants:**

Mass used (g): 153.8 grams

Height of the system (m): 2.134 meters

**Table(s)**

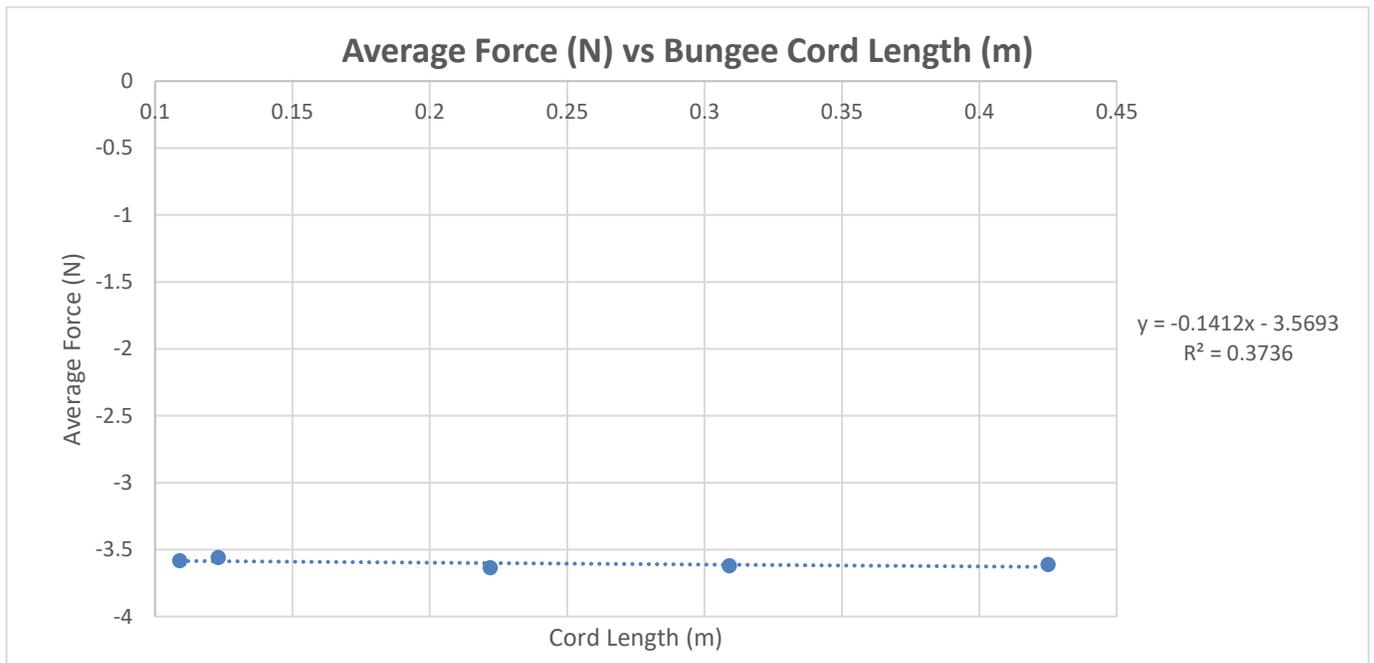
Length (m) ( $\pm 0.001$ m)	Max Force (N)
0.109	-3.61
	-3.57
	-3.60
	-3.58
	-3.56
0.123	-3.52
	-3.59
	-3.52
	-3.59
	-3.58
0.222	-3.61
	-3.68
	-3.65

	-3.61
	-3.63
0.309	-3.61
	-3.61
	-3.66
	-3.58
	-3.65
0.425	-3.59
	-3.61
	-3.59
	-3.61
	-3.66

**Table 1:** Raw force data for each length of bungee cord tested. Note that force values are given as a negative number – this is simply indicative of the direction of the force (downward) in relation to the force sensor.

Length (m) (±0.001 m)	Average Force (N)
0.109	-3.584
0.123	-3.560
0.222	-3.636
0.309	-3.622
0.425	-3.612

**Table 2:** Table of average forces for each length of bungee cord tested. Force is negative in accordance with the directionality of each force.



**Figure 1:** Graph of the average force for each length of bungee cord versus the cord length. Note that the force values have been kept negative in accordance with the direction of the force. Overall, the graph gives a very linear, horizontal line. This is evident from the slope, which approaches zero.

**Equation of Trend Line:**  $F = -0.141L - 3.569$

Use *Excel* regression analysis on any graph that has a **linear** fit only (see EG), to obtain:

uncertainty for slope= 0.10554 % uncert= 74.9 %

uncertainty for y-intercept= 0.02802 % uncert= .785 %

### Experimental Value of Interest

value obtained = Coefficient on  $L$  (slope)

uncertainty of experimental value(s) = 0.10554 % uncert= 74.9 %

name the technique used for propagation of uncertainty (see *UG*), or where/how uncert was obtained: this uncertainty was obtained through linear regression analysis

**Summary of the Results:** We found that the force of a falling mass given different lengths of bungee cord is modeled by the equation  $F = -0.141L - 3.569$ . The uncertainty on the slope and y-intercept were found through linear regression, and the uncertainty for the slope is quite high while the uncertainty in the y-intercept is fairly low.

**DISCUSSION:** *What do you make of your results? Evaluate them.*

#### Error analysis:

% error of the slope: 74.9%

While the percent uncertainty in the slope is rather high, this is most likely not an actual uncertainty but a product of the horizontal line. Because the slope is close to zero, and the two numbers are very small, any small change in uncertainty results in a wide change in the percent uncertainty. Something that would likely allow for a more accurate determination of the uncertainty in the slope value would be to test many more lengths of cord, and get a more accurate trend line. Because we were determining an unknown value, we do not have any calculations for percent error. To test the error in the force, we would need to know the value of  $k$  for the spring to be able to calculate the expected force for each length. We could then compare this value to the value of force calculated at each length to give us an error for the experiment.

**Sources of uncertainty:** Because the cord most likely does not act like an ideal spring, there is probably variation in the spring constant,  $k$ , that varies the force measured during each drop. There was also most likely variation in the drop itself, though we attempted to limit this error as much as possible by marking the drop point and only having one person drop throughout the entire experiment.

We found that the average restoring force for a mass on a bungee cord does not vary significantly across different lengths of bungee cord used. Rather, the restoring force for a bungee cord remains constant at a particular height regardless of the length of bungee cord used. This contradicts what we hypothesized. Upon further examination of the pertinent equations involved, Hooke's Law provides an answer for the constant restoring force. If  $F = kx$ , changing the length of the cord allows for a proportional change in  $x$ , the displacement of the cord. Because  $x$  changes proportionally, the force  $F$  remains unchanged for each drop. This force  $F$  is also well under the maximum  $3mg$  that is required for our egg to survive the bungee jump, given an approximately 150 gram egg.

**CONCLUSION:**

In this experiment, we determined that the force required for restoration of a bungee cord does not change with the length of the cord. Rather, the restoring force of the bungee cord remains the same despite changes in length. This restoring force is also under our limit for the egg drop,  $3mg$ , assuming a 150 gram egg. These data are important for our experiment, as we now know that the force on the egg will remain unchanged regardless of the length of bungee cord we use to drop our egg. We now only need to experimentally determine the appropriate length of cord to ensure that our egg can get as close to the ground as possible from a certain height without colliding with the ground.

**On my honor, I have neither given nor received any unacknowledged aid on this assignment.**

***Pledged: Josh Frost***