# *Lab Report Outline—the Bones of the Story*

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***TITLE:*** Determining the Effect of a Falling Mass on the Maximum Stretch of a Bungee Cord

## *ABSTRACT:*

In this experiment, we demonstrate an empirical model for the relationship between the maximum stretch of a particular length of bungee cord and the amount of applied mass. Our initial hypothesis assumed that these quantities would relate in a manner similar to the relationship between stretch and mass implied by the classical work-energy theorem for an object attached to a vertical ideal spring, or $x = \sqrt{\frac{2gh}{k}}\sqrt{m}$, where x is the stretch of the ideal spring, g is the acceleration due to gravity (9.81 m/s$^2$), k is the spring constant, and m is the mass. This model implies a direct relationship between maximum stretch and the square root of the applied mass. To determine our model, we attached various masses to a 0.51 m ± 0.002 m bungee cord and dropped them from a height 2 m above the ground, which was coincident with the bungee cord's anchoring point. Using a meter stick and an iPad with a highspeed camera application, we determined the maximum length achieved by the cord when subjected to each mass. By subtracting the initial length of the cord from these values, we found the maximum stretch of the cord for each mass. After plotting each maximum stretch against its corresponding mass, we used regression analysis to determine an empirical model of $x = 11.81m$, where x is the maximum stretch of the cord and m is the applied mass. The coefficient 11.81 m/kg ± 0.3% quantifies the maximum amount of stretch experienced by the cord per applied kilogram. While no theoretical value for the maximum stretch per applied kilogram, we evaluate the reasonability of our results by examining the implications of the 0.3% uncertainty in the scenario provided by the bungee challenge (i.e., a 9 m drop). Since 0.3% of 9 m is only 2.7 cm, our model is useful in the bungee challenge except for the finest cord length calculations. Our results do not confirm our hypothesis since x is directly related to the first power of m rather than its square root. However, our results are still relevant to the bungee challenge since they demonstrate the proper relationship between stretch and mass for our bungee cord.

## *INTRODUCTION:*

In a bungee jumping scenario, it is important that the participating object avoid contacting the ground. Since bungee cords are elastic, they necessarily stretch when a subjected to a force. This is the case when an attached mass fall under the influence of gravity. If the cord stretches so far that its overall length is greater than the height of the jump, the object will make contact with the ground and inevitably experience damage. This experiment seeks to develop an empirical model which quantifies the relationship between the amount of mass (m) participating in a bungee jump and the maximum stretch (x) of the cord, so as to provide a mathematical method for preventing such catastrophes.

Relevant equation(s) specific to this experimental purpose or setup, identifying variables:

- **Equ. 1: The Classical Work-Energy Theorem for an Object Falling while Attached to a Vertical Spring.** Because gravity and the force applied by a spring on an object are both considered conservative forces, the total amount of energy at the top of a given drop and the total amount of energy at the bottom of the drop must be equal. For an object attached to a vertical spring, all energy exists as gravitational potential energy at the top of the drop. At the bottom, all energy is converted to spring potential energy.

$$mgh = \frac{1}{2}kx^2$$

where m is the mass of the falling object, g is the acceleration due to gravity near the surface of the earth, or 9.81 m/s$^2$, k is the force constant of the spring, and x is the maximum stretch of the spring.

- **Equ. 2: The Ideal Relationship between Stretch and Mass as Predicted by the Classical Work-Energy Theorem.** By rearranging the Classical Work-Energy Theorem equation for an object falling while attached to a vertical spring, we can show that the stretch of a vertical spring varies directly with the square root of the mass of the attached object.

$$x = \sqrt{\frac{2gh}{k}}\sqrt{m}$$

where m is the mass of the falling object, g is the acceleration due to gravity near the surface of the earth, or 9.81 $m/s^2$, k is the force constant of the spring, and x is the maximum stretch of the spring.

Basis or brief theoretical background, providing enough context that the reader understands where the equation(s) are from:

In an attempt to describe the dynamics of systems that are beyond the scope of simple kinematics, physicists have developed the classical work-energy theorem, which states that the total amount of non-conservative work acting on a system is equal to the change in mechanical energy it experiences. While this relationship may seem esoteric, it provides for a particularly rigorous consideration of motion, especially for conservative systems. When no amount of non-conservative work is done on an object, the total amount of mechanical energy is said to be constant, meaning that it must be the same at all times during an event. When an object is dropped from a particular height while attached to a vertical spring, it experiences two forces: its own weight and the force of the spring. Both of these forces are considered conservative. Therefore, in this situation, the total amount of non-conservative work is zero, and the total amount of mechanical energy at the top of the drop must equal to the total amount of mechanical energy at the bottom of the drop. Since the object begins at rest and comes to a turning point at the bottom of the drop, the total kinetic energy in both of these locations is zero. With this in mind, all energy must exist in the form of potential energy at these two locations. At the top of the drop, only gravitational potential energy is present. Likewise, at the bottom, only spring potential energy is present. Setting the equations for these two types of energy equal to one another produces a dynamic model for an object falling while attached to a vertical spring (Equ. 1 and Equ. 2).

Hypothesis (or expectations):

We hypothesized that falling masses would affect the maximum stretch of the bungee cord in a manner similar to that which is predicted by the classical work-energy theorem for an object falling when attached to a vertical ideal spring (Equ. 2). However, since our previous experiments had already demonstrated that our bungee cord acts similar to, but not exactly like, an ideal spring, we were expecting the model to vary in some way (e.g., the addition of terms or factors that are not found in the basic classical work-energy theorem model). At the very least, we expected the stretch of the cord to increase as greater masses were applied.

## ***METHODS***:

Describe the overall method and its rationale in a sentence or two:

In order to determine the relationship between the maximum stretch of the bungee cord and the amount of falling mass, we established a particular length of cord and proceeded to attach and drop different masses from a consistent height, using motion capturing software and a meter stick to determine how far the cord stretched. By plotting each stretch against the mass that produced it, we found a mathematical model for the relationship between the two quantities.

**Diagrams:**

**Fig. 1: Experimental Setup before Dropping Mass.** The mass was dropped from a position coincident with the top of the meter stick. The unstretched length of the cord was kept constant at 0.51 m ± 0.002 m.

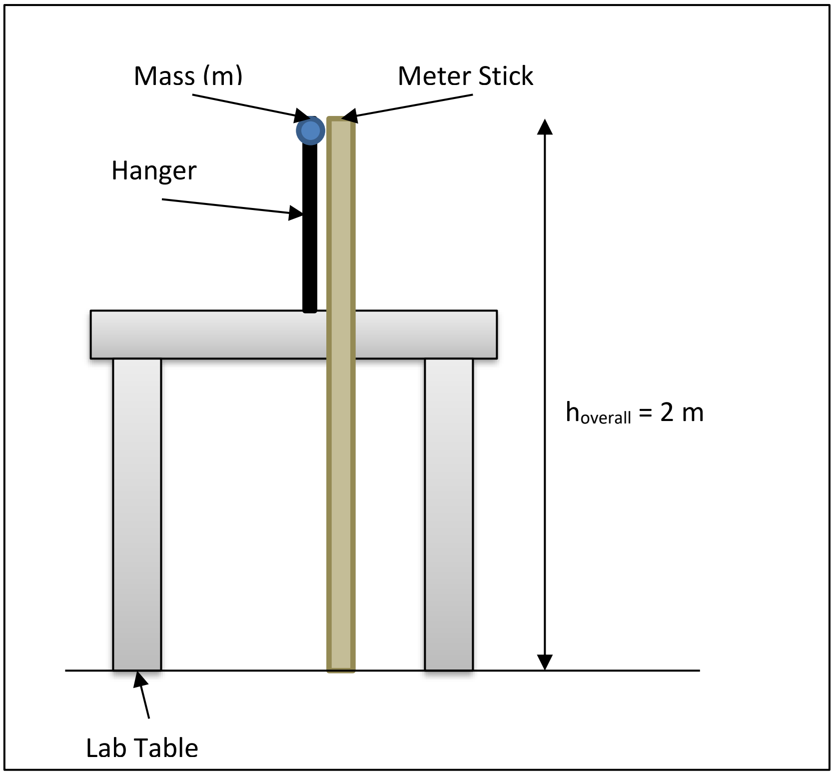



**Fig. 2: Experimental Setup during Drop.** The mass falls along the length of the meter stick and stretches the bungee cord.

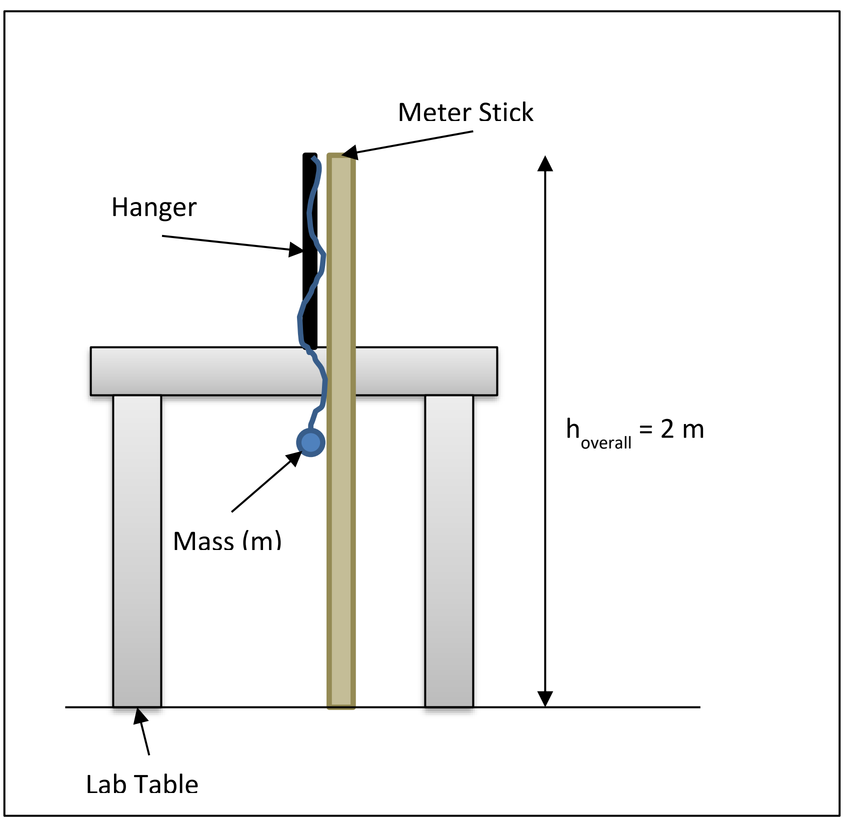
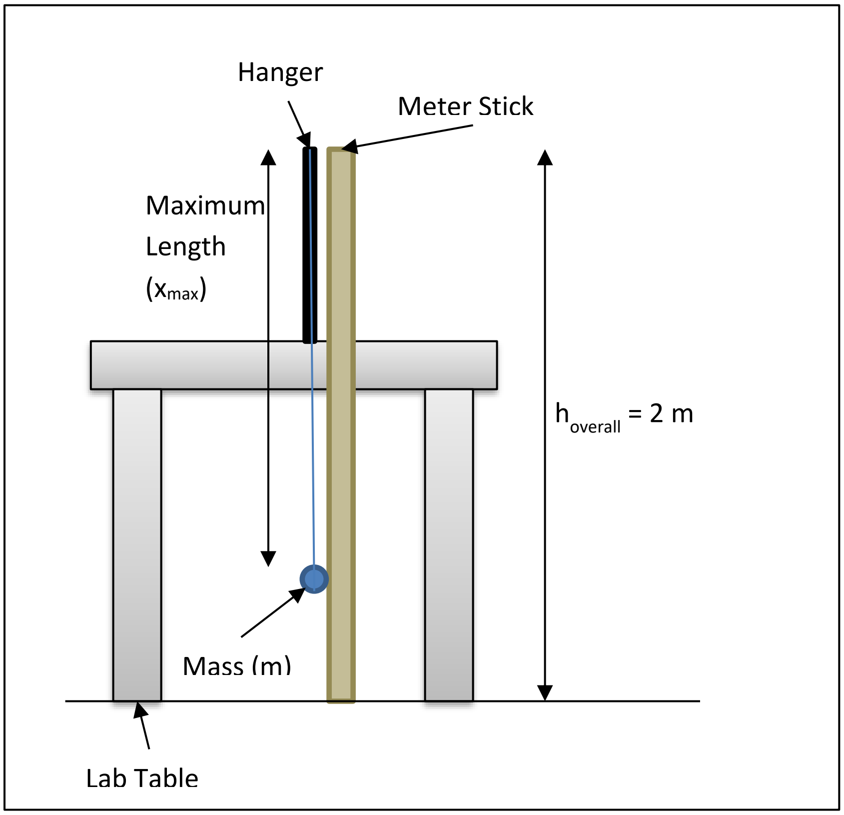


**Fig. 3: Experimental Setup at bottom of Drop.** The maximum stretch is found by subtracting the original length of the cord ($x_L$) (not shown) from the maximum length of the cord ($x_{max}$).




**Describe setup**:

We attached a meter stick to the face of a laboratory table with the 0 m mark facing away from the ground. Immediately next to the meter stick, we attached a hanger to the top of the table and hung our bungee cord from it, ensuring that the point at which the cord was secured (the point from which we intended to drop each mass) was coincident with the top of the meter stick. When we dropped each mass, we used an iPad to film the object as it fell. Using a high-speed camera application, we could determine how far each mass fell by examining the frame at which the mass switched from traveling downward to traveling upward.

**Describe procedure**:

1. We attached the bungee cord to the hanger, ensuring that its initial length ($x_L$) was identical to the length of cord we used in Bungee I (0.51 m ± 0.002 m)[1]. The k-value for this particular length of cord is approximately 1.74 N/m ± 0.02 N/m.
2. We attached a meter stick to the face of the table immediately next to the cord (i.e., immediately next to the path each mass would travel when dropped). We adjusted the height of the hanger to ensure that the point at which the bungee cord was secured to the hanger was coincident with the 0 m mark on the meter stick.
3. We hung a mass from the cord and dropped it from the point at which the cord was secured to the hanger.
4. Using an iPad and a highspeed camera application, we determined the maximum length of the cord for that given mass by examining how far down the meter stick the mass traveled at the moment when it began to travel back upward. For each mass, we repeated this measurement until we had collected five satisfactory distances. We averaged these distances to determine the average maximum length of the cord ($x_{max}$) for that mass.
5. We repeated steps 3 and 4 for four other masses.

[1] See Jackson Thigpen's journal article for more detail.

6. We subtracted the initial length of the cord ($x_L$) from the average maximum length for each mass to determine the stretch (x) of the cord for each mass.
7. We plotted the stretch (x) against mass to produce our dynamic model.

## *RESULTS*:

**Introduce the Results section** in a sentence or so, to give the reader context—data collected, and how it is analyzed to get the relevant result:

Using the methods outlined in our procedure, we collected and averaged the maximum stretch (x) experienced by our bungee cord when subjected to a variety of falling masses. By plotting each stretch value against its corresponding mass, we created a model which predicts the stretch of the cord for any given mass and determined the amount of stretch per applied kilogram for the length of cord we investigated.

**Tables**:

**Fig. 2: Maximum Length of Cord for Each Mass.** We dropped each mass five times from the top of the meter stick, using an iPad to film and analyze the maximum length the cord achieved ($x_{max}$) during the drop.

| mass of falling object **m** (kg) (± 0.02%) | trial 1 maximum length $\mathbf{x}_{max,1}$ (m) (± 0.01 m) | trial 2 maximum length $\mathbf{x}_{max,2}$ (m) (± 0.01 m) | trial 3 maximum length $\mathbf{x}_{max,3}$ (m) (± 0.01 m) | trial 4 maximum length $\mathbf{x}_{max,4}$ (m) (± 0.01 m) | trial 5 maximum length $\mathbf{x}_{max,5}$ (m) (± 0.01 m) | average maximum length $\mathbf{x}_{max,ave}$ (m) (± 0.01 m) |
|---:|---:|---:|---:|---:|---:|---:|
| 0.05 | 1.10 | 1.11 | 1.11 | 1.10 | 1.10 | 1.11 |
| 0.06 | 1.20 | 1.22 | 1.22 | 1.22 | 1.21 | 1.22 |
| 0.07 | 1.34 | 1.33 | 1.33 | 1.33 | 1.34 | 1.34 |
| 0.08 | 1.46 | 1.45 | 1.46 | 1.46 | 1.46 | 1.46 |
| 0.09 | 1.57 | 1.57 | 1.58 | 1.56 | 1.56 | 1.57 |

The uncertainty for our mass values is the standard uncertainty for laboratory masses (i.e., 0.02%). The uncertainty for each maximum length value stems from the smallest length value we could determine with confidence when reviewing the iPad footage of each fall.

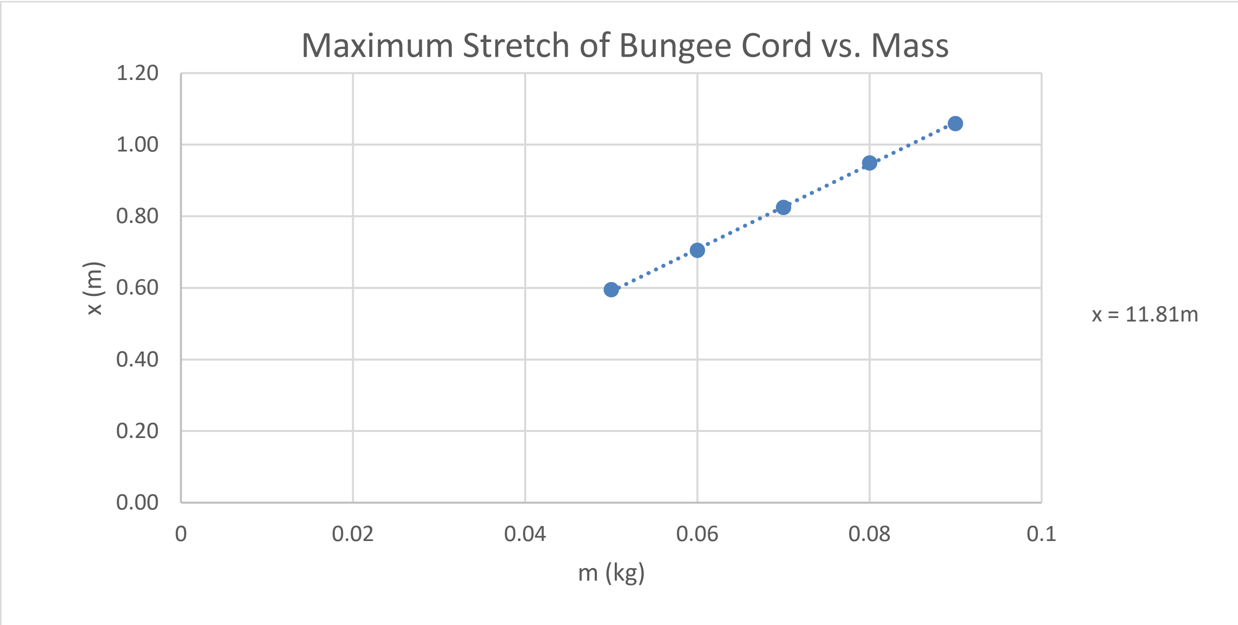
**Fig. 3: Stretch of Cord for Each Mass.** We subtracted the original length of the cord (0.51 m ± 0.002 m) from each average length ($x_{max,ave}$) found in Fig. 2 to determine the maximum stretch (x) of the cord for each mass.

| mass of falling object **m** (kg) (± 0.02%) | maximum stretch **x** (m) (± 0.01 m) |
|---:|---:|
| 0.05 | 0.60 |
| 0.06 | 0.71 |
| 0.07 | 0.83 |
| 0.08 | 0.95 |
| 0.09 | 1.06 |

The uncertainty for our mass values is the standard uncertainty for laboratory masses (i.e., 0.02%). The uncertainty the maximum stretch was found by propagating the uncertainty in the average maximum lengths from Fig. 1 (± 0.01 m) and the original length of the cord (± 0.002 m) using the quadratic sum method and rounding according to convention.

**Graph**:

**Fig. 4: Maximum Stretch of Bungee Cord vs. Mass.** This model indicates that the maximum stretch of the bungee cord (x) varies directly with the first power of mass (m).




Since there was no appreciable difference between the linear regression model produced when the y-intercept was allowed to vary and the linear regression model produced when the y-intercept was set through the origin (0,0), we decided to set the y-intercept through the origin.

**Equation** of the curve-fit from the graph:

$x = 11.81m$

**Use *Excel* regression analysis** on any graph that has a **linear** fit only (see EG), to obtain:

uncertainty for slope = 0.03 m/kg  % uncert = 0.3 %

uncertainty for y-intercept = N/A[2]  % uncert = N/A

**Identify experimental value(s) of interest**, why it is of interest, and how/from where obtained, briefly:

The primary experimental value of interest is the amount of cord stretch per applied kilogram, 11.81 m/kg ± 0.3 %, which is the slope of the model we obtained for the relationship between the maximum stretch of the cord and the amount of applied mass.

value obtained = 11.81 m/kg

uncertainty of experimental value(s) = 0.03 m/kg  % uncert = 0.3%

name the technique used for propagation of uncertainty (see *UG*), or where/how uncert was obtained:

The uncertainty of the amount of cord stretch per applied kilogram was found using Excel's regression analysis capabilities.

[2] Since we decided to force the intercept through the origin, there is no uncertainty for the y-intercept.

**Summarize Results (just the facts)**—give the important, relevant results, and why/how they are relevant to the purpose, in a sentence or two, including main equations and quantitative results:

By developing a linear model, $x = 11.81m$, for the relationship between the maximum stretch of the cord and the applied mass, we were able to determine that the amount of cord stretch per applied kilogram is 11.81 m/kg ± 0.03 m/kg. This result is significant because it allows us to accurately predict how far the bungee cord will stretch when a particular mass is dropped, which is critical in ensuring that our egg will refrain from contacting the ground during the bungee challenge.

## *DISCUSSION*:

**Error analysis**:

Amount of Stretch per Applied Kilogram = 11.81 m/kg ± 0.3%

If no values are available for comparison, **determine "acceptability" of uncertainty in your value(s)** according to your needs**. AND determine a test** of your value(s) for "error" -- e.g. use your result to predict something, and then measure it (if time permits), or briefly describe how you would test it:

Since there is no ideal value for the amount of cord stretch per applied mass, a percent uncertainty vs. percent error evaluation of the accuracy of our value is not possible. However, we can quantify and evaluate the limitations brought about its uncertainty by examining how it relates to the overall bungee challenge. Our data suggests that there is a 0.3% error in the relationship we obtained for the distance the cord stretches per kilogram of applied mass. Assuming that the bungee challenge will take place from a height of three stories (the top balcony in the Great Hall) and that the height of a single story is approximately 3 m, we can predict that the bungee challenge will take place over a distance of approximately 9 m. 0.3% of 9 m is 0.027 m or 2.7 cm. 2.7 cm is a fairly small distance when compared to 9 m; however, we must be aware of this limitation when attempting to position our egg as close to the ground as possible without making direct contact.

Our results can also be assessed by comparing our experimental model to the theoretical dynamic model for an object falling when attached to an ideal spring. As demonstrated in our introduction, the classical work-energy theorem suggests that stretch of a spring will vary directly with the square root of the applied mass, that is, $x = \sqrt{\frac{2gh}{k}}\sqrt{m}$, where x is the stretch of the spring, g is the acceleration due to gravity (9.81 m/s$^2$), k is the spring constant of the particular spring used, and m is the mass of the attached object. However, our results suggest that x varies directly with the first power of m. This comparison does not necessarily indicate that our results are inaccurate; however, it does indicate that our bungee cord does not act like an ideal spring.

**Sources of uncertainty**:

There are a number of sources of uncertainty that could have affected the results of our experiment. First, as mass is applied repeatedly to a bungee cord, it inevitably begins to lose some of its elastic characteristics. Over time, the cord's molecules change so that it no longer returns to the exact same position when all loadings are eliminated. This is perhaps the most significant source of error present in our experiment. Limiting the amount of time mass is applied to the cord is the only real way to eliminate this source of uncertainty. Another possible source of uncertainty pertains to the method by which we recorded the maximum length of the bungee cord (i.e., recording the falling object with an iPad and assessing, frame by frame, its position). Since we did not have a tripod to hold and stabilize the iPad it is possible that parallax error could have affected the precision of our distance measurements. Adjusting our experimental setup to include a tripod for the iPad would eliminate this source of uncertainty. Finally, the meter stick, which we used as a scale in our iPad videos to determine the maximum length of the cord, could have deviated slightly from the vertical position. While this source of uncertainty is certainly minimal, especially since we attached the meter stick to the edge of a laboratory table, which is nearly vertical, adjusting our procedure to include leveling the meter stick would eliminate any doubt as to whether the meter stick was oriented properly.

In a couple sentences**, describe whether your main results support your hypothesis.** How well were the results in agreement with theory, expectations, or otherwise deemed "acceptable"? Why/how so, or not?

We predicted that the dynamic model produced by our experimentation would be similar to the classical work-energy theorem formulation for an object falling when attached to an ideal spring, with the possible addition of a term or factor to account for the fact that our bungee cord does not act a perfectly ideal spring. In particular a direct relationship between the maximum stretch of the cord and the square root of the falling mass would be present. However, our results suggest that the maximum stretch of the bungee cord is directly proportional to the first power of the amount of dropped mass. Therefore, we cannot report that our experiment confirms our hypothesis.

***CONCLUSION***:

Clearly and definitively state the experimental outcome(s) in terms of your question or purpose:

We initially designed this experiment to determine an empirical model for the relationship between the maximum stretch of a given length of bungee cord and the amount of mass dropped while attached to said cord. By evaluating the relationship between the cord's maximum stretch (as determined by subtracting its maximum length from its original length) and various falling masses, we determined that the two quantities are related by the model $x = 11.81m$, where x is the maximum stretch and m is the mass. The coefficient 11.81 m/kg ± 0.3% quantifies the stretch of the cord per kilogram of applied mass, a critical value in determining how the bungee cord will respond when any mass is applied.

Implications of these conclusions (e.g. the significance to larger questions), or next steps proposed:

Using our empirical model and the amount of cord stretch per kilogram value, we can now predict the maximum distance a 0.51 m ± 0.002 m bungee cord will stretch when any mass is applied.[3] This result is critical to successful completion of the bungee challenge since it allows us to determine the maximum length of the bungee cord for any applied mass. The maximum length of the cord must be less that the height of the drop for our egg to complete its jump without experiencing any damage. The logical next step for experimentation is to examine how this empirical model changes as the initial length of the cord is varied, since the elastic properties of a bungee cord are dependent upon the length of cord used.

**Report Outlines are *individual assignments*. Cite any work not your own, acknowledge any aid, and pledge the report:**
**On my honor, I have neither given nor received any unacknowledged aid on this assignment.**

***Pledged:*** *Matthew O'Neal Withers*

[3] Applied masses must not exceed the elastic rating of the cord.