TITLE: Determining the Effect of Unstretched Length of a Cord on its Spring Constant *k*
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ABSTRACT:

To characterize the behavior of an elastic cord, the cord can be approximated to behave like a series of springs attached to one another. Hooke's Law can then be applied and the spring constant *k* can be determined. This constant is necessary to predict how a certain spring or cord will behave in a given set of conditions. However, the value of k is dependent on multiple factors, one of which is the length of the cord or spring itself. This experiment tested the effects of unstretched length, or the equilibrium length of the cord when no mass is attached, on k. To find k with respect to the unstretched length of the cord, a constant mass was dropped from a two-stranded elastic cord of various unstretched lengths, ranging from 0.33 m to 0.97 m, and the total distance travelled was measured. Then by using the displacement of the egg, an average k value for each unstretched length was determined. Unstretched lengths were plotted against the average k values for each unstretched length, and a relationship between the two variables was determined to be $k= 3.506l^{-0.777}$. Percent uncertainty in k was determined to be 11%. The hypothesis that k would decrease as length increased was shown to be true. Ultimately, these k values will allow for the best bungee experience by using them to determine the length that will allow the egg to get as close as possible to the ground without breaking.

INTRODUCTION:

The ideal bungee jump experience gives the jumper the longest free-fall possible and reaches the maximum possible speed. The bungee challenge attempts to simulate a real life bungee jump using an elastic cord and a raw egg attached to the cord through a harness. To have a successful bungee jump, the cord must be fully characterized to know how it would behave in this situation, giving an ideal bungee experience while also keeping the egg intact. To characterize the cord, Hooke's Law was used to determine a spring constant *k* that would allow us to predict the distance the egg would travel during the jump. However, this spring constant depends on numerous factors, including the equilibrium length of the cord.

The purpose of this experiment was to determine how the spring constant of the elastic cord *k* changed as the unstretched length of the cord changed. Unstretched length is defined as the length of the cord from the point at which the cord itself was hung on the metal rod and the point where the mass is attached to the cord. Unstretched lengths were measured with no mass attached to the cord. By knowing how *k* values change with variation of this length, it will be possible to determine what length the cord must be to allow the egg to get close to the ground without breaking. It is hypothesized that *k* will decrease as unstretched length increases.

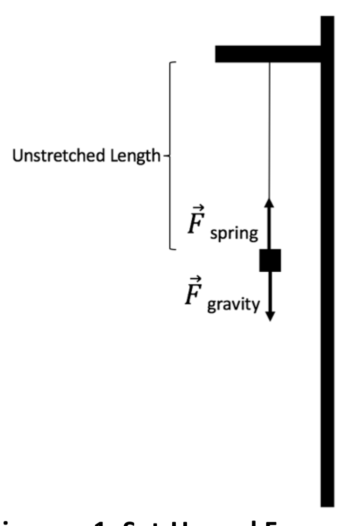


Diagram 1: Set-Up and Forces. A diagram of the set-up of the experiment and the forces in the system.

Brief Theoretical Background: By using the Conservation of Work and Energy Theorem, an equation for k was found to be:

$$k=\frac{-2mg}{x}$$

where m is the mass hanging from the cord, g is acceleration due to gravity, and x is the vector displacement of the hanging mass. By finding x experimentally and maintaining a constant mass, experimental k values were found for each unstretched length.

The derivation of the equation for k is from the Conservation of Work and Energy Theorem, which states:

$$W_{nonconservative}=\Delta PE+\Delta KE$$

where W is the total work done by nonconservative forces in the system, ΔPE is the change in potential energy of the system, and ΔKE is the change in kinetic energy of the system. Because the system both starts and stops at rest, $\Delta KE=0$. Because the only forces in the system are the force of gravity (or the weight of the hanging mass) and the force of the spring (in this case, the elastic cord), both of which are conservative forces, $W_{nonconservative}=0$. Since both $W_{nonconservative}=0$ and $\Delta KE=0$, $\Delta PE=0$, therefore work of the system can be stated as $W_{conservative}= -\Delta PE_{total}=0$.

In this system, there are two forms of potential energy:

$\Delta PE_{gravity}$: mgh

where h is defined as the difference between the total distance travelled and the unstretched length of the cord, and

$\Delta PE_{spring}= \frac{1}{2} kx^2$

where k is the spring constant value for this cord that is approximated to behave like a spring and x is the vector displacement of the hanging mass, which is defined as the difference between the total distance travelled and the unstretched length of the cord.

Simplification of these equations results in:

$$k=\frac{-2mg}{x}$$

The above equation was used to determine experimental k values using the displacements for each free fall.

METHODS:

Overall method:

A constant mass of 0.171 kg was attached at various unstretched lengths of an elastic cord. The distance travelled by the hanging mass was filmed on an iPad and recorded. Displacements were calculated and used to find k values for each unstretched length using the previously mentioned equation for k. Three free falls were performed for each drop, and the average k for each unstretched length was calculated.

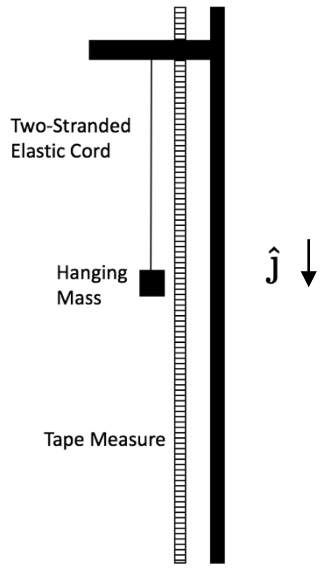


Diagram 2: Experimental Set-Up. A mass was hung from an elastic cord attached to a metal rod. A tape measure was held with the rod at the same height as the cord.

Describe Setup: A double-stranded elastic cord was hung from a metal rod system (Diagram 2). A constant mass of 0.171 kg was attached to the cord at the bottom of the length defined as the unstretched length. A tape measure was held from the same height from which the cord was hung and used to measure the distance travelled by the hanging mass when it was dropped.

Procedure:

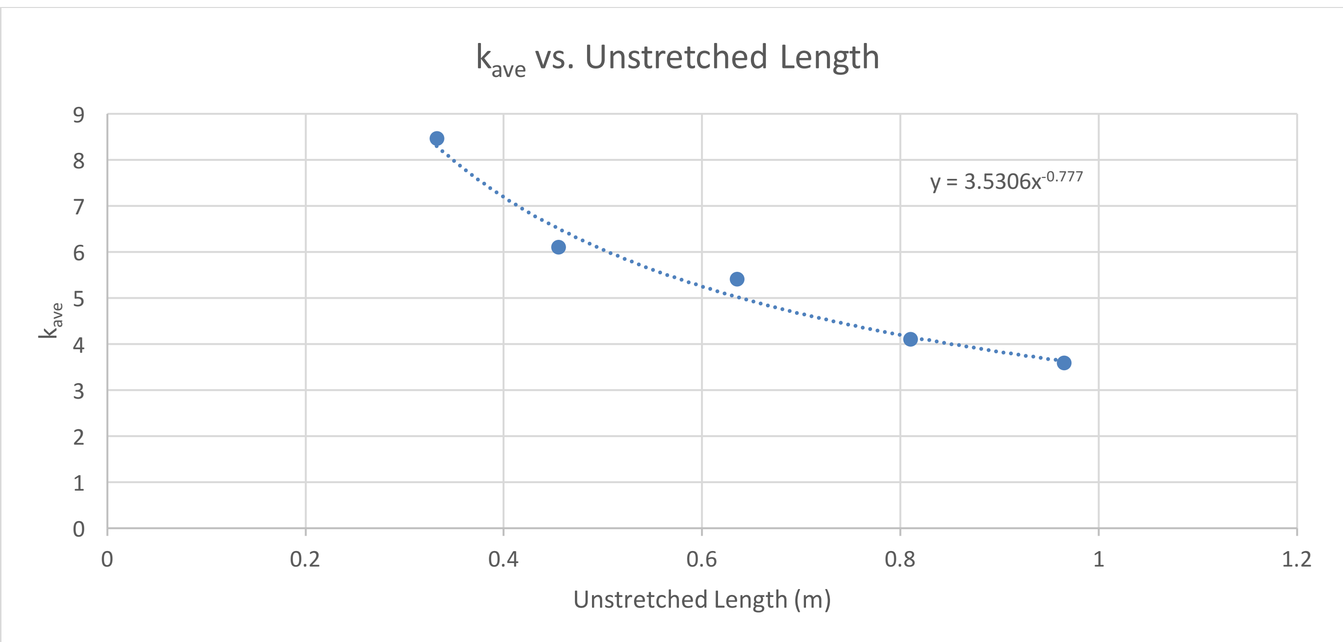
- A double stranded cord was hung from a metal rod.
- A knot was tied at various distances from the point at which the cord was hung. The distance from the point at which the cord was hung and the knot was defined as the unstretched length.
- The unstretched length was measured with no mass attached to the cord.
- A mass of 0.171 kg was attached to the cord at the bottom of the unstretched length and released from rest.
- A video of the fall was recorded on an iPad.
- Using the tape measure in the background of the video, the videos were watched to determine the final distance reached by the hanging mass.
- This procedure was performed three times for each of the five unstretched lengths: 0.33m, 0.46m, 0.64m, 0.81m, and 0.97m.
- By finding the difference between the total distance travelled and the unstretched length, the displacement was calculated for each trial.
- Displacement values were plugged into the equation: $k = \frac{-2mg}{x}$
- Experimental *k* values were determined for each trial.
- The average *k* value across the three trials per each height was calculated.
- Unstretched lengths were plotted against average *k* values to determine a mathematical relationship between the two variables.

RESULTS:

Introduction to Results: Displacement from various unstretched lengths during the fall of a hanging mass was calculated by finding the difference between the total distance travelled by the mass and the unstretched length. These displacements were graphed versus the unstretched length of the cord to determine a relationship between the two variables.

Mass (kg) ±0.1kg	Unstretched Length (m) ± 0.02 m	Displacement (m) ± 0.03 m	Experimental k	k_{ave}
0.1714	0.3325	-0.3875	8.678369032	8.463617042
0.1714	0.3325	-0.3975	8.460045283	
0.1714	0.3325	-0.4075	8.25243681	
0.1714	0.455	-0.545	6.1704	6.096281081
0.1714	0.455	-0.555	6.059221622	
0.1714	0.455	-0.555	6.059221622	
0.1714	0.635	-0.615	5.468078049	5.409751883
0.1714	0.635	-0.625	5.3805888	
0.1714	0.635	-0.625	5.3805888	
0.1714	0.81	-0.805	4.177475776	4.101775869
0.1714	0.81	-0.825	4.076203636	
0.1714	0.81	-0.83	4.051648193	
0.1714	0.965	-0.93	3.615987097	3.577988371
0.1714	0.965	-0.955	3.521327749	
0.1714	0.965	-0.935	3.596650267	

Table 1: Raw Data. Raw numerical data for unstretched length and displacement. K values were calculated for each unstretched length, and the average k was found across the three k values for each unstretched length.



Graph 1: Average k and unstretched length. Average k values were plotted with their corresponding unstretched length. A trendline was fit to the graph, and the equation for this line was found to be $k=3.5306l^{-0.777}$.

Equation: $k=3.5306l^{-0.777}$

Propagation-of-uncertainty analysis:

uncertainty in coefficient(s)= ± 0.108 % uncertainty= 11%

Uncertainty in displacement was first found to be ± 0.03, as displacement was affected by uncertainty in the measurement of the total distance travelled and the uncertainty in the unstretched length, both of which had a ± 0.02 m uncertainty associated with them. The displacement uncertainty was then used to find uncertainty in *k*. Since *k* values were calculated with the equation, $k=\frac{-2mg}{x}$, uncertainty was propagated using the formula $\sqrt{(n\frac{\Delta a}{a})^2 + (m\frac{\Delta b}{b})^2}$.

Experimental value of interest: The experimental value of interest is *k* with respect to unstretched length, which is derived from the graph of k_{ave} versus unstretched length in the equation $k=3.5306l^{-0.777}$. This equation allows for the calculation of any k value for any unstretched length of this elastic cord.

uncertainty of experimental value(s) = $\pm$ 0.025 % uncertainty= 2.5%

Propagation of Uncertainty: The formula $\sqrt{(n\frac{\Delta a}{a})^2 + (m\frac{\Delta b}{b})^2}$ was used to find uncertainty in the experimental model of k, $k=3.5306l^{-0.777}$.

Summary of Results

- A constant mass was hung from a cord of different unstretched lengths.
- The displacement of the mass from each unstretched length was recorded.
- The displacement was used to calculate the *k* value with respect to the unstretched length of the cord.
- The average *k* value across three trials per unstretched length was calculated and plotted with its corresponding unstretched length.
- A trendline was fit to the plotted points, and an equation relating unstretched length and average *k* value was found to be $k=3.5306l^{-0.777}$.

DISCUSSION:

Error analysis:

While there is no accepted *k* value to compare these results to, it is possible to determine whether or not these results are accurate. First, in comparison to similar experiments in the Bungee Journal, these results demonstrate the same trend seen by other groups. As the unstretched length of the cord increased, *k* values decreased, ultimately resulting in a larger displacement.

Second, this model can be compared to the accepted theoretical model that $k(x_L) \propto \frac{1}{x_L}$, which demonstrates that value of k with respect to the equilibrium (unstretched) length is proportional to $\frac{1}{x_L}$, or $x_L^{-1.00}$. For the elastic cord tested in this experiment, the constant of proportionality according to this accepted model is 3.489, meaning that according to the accepted model, for this elastic cord, $k=3.489l^{-1.00}$. The constant of proportionality was found by approximating the value of k for an unstretched length of 0.97 m to be the value of k for an unstretched length of 1.0 m. These values were then plugged into the equation for a general $k(x_L)=\frac{k_R x_R}{x_L}$, and $k(x_L)$was solved for. The experimental model can be tested for accuracy by comparing the percent error to the percent uncertainty of the experimental model.

Uncertainty vs. error:

For an average unstretched length (0.64 m, the average of the set of lengths tested in this experiment), the experimental model states that the k= 5.41. The theoretical model states that k= 5.45. Using these values, the percent error is found to be 0.9%. Because the percent error is smaller than the percent uncertainty in k with respect to unstretched length (2.5%), in comparison to the theoretical model, the experimental model is accurate. However, the percent error is likely slightly larger than 0.9% due to the fact that the constant of proportionality is based on an approximation, therefore it is possible that the model is inaccurate.

Determining a test:

The trendline equation, $k=3.5306l^{-0.777}$, can also be tested for accuracy by using it to predict a new value. To test this equation, a starting unstretched length of the elastic would be determined and used to find the *k* value. Then, this *k* value could be used to predict the displacement of a certain mass using the equation initially used to find *k* values, $k=\frac{-2mg}{x}$. Should the prediction be accurate within the uncertainty of *k*, the equation for *k* with respect to unstretched length would be deemed accurate.

Sources of uncertainty:

There are multiple possible sources of uncertainty. First, a high degree of difficulty was associated with measuring the distance travelled by the mass during the free fall. The tape measure was hard to read in the videos of the free fall, therefore an uncertainty of $\pm$ 0.02 m was associated with every measurement taken by the experimenters, including the unstretched length and the distance travelled in the fall. Second, air resistance is not accounted for in this experiment. This force is in the opposite direction of the fall, which would cause a smaller displacement and therefore a larger *k* value. Therefore, the displacements used may not directly relate to the *k* value calculated from the displacement, making the equation for *k* less accurate.

The results ultimate supported the hypothesis, since *k* values decreased as unstretched length increased. An important consequence of this trend is that displacement also increases as unstretched length increases. This trend will be an important factor in the final Bungee Challenge.

CONCLUSION:

Experimental outcome:

This experiment resulted in an equation that relates the spring constant of an elastic cord k to the unstretched length of a cord at rest. This equation, $k=3.5306l^{-0.777}$, can be used to predict the k value for any unstretched length of this elastic cord.

Implications of these conclusions:

This k value can be used to predict the displacement of a certain mass that falls from a certain unstretched length of the elastic cord. These calculations will be important for the bungee challenge as they will allow a determination of the starting unstretched length of the cord that will give the greatest displacement without breaking the egg. By accounting for the uncertainty in k in the calculations predicting the displacement (for example, account for only 90% of the height it will be dropped from as opposed to the entire height), it would be possible to get an accurate prediction of the displacement of the egg during the bungee challenge. Further work would include testing the model for accuracy by using the equation for k with respect to unstretched length to predict the displacement of a certain mass.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.