Lab Report Outline—the Bones of the Story

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TITLE: Determining how the dynamic spring constant (k) varies with dropped mass

ABSTRACT

In order for the bungee jump to be a success, there needs to be a good prediction of how much the bungee cord will stretch and how much acceleration it will apply to the egg at the end of the jump. By modeling the cord as an ideal Hooke's Law system and using the Conservation of Work-Energy (CWE) theorem, an estimate of the dynamic spring constant (*k*) of the bungee cord can be calculated. In order to do this, 12 different masses were attached to the bungee cord and dropped, and the vertical displacement from each trial was used to calculate the dynamic *k* of the cord given a certain mass. Using a force monitor during these trials, the maximum force during the jump was recorded and the maximum acceleration that the egg underwent was calculated. We find that the dynamic *k* of the bungee cord is constant in mass, and that the maximum acceleration undergone by the mass is decreasing in mass until converging to 22 m/s^2. This estimate of the dynamic *k* and maximum acceleration will be crucial when calculating what length of cord we want to use during the bungee jump.

INTRODUCTION

During Bungee Lab 1 we modeled the bungee cord as a linear spring to estimate the static spring constant

$$k_0 = \frac{mg}{x_0}$$

of the bungee cord and found that the spring constant was decreasing with additional mass. The purpose of the experiment this week was to evaluate the dynamic spring constant, *k*, of the bungee cord to help decide which length of bungee cord to use during the bungee egg jump.

The potential energy at the top due to gravity is $PE_{top} = mgh$. Modelling the cord as an ideal Hooke's Law System, the kinetic energy at the instant when the mass stops falling can be calculated as

$$KE_{bottom} = \frac{1}{2}kx^2$$

The only forces acting on the experimental mass are gravity and the bungee cord modelled as an ideal Hooke's Law System. This means that all of the forces in the experimental system are conservative, and the CWE theorem can be applied

$$(PE + KE)_{top} = (PE + KE)_{bottom} \Leftrightarrow mgh = \frac{1}{2}kx^2 \Leftrightarrow k = \frac{2mgh}{x^2} = \frac{2mgx_{max}}{(\Delta x)^2}$$

In the above equation, k is the dynamic spring constant in Newtons per meter, m is the mass in kilograms, g is the force of gravity in meters per seconds squared, x_{max} is the distance from the drop point to the cord's maximum extension in meters, and Δx is the displacement between the static cord equilibrium with the hanging mass (x_0) and x_{max} in meters. Since the static *k* was decreasing in mass due to the x_0 value in the denominator, we expect that since we have $(\Delta x)^2$ in the denominator, this will linearize the data and give us a constant dynamic *k*.

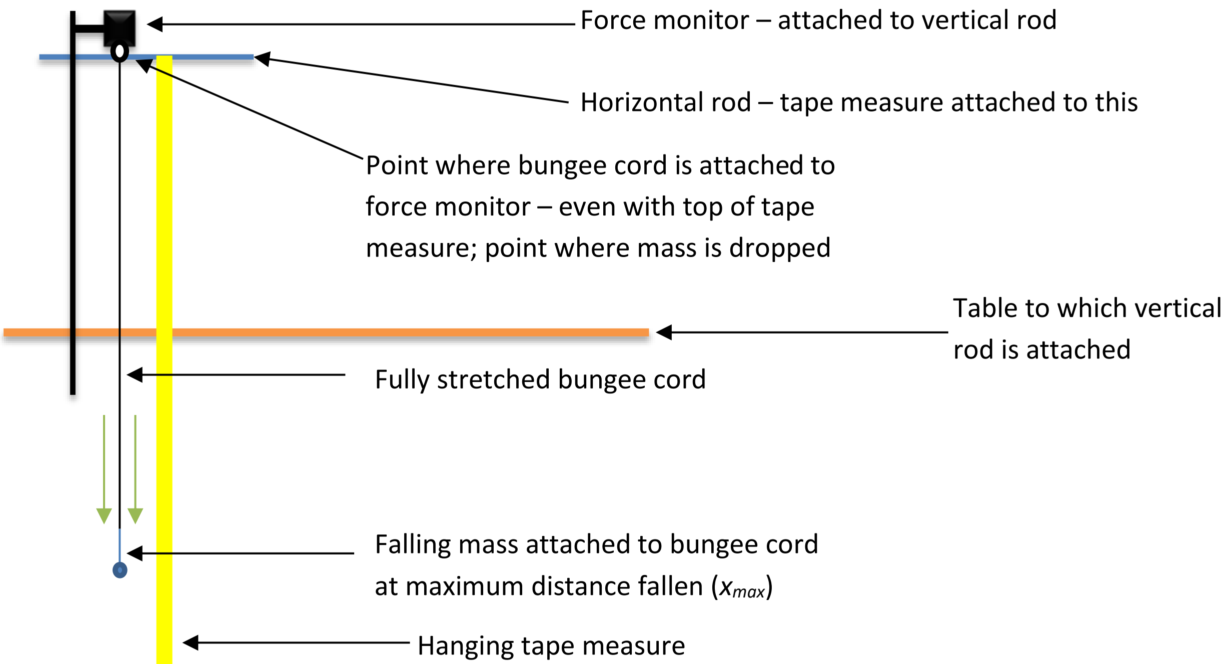
We were also interested in the maximum acceleration that the egg will undergo during the bungee jump, so we used a force sensor to measure F_{max} and calculate the maximum acceleration using Newton's Second law:

$$F_{max} = ma_{max} \Leftrightarrow a_{max} = \frac{F_{max}}{m}$$

METHODS

We tied the bungee cord to a force monitor fixed atop a metal rod and taped a tape measure in place parallel to the mass's predicted falling path. We then dropped various masses attached to the bungee cord from a point even with the rod. We used Capstone to record the maximum amount of force exerted during the fall and slow-motion video to determine the maximum distance that the mass fell.

Figure 1: Diagram of Lab Setup



Procedure:

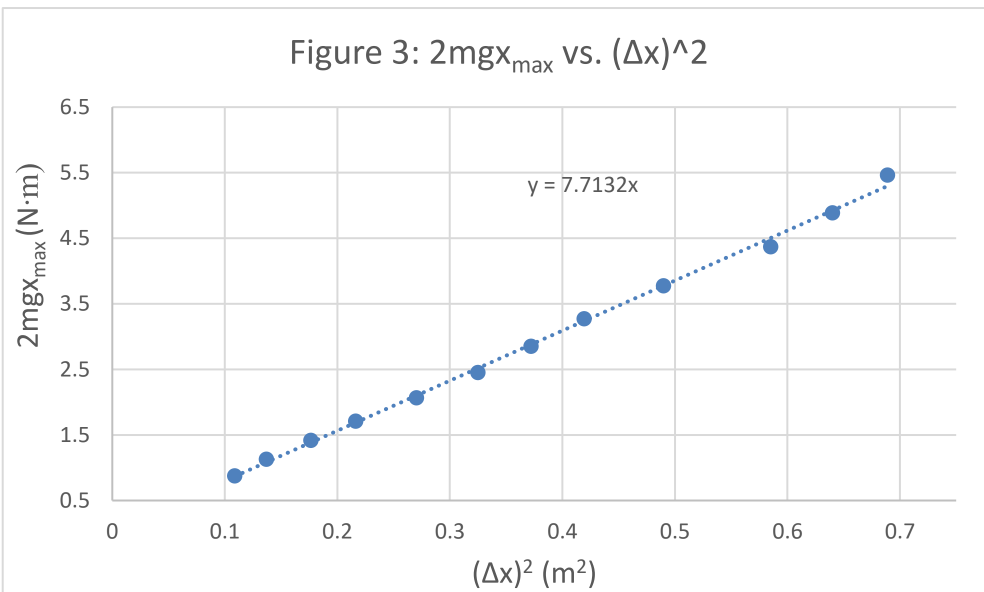
First, attach a long metal rod perpendicularly to the table and then attach a horizontal bar with knobs near top of vertical bar. Next, tape the tape measure to the horizontal bar. After that, attach the force monitor to the vertical bar above horizontal bar such that the ring at the bottom of the force monitor is even with the horizontal bar. With the force monitor in place, measure out a .49m length of bungee cord, tie a very small knot with which to attach it to the force monitor, and loop knot around the ring at the bottom of the force monitor. After this, hook the mass holder (m=0.05 kg) through a small loop at the bottom of the hanging bungee cord. Then, put the trial mass on the holder and raise the top of the mass holder until it is even with the ring on the force monitor. Lastly, start capstone, drop the mass, and record the expected point of maximum stretch with slow-motion video.

RESULTS

This experiment resulted in a constant k value of 7.71. We also found that acceleration is decreasing in mass until it begins to converge to 22 m/s^2 at 0.11 g. Maximum acceleration was calculated from the information on maximum force that was found from the force monitor which fed through the Capstone program. x_0 was found by hanging the weight from the metal bar and measuring how far the bungee cord stretched. The data on x_{max} was gathered by recording the bottom of the mass's fall with a slow-motion app, and lining the mass up with the tape-measure at its lowest-point. Using x_0 and x_{max}, we were able to calculate Δx which allowed us to calculate k.

Figure 2: Experimental Data and Calculations

Mass (±0.001 kg)	x_0 (±0.001 m)	x_{max} (±0.01 m)	Δx (±0.01 m)	k (±1% N/m)	F (±0.01 N)	a (±0.05% m/s²)
0.05	0.56	0.89	0.33	8.02	1.41	28.20
0.06	0.59	0.96	0.37	8.26	1.59	26.50
0.07	0.61	1.03	0.42	8.02	1.75	25.00
0.08	0.625	1.09	0.47	7.91	1.9	23.75
0.09	0.65	1.17	0.52	7.64	2.07	23.00
0.1	0.68	1.25	0.57	7.55	2.23	22.30
0.11	0.71	1.32	0.61	7.66	2.43	22.09
0.12	0.7425	1.39	0.65	7.81	2.63	21.92
0.13	0.78	1.48	0.70	7.70	2.84	21.85
0.14	0.825	1.59	0.77	7.46	3.01	21.50
0.15	0.86	1.66	0.80	7.63	3.28	21.87
0.16	0.91	1.74	0.83	7.93	3.49	21.81



The linear relationship from the graph is $2mgx_{max} = 7.71(\Delta x)^2$ and the uncertainty for the slope is .05 or 0.6%. We know that *k* is the coefficient of $(\Delta x)^2$ in the graph equation, so the bungee cord's dynamic *k* value, the experimental value of interest, is a constant 7.71 N/m (±.05 N/m). We obtained the uncertainty of our *k* value through a regression analysis, and all other uncertainties are measurement uncertainties. From Figure 2, we see that acceleration is decreasing with mass, until it begins to stabilize around 22 m/s^2. The most important take-away for us, however, is that the dynamic *k* is constant for the falling mass.

DISCUSSION

After conducting the experiment, the results support our initial hypothesis as the dynamic *k* of the bungee cord is constant and independent of the size of the dropped mass. This also means that our modeling of the bungee cord as an ideal Hooke's Law System was a correct choice. We do not have a known *k* value for the cord, but given the strong, linear relationship exhibited in Figure 3 and 0.6% uncertainty of our linear coefficient, we are quite certain that our *k* value of 7.71 N/m is accurate, which will be very important for us when we calculate the length of cord with which to drop the egg.

The percent uncertainty for the estimate of dynamic *k* is low at 0.6%. This is about as low as we can expect to get the uncertainty given the imprecise method of measuring x_{max}. This measurement of x_{max} is the primary source of uncertainty for the estimate of *k*. We tried to minimize this uncertainty by conducting multiple trials for masses with outlier Δx's and recording video from multiple angles. Another main source of uncertainty was the flight path of the falling mass. We assumed that it fell directly downwards, but even small deviations off a downwards line would introduce uncertainty into the estimate. We were very precise in how we dropped the mass and redid any trials where even the smallest item went wrong. The measurements of the masses and the initial displacement are both very precise, and we do not believe they contributed very much uncertainty to the estimate of dynamic *k*. We will test the estimate of dynamic *k* during the bungee egg jump. The percent uncertainty for the estimate of maximum acceleration is low at 0.5%, and we are very confident in this result.

CONCLUSION

The primary purpose in this experiment was to see how dynamic *k* varied with mass, and we were successful in this regard as we were able to establish that dynamic *k* is, in fact, constant as mass varies. The secondary purpose was to understand how acceleration varied with mass, and again we were successful as we found that it decreases before converging to 22 m/s^2. This knowledge will allow us to more accurately calculate what length of bungee cord we should use in the bungee egg jump.

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On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Christopher Curfman