Determining the Relationship Between the Spring Constant and Length of Bungee Cord

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ABSTRACT:

This report presents an experimental determination of the spring constant for a bungee cord using Hooke's Law. Multiple experiments were performed measuring the displacement of a bungee cord due to various hanging masses $m_{hanging}$ across multiple initial bungee lengths x_i. Displacement was measured by subtracting the stretched length from the initial length of bungee cord. Five different initial lengths of bungee cord were tested across five different masses in order to determine how stretch length varies. Experimental data was used to determine the equation $k = 1.4149(1/x_0) - 0.3268$ where k is the spring constant for the bungee cord and varies as a function of inverse length of initial bungee cord. We can use this equation to predict the spring constant of our bungee cord at a given unstretched length, assuming the bungee cord behaves like an ideal spring.

INTRODUCTION:

The purpose of this lab was to experimentally derive an equation relating the spring constant k to the initial length x_i of our bungee cord. Assuming the bungee cord behaves like an ideal spring, Hooke's Law states that $F_{spring} = -kx$ where $F_{spring} = 9.81m_{hanging}$ and x is the displacement of the bungee cord due to the hanging mass. Using Hooke's Law we can determine the spring constant k across varying lengths of bungee cord and derive an equation relating k to the length of bungee cord.

Rearranging Hooke's Law equation shows that $k = -F_{spring}/x$, thus for a bungee with a larger x_i we expect a smaller k value. Thus, we expect the spring constant k to vary inversely with bungee length.

METHODS:

<u>Experiment 1:</u> Determining the spring constant k as a function of original bungee length x_i.

Figure 1 outlines the general experimental set up for determining the spring constant k as a function of original bungee length x_i. Data was collected by measuring the displacement of the bungee cord due to the gravitational force of a hanging mass $m_{hanging}$. Multiple trials were run with varying bungee lengths and hanging masses. Experimental data was used to derive an equation relating the spring constant k to the initial bungee cord length x_i.

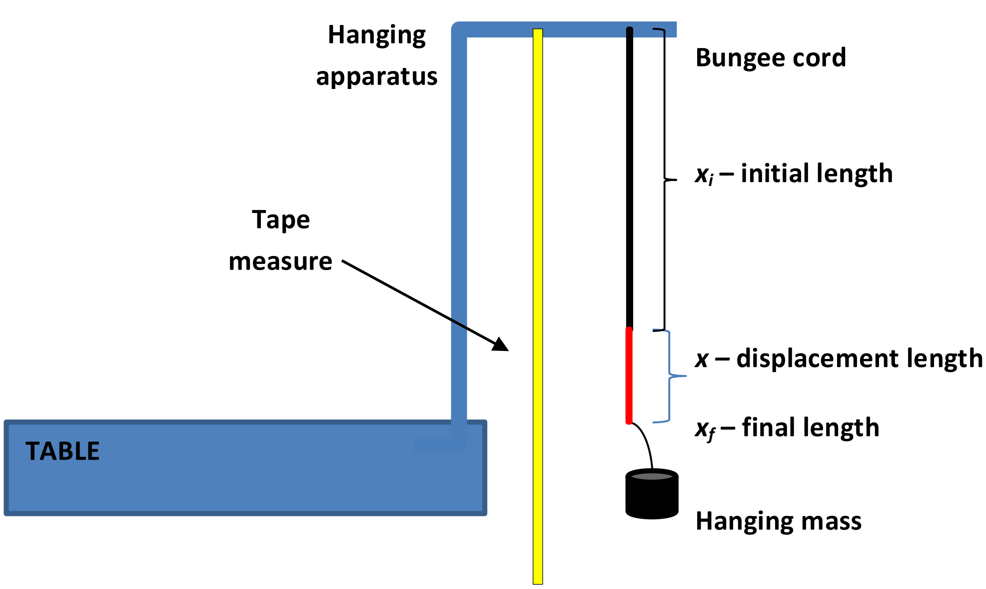

Figure 1: Diagram of initial experimental setup of the bungee cord and hanging apparatus.

- STEP 1: Attach the bungee cord to an apparatus appropriate for hanging masses using a secure knot.
- STEP 2: Tie a knot in the bungee cord and measure the initial bungee length from the first knot to the second. Record.
- STEP 3: Attach a hanging mass to the lower knot and measure the total bungee length. Record.
- STEP 4: Repeat with four more varying hanging masses. Record total bungee length for each trial.
- STEP 5: Untie the lower knot and tie at a new location. Measure the new initial bungee length from the first knot to the second.
- STEP 6: Repeat Steps 3 & 4 for the new total length.
- STEP 7: Repeat Steps 5 &6 until data has been gathered for five different initial bungee lengths.

RESULTS:

Data collected relates the spring constant for the bungee cord (k) to the inverse of initial bungee cord length (x_i). Data was gathered by measuring displacement of the bungee cord by the gravitational force of a hanging mass ($m_{hanging}$). Displacement due to various hanging masses across initial bungee cord length were measured as listed in Table 1.

Table 1: Displacement for each initial bungee length due to the gravitational force of the hanging mass for all five initial bungee cord lengths x_i.

	Cord Length x_i (m, ± 0.01 m)	0.10	0.25	0.40	0.55	0.70
Hanging Mass (kg)	F_{grav} (N)	**Displacement at each initial length of bungee x** (m, ± 0.01 m) $x = x_f - x_i$				
0.05	0.4905	0.020	0.029	0.041	0.058	0.075
0.07	0.6867	0.045	0.076	0.109	0.149	0.197
0.09	0.8829	0.071	0.119	0.173	0.242	0.319
0.11	1.0791	0.100	0.160	0.237	0.329	0.434
0.13	1.2753	0.126	0.203	0.305	0.422	0.555

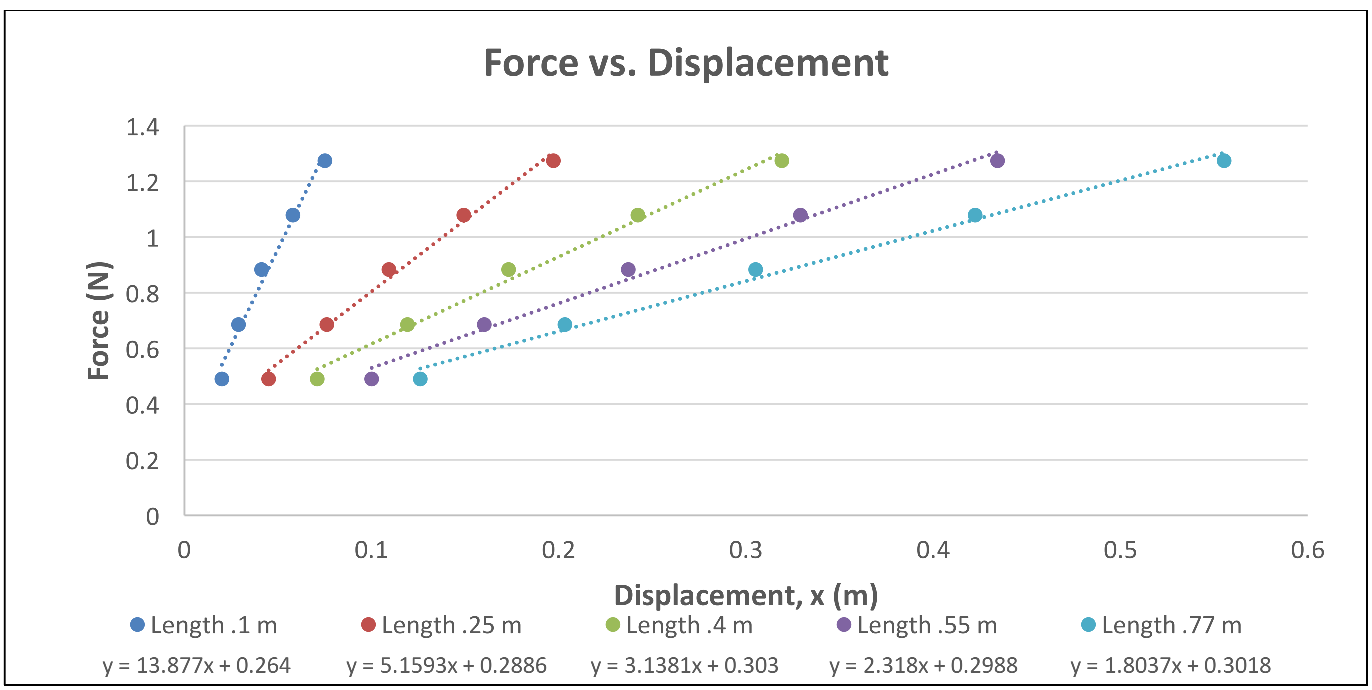

Figure 1: Force vs. Displacement. The linear relationship between the displacement and the gravitational force of a hanging mass for each length of bungee tested. Slope is the spring constant (k) for each respective length of bungee.

Table 2: Spring constants *k* for each length of bungee cord tested. Spring constant values were derived from the linear equations in Figure 1.

Length of Bungee Cord *x* (m, ± 0.01 m)	**Spring Constant *k*** (N/m)
0.10	13.877
0.25	5.1593
0.40	3.1381
0.55	2.3180
0.70	1.8037

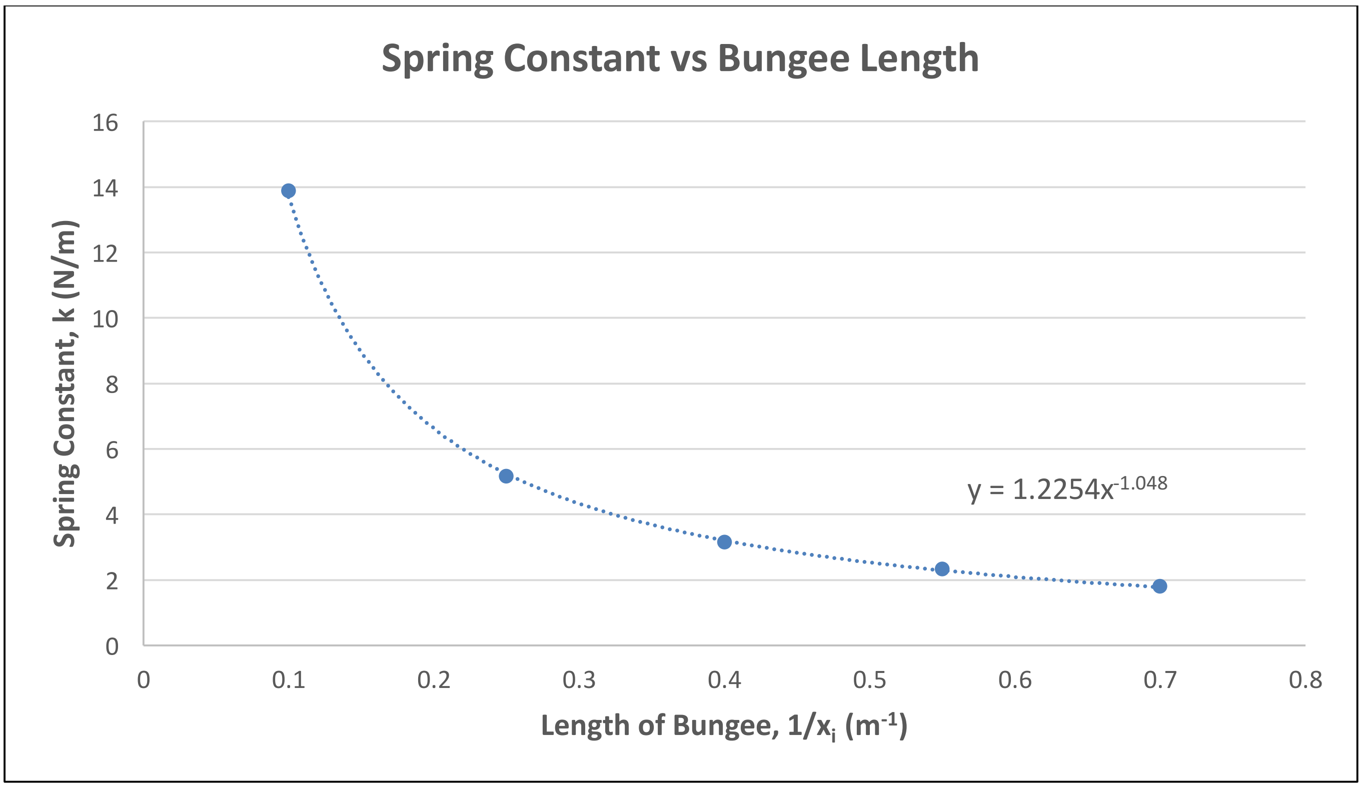

Figure 2: Spring constant vs. Bungee length. The power function for the relationship between the spring constant *k* and the initial bungee length x_i. Spring constant decreases at a rate of 1.048 as the length of the bungee increases.

In order to obtain a linear relationship between the two variables, spring constant *k* was plotted against the inverse of the initial bungee length $1/x_i$ (Figure 3).

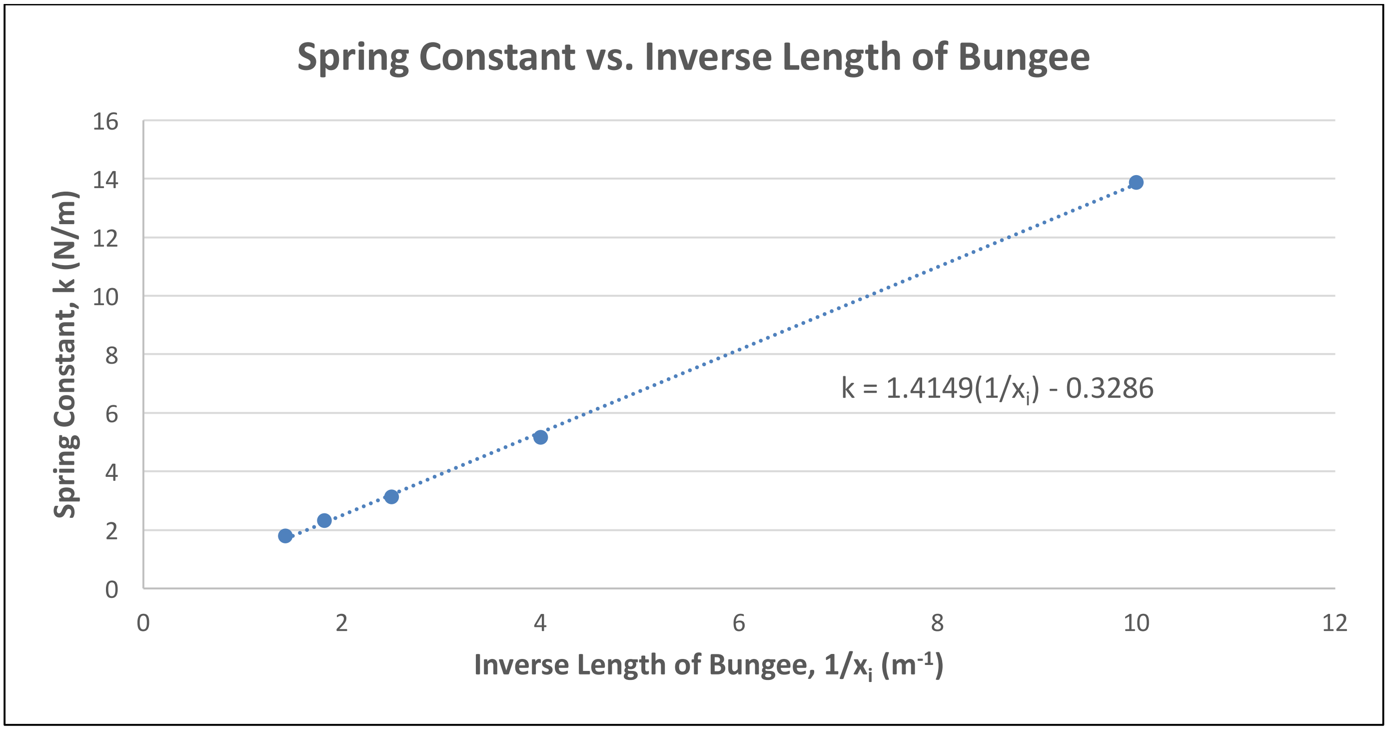

Figure 3: Spring Constant vs. Inverse Length of Bungee. The linear equation found to display the relationship between the spring constant, *k*, and the inverse of the initial length of bungee cord, $1/x_i$.

The equation $k = 1.4149(1/x_i) - 0.3286$ is the experimentally derived equation for the relationship between the spring constant and initial bungee length, where the constant *1.4149* is the proportionality constant that relates the spring constant to the initial bungee length. Linear regression analysis showed the standard error of the slope to be $\pm$ 0.0193.

DISCUSSION:

Experimental data showed the spring constant *k* is inversely related to the initial bungee length with the proportionality constant *1.4149 (*$\pm$*0.0193)*. Sources of uncertainty include the effect of friction on oscillatory behavior. Future studies could examine the oscillatory behavior of the bungee cord and determine the effect of friction on the bungee cord. Additional sources of uncertainty include the knots tied in order to attach 1) the cord to the apparatus and 2) the hanging mass to the cord.

The data supports our hypothesis that as the initial bungee length increases, the spring constant decreases. Experimentally, this means that for a bungee with a larger x_i, we expect to see a larger displacement *x* due to the net force acting on the cord. The experiment confirmed this.

CONCLUSION:

This experiment determined the spring constant *k* of a bungee cord as a function of the initial length of bungee cord. Knowing the spring constant enables us to predict the oscillatory behavior of the bungee cord.

Future studies could determine the actual oscillatory behavior and compare it to the predicted behavior. Additionally, we could address the assumption that the bungee cord behaves as an ideal spring and obeys Hooke's Law by analyzing the experimental oscillatory behavior.