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TITLE: Utilization of Hooke's Law to Determine the Spring Constant of a Portion of Bungee Cord

ABSTRACT:

The purpose of this initial experiment is to find a spring constant of a portion of the bungee cord given to our group and find an equation for the spring force the bungee applies to a hanging mass at equilibrium. This provides us with the tools necessary to predict the spring constant at varying lengths and eventually calculate the conditions necessary for a successful bungee jump in week 12. A 0.915m of un-stretched bungee cord was hung from a bar and various known masses were attached to the bottom end of the bungee. The change in elongation of the bungee cord was recorded for each mass and the equation $\boldsymbol{F_{Spring} = 1.475x + 0.248}$ was formulated from regression analysis. This provided us with a spring constant of 1.475 ($\pm$ 0.111) N/m for a 0.915m portion of our bungee cord. We were unable to determine the acceptability of this experimental value due to time constraints, but proposed a method for its determination, as well as, plans for future experiments to determine the spring constant at varying lengths of the bungee cord.

INTRODUCTION:

The purpose of this initial experiment is to determine the spring constant of the bungee cord in question at 0.915m. By finding the spring constant for this system we can predict how the bungee cord will react in the future. Thus, with further experimentation we can later determine how the spring constant changes as length of the cord varies.

Relevant equation(s), identifying variables:

$\boldsymbol{F_{Spring} = -kx}$ **:** Where **k** = the spring constant at cord length 0.915m and **x** = the distance past the equilibrium the cord is stretched. This equation is Hooke's law.

$\boldsymbol{F_{Gravity} = mg}$ **:** Where **m** = the mass of the object in question and **g** = the acceleration of gravity.

$\boldsymbol{F_{total} = F_{Gravity} - F_{Spring}}$ **:** By adding the two forces together we can find the total force on the system. If the object in question (the hanging mass) is at rest the forces will add to zero.

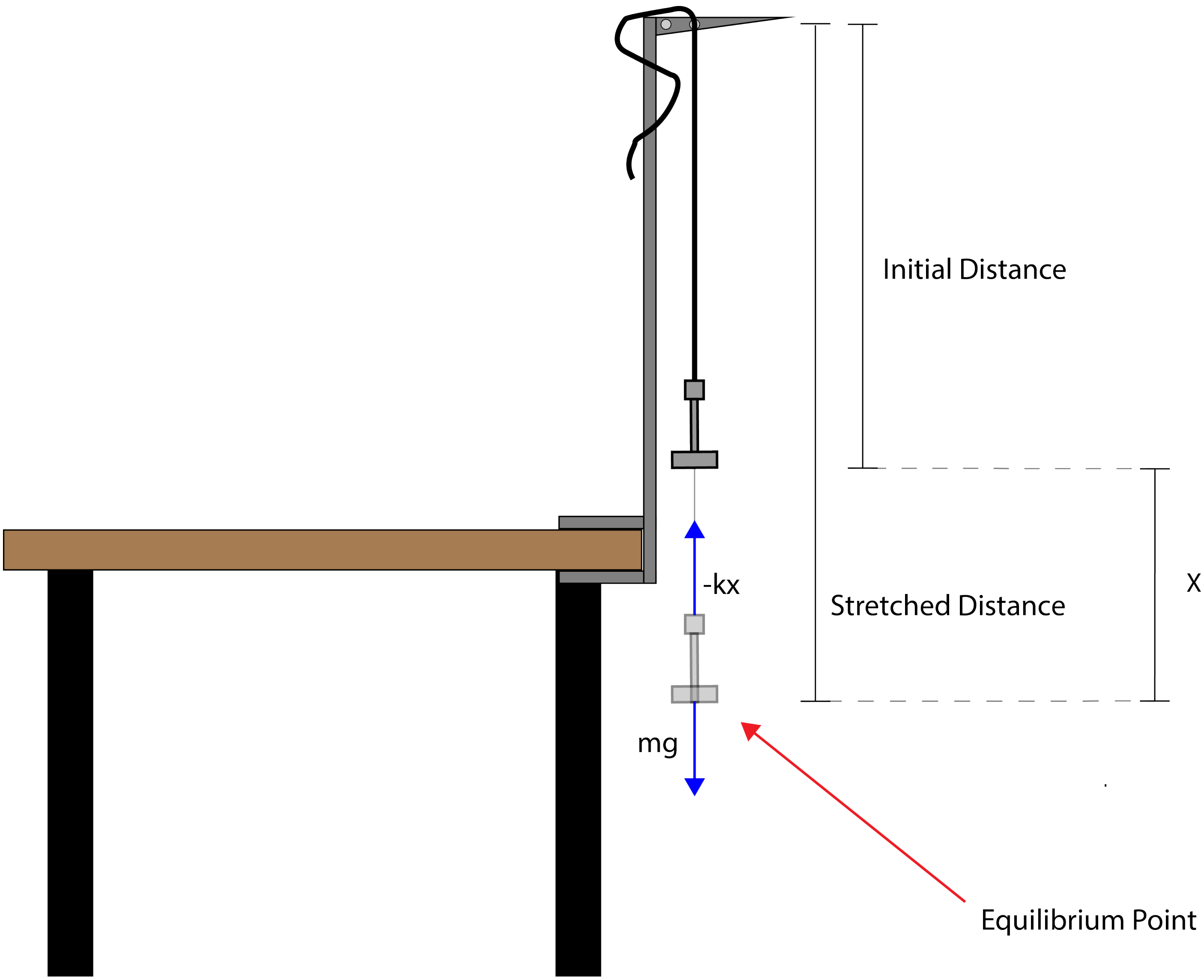
Theoretical Background:
Utilizing our Newtonian equations for force and Hooke's Law for the behavior of an object in a system containing a spring, an estimate for the spring constant should be able to be determined through experimentation. The spring constant will allow us to predict the amount of force the bungee will exert on an object when the cord is stretched to a certain length. If the object hanging from the spring is not at motion, we can set the weight and spring force equations equal to each other to find the spring constant at the length being tested, because net total force on the hanging mass will be zero. As the length of the bungee changes so will the spring constant due to the increased distance for the cord to elongate. The bungee cord will also not behave like a perfect spring, but will exhibit hysteresis as different masses are hung from the bungee cord. This will cause the bungee's spring constant to change over time. While we have no way to measure this hysteresis, we account for it experimentally (See Methods).

Hypothesis:
The distance of stretch by the bungee cord at length 0.915m will be modeled in a linear equation similar to Hooke's Law equation, which will have a slope of the bungee's spring constant. This will allow us to predict the equilibrium of the system for any given mass, as well as the force exerted by the bungee on the hanging mass.

METHODS:

By finding the distance of stretch by the bungee cord at a certain starting length under the force of varying hanging weights, we will be able to determine an equation and a spring constant that can predict the elongation of the bungee cord at any certain length with any hanging mass by applying Newton's laws and Hooke's law. It is predicted that the bungee cord will demonstrate hysteresis, or a change in elastic behavior over time as weights are consecutively hung from it. We accounted for this change in elastic behavior by measuring the distance of elongation in two separate rounds and then averaging the data to produce a more accurate estimate of the bungee's spring constant.

Diagram:



Procedure:

- Knots that formed loops were tied at the end of the bungee and at 0.915m from the end.
- The bungee was hung from a bar from the knot at .915m and a weight was hung from the free end.
- Distance from center of the bar to bottom of hanging mass was recorded
- Hanging mass was then changed in increments of 0.05kg and distances from center of the bar to bottom of hanging mass were measured and recorded.
- This process is repeated twice to account for the hysteresis of the bungee cord.
- Recorded data of length of bungee at each weight from the "stretched" trial and the "non- stretched" trial was averaged together and then regressed (weight vs. distance) using excel to find the equation for the spring constant of the spring.

RESULTS:

The collected data, weight of hanging mass and length of bungee at each specific weight was analyzed by taking the difference in length at each weight from the first length recorded to find how the length of the bungee changed with respect to the weight of the hanging mass. This was the basis for the regression analysis used in this section to find the equation for the spring constant at this specific length of bungee.

Figure 1: Change in Length of Bungee with respect to Weight of Hanging mass. The initial length of the bungee cord was taken with a hanging mass of .05003 N. The change in length of bungee at each equilibrium point for each hanging mass is represented in the right hand column. Uncertainty in weight of hanging mass stems from the raw uncertainty of the weights being used in the experiment and the uncertainty for length is the raw uncertainty.

Weight of Hanging Mass (N) (±0.00001 N)	Average Distance from Initial Length (m) (±0.005 m)
0.05003	0.000
0.54053	0.188
0.63863	0.239
0.73673	0.294
0.83483	0.356
0.93293	0.387
1.03103	0.496
1.12913	0.579
1.22723	0.676
1.32533	0.762
1.42343	0.866

Figure 2: Spring Force with respect to Average Distance from Initial Length. The slope, 1.475 (± 0.111) N/m, is the spring constant of the system. Spring Force is on the Y – axis because it counters the weight of the mass, thus allowing the system to rest at equilibrium.

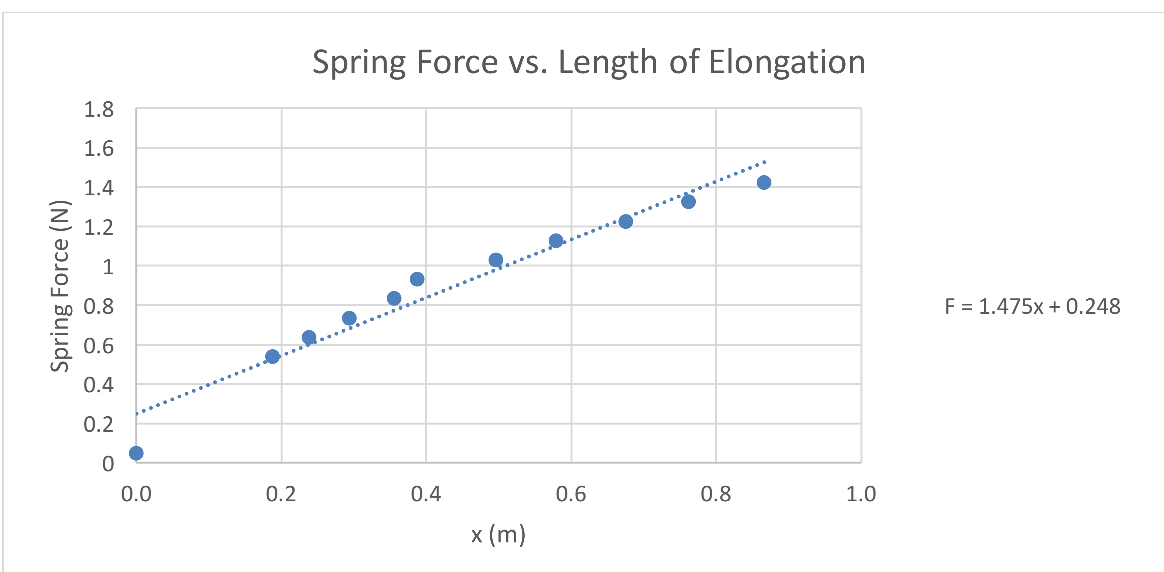


Figure 3: Regression Analysis table. The results of the regression analysis show the Spring Constant to be 1.475 ($\pm$ 0.111) N/m and the Intercept for the Spring Force equation to be 0.248 ($\pm$ 0.056) N. This gives us the equation of the Spring force to be $\boldsymbol{F_{Spring} = 1.475x + 0.248,}$ where x is the change in length of the bungee cord from the original measure.

Value	Coefficients	Standard Error
Intercept (N)	0.248	0.056
Spring Constant (N/m)	1.475	0.111

The values from the regression analysis provides us with the following uncertainty values and percentages. Total uncertainty and Total Percent Uncertainty of the force equation is calculated through the quadratic sum method of uncertainty propagation.

- Uncertainty for Slope (Spring Constant) = $\pm$ 0.111 N/m
- Percentage Uncertainty for Slope (Spring Constant) = 8%
- Uncertainty for Y-intercept of Spring Force Equation = $\pm$ 0.056 N
- Percentage Uncertainty for Y- Intercept of Force Equation = 22%
- Total uncertainty of the Force equation = $\pm$ 0.124 N
- Total Percent Uncertainty of Force Equation = 23%

Experimental Value of Interest.
The experimental value of interest in this experiment is the spring constant or the slope of the spring force equation of the bungee cord at un-stretched length of 0.915m. The spring constant was determined to be 1.475 ($\pm$ 0.111) N/m. This value represents how the force of the bungee on the hanging mass changes as bungee is stretched.

Results Summary
By measuring the distance the bungee cord stretched at equilibrium by attaching different weighted masses we were able to determine the spring constant of the system with un-stretched bungee cord length 0.915m. Utilizing regression analytics we determined that the equation, $\boldsymbol{F_{Spring} = 1.475x + 0.248,}$ represents the Spring force in the system where 1.475 ($\pm$ 0.111) is the spring constant.

DISCUSSION:

Due to the fact that there is no accepted value for the spring constant of the bungee cord used in our system comparing the percent uncertainty against the percent error to determine the acceptability of our experimental spring constant cannot be done. Alternatively, a test of the experimental spring constant's viability as predictor the equilibrium point must be done. This test would consist of choosing a weight and predicting the equilibrium point of the system from our equation, $\boldsymbol{F_{Spring} = 1.475x + 0.248,}$ then finding the actual equilibrium point of our system. If the measured equilibrium point lies within our calculated total percent uncertainty for our spring force equation of 23% we will be able to state that our experimental spring constant is acceptable. The 0.248 in our equation is a value produced by the regression analysis, that allows our linear equation to best fit the data, and is solely an experimental value. Sources for uncertainty in our experiment are unfortunately abundant, with the majority lying in the bungee cord itself. The hysteresis of the bungee cord's material poses difficulties for accurate measurement as the elasticity of the cord seemed to increase as we conducted our experiment. This would lead to varying spring constants for the bungee cord after continued use of the cord. We attempted to account for hysteresis by conducting a second round of measurements on the bungee cord after our first data collection. This poses difficulties for further experimentation, however, as there is no way for us to accurately determine the elasticity of the bungee before running future experiments. Another source of uncertainty will stem from the size of the knots and loops. We attempt to minimize this by making the loops as small as possible to maintain consistency throughout experimentation. The bungee cord also seemed to elongate as we were taking measurements of the equilibrium point of the system. This made accurate measurement difficult. With these sources of uncertainty taken into consideration we will continue experimentation on the bungee cord to determine how the spring constant of the cord changes as the length of the cord does. After determining whether our experimental force equation confirms our hypothesis or not through conducting the for mentioned acceptability test we will conduct our same experiment with varying lengths of the bungee to find an equation for the relationship between cord length and the system's spring constant. This will then allow us to determine the forces at work on a mass released from non-equilibrium height attached to the bungee cord and calculate the appropriate length of cord to use for our final bungee test.

CONCLUSION:

In this experiment we measured the distance of elongation at equilibrium of a 0.915m section of our bungee cord as varying masses were hung from it. This experiment and its results served as a stepping stone for further insight into the behavior of our bungee cord by producing a Hooke's Law equation for the spring force of the bungee cord, $\boldsymbol{F_{Spring} = 1.475x + 0.248}$, at 0.915m of cord. While confirmation of our hypothesis requires one additional test for acceptability, as mentioned, the basis of the experiment provides us with a method for determining the spring constant for various lengths of bungee cord. This will ultimately allow us to calculate and predict the conditions necessary for a successful bungee jump in week 12.