

## Investigation of the spring constant of a bungee cord

### **ABSTRACT:**

There are two main conditions for our bungee jumping egg: the egg must not hit the ground and the force of the deceleration of the egg must not be too great to cause damage. This experiment aims to estimate the displacement of the egg at different bungee cord lengths. Using Hooke's law to find the spring constant at different lengths, an elastic cord tied around a metal bar was measured relaxed at five different heights and its displacement was measured after various masses were added to the cord. A plot of the various masses versus displacement at different cord lengths yielded different spring constants. The different lengths versus the spring constant was then plotted to derive the equation,  $y = 10.913x^{0.936}$ , suggesting a decrease in spring constant as the length of the cord increases. This may be used to predict the spring constant of our actual bungee cord length needed for modeling the force of bungee cord in the following lab.

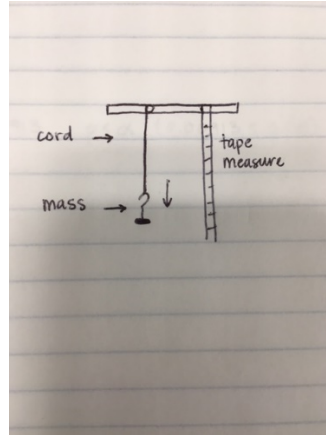
### **INTRODUCTION:**

The purpose of this experiment was to determine the properties of our elastic cord to ensure that an egg can be safely dropped from a predetermined height without breaking. In order to do this, the Hooke's Law ( $F = k \cdot \Delta x$ ) was used to find the spring constant of the elastic cord. There are two forces on the egg when the bungee cord begins to stretch: the force of gravity ( $m \cdot g$ ) and the force of the spring going in the opposite direction, which is  $F = k \cdot \Delta x$ . Since acceleration is constant,  $k \cdot \Delta x = m \cdot g$ , therefore we can assume a linear relationship between  $\Delta x$  and  $m \cdot g$ , whose relationship when graphed yields the slope  $k$ . Since we will be manipulating the length of the bungee cord to optimize a safe free-fall, we determined a  $k$  at different lengths by plotting the relationship between  $\Delta x$  and  $m \cdot g$  at different masses for each length. Next we plotted derived  $k$  values for their respective lengths to derive our final equation. We hypothesized that  $k$  would decrease as length ( $L$ ) increased.

### **METHODS:**

The point of this method is to measure the displacement of the cord in order to find the spring constant.

- An elastic cord was tied around a screw on a level bar, with a knot small enough to hold the hook of the mass.
- Attaching a tape measure next to the elastic cord, we measured the unstretched cord to 0.84 m.
- We then added masses 0.9 kg, 1.1 kg, 1.3 kg, 1.5 kg, and 1.7 kg to the hook of the elastic cord and measured displacement from 0.84 m at each mass
- We repeated this procedure for unstretched cord lengths of 0.64 m, 0.42 m, 0.22 m and 0.12 m.
- We then plotted the relationship between displacement at different weights ( $m \cdot g$ ) at different lengths
- The slope of the derived equation from each graph was the  $k$  constant for different lengths
- Finally, we plotted the relationship between the  $k$  constant and the different lengths to derive the equation  $y = 10.913x^{0.936}$



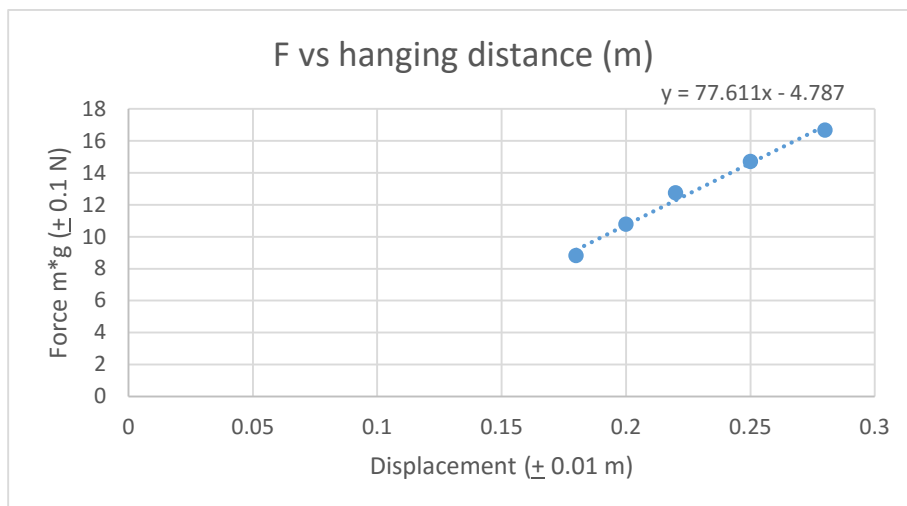
**Diagram of our experiment.** Various masses were added to the “mass” to increase the weight, therefore stretching our bungee cord. The cord was measured against the tape measure next to it.

**RESULTS:**

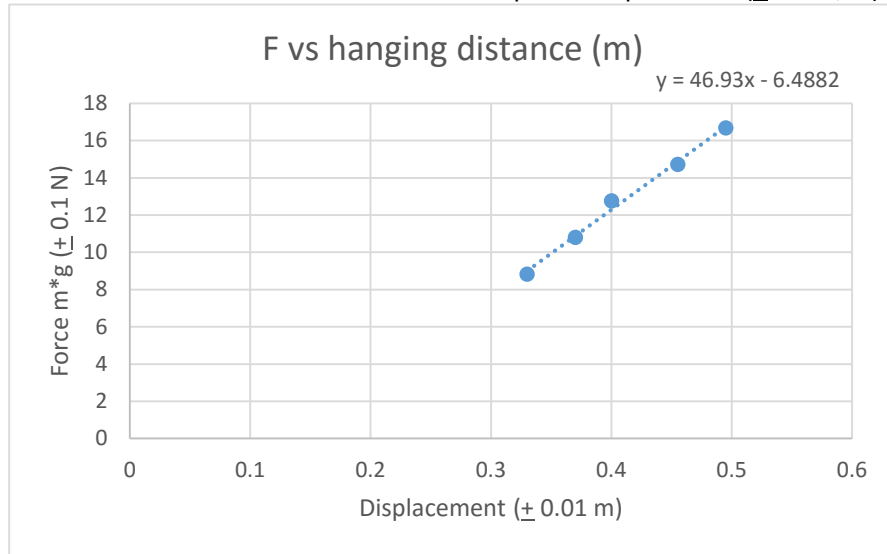
Our tables show  $\Delta x$  versus  $m \cdot g$  for different unstretched cord lengths. This relationship is graphed to derive the slope or k value. Finally, the relationship between these k values and unstretched cord lengths is depicted in Fig. 2.

Relaxed Height = 0.12 m		Relaxed Height = 0.22 m		Relaxed Height = 0.42 m		Relaxed Height = 0.64 m		Relaxed Height = 0.84 m	
Hang. Dist. ( $\pm 0.01m$ )	$m \cdot g (\pm 0.1 N)$	Hang. Dist. ( $\pm 0.01m$ )	$m \cdot g (\pm 0.1 N)$	Hang. Dist. ( $\pm 0.01m$ )	$m \cdot g (\pm 0.1 N)$	Hang. Dist. ( $\pm 0.01m$ )	$m \cdot g (\pm 0.1 N)$	Hang. Dist. ( $\pm 0.01m$ )	$m \cdot g (\pm 0.1 N)$
0.18	8.8	0.33	8.8	0.61	8.8	0.99	8.8	1.22	8.8
0.20	10.8	0.37	10.8	0.68	10.8	1.06	10.8	1.35	10.8
0.22	12.8	0.40	12.8	0.77	12.8	1.18	12.8	1.50	12.8
0.25	14.7	0.46	14.7	0.85	14.7	1.32	14.7	1.67	14.7

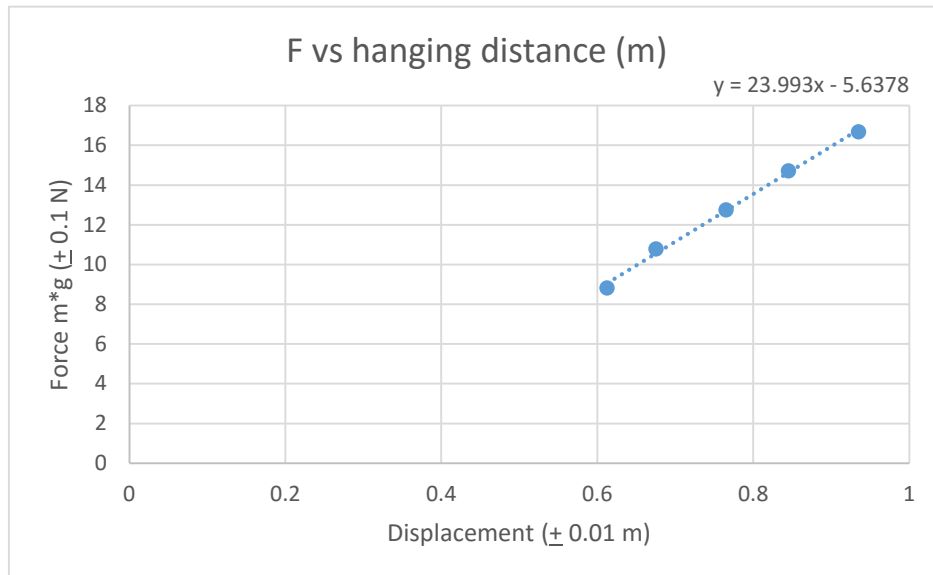
**Table 1. Stretched distances at different masses for varying unstretched cord lengths.** The total length of the cord was subtracted from starting unstretched length at different masses. This displacement is compared to weight derived by multiplying gravitational acceleration to mass.



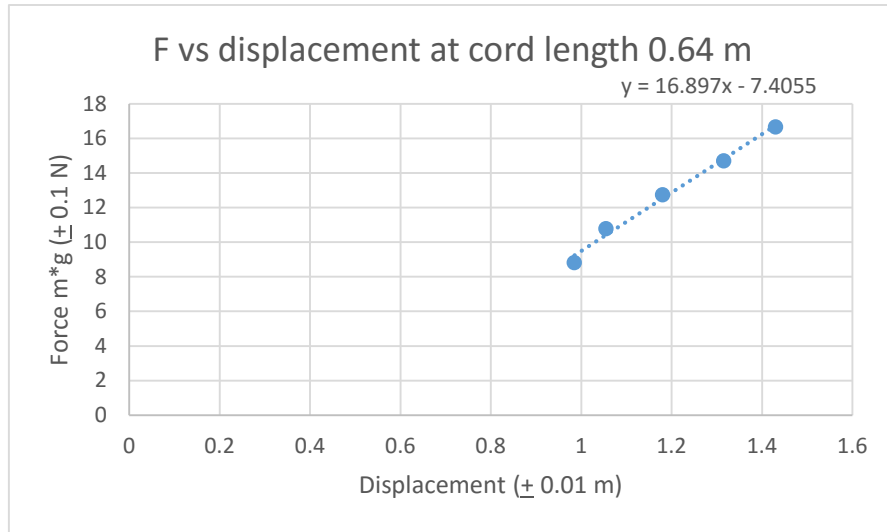
**Fig. 1a.** The force ( $m \cdot g$ ) vs its corresponding stretch distances at initial unstretched cord length of 0.12 m. k value is determined from the slope and equals  $77.6 (\pm 4.7 \text{ N/m})$ .



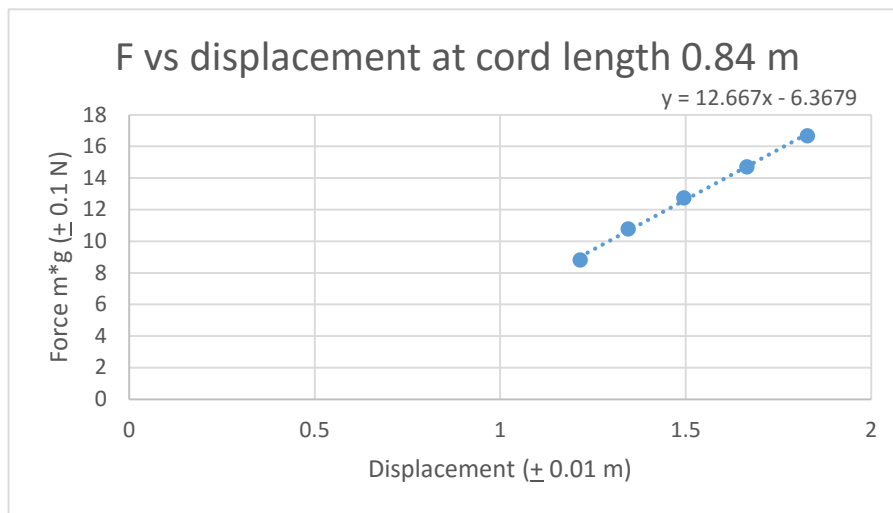
**Fig. 1b.** The force ( $m \cdot g$ ) vs its corresponding stretch distances at initial unstretched cord length of 0.22 m. k value is determined from the slope and equals  $46.9 (\pm 2.3 \text{ N/m})$ .



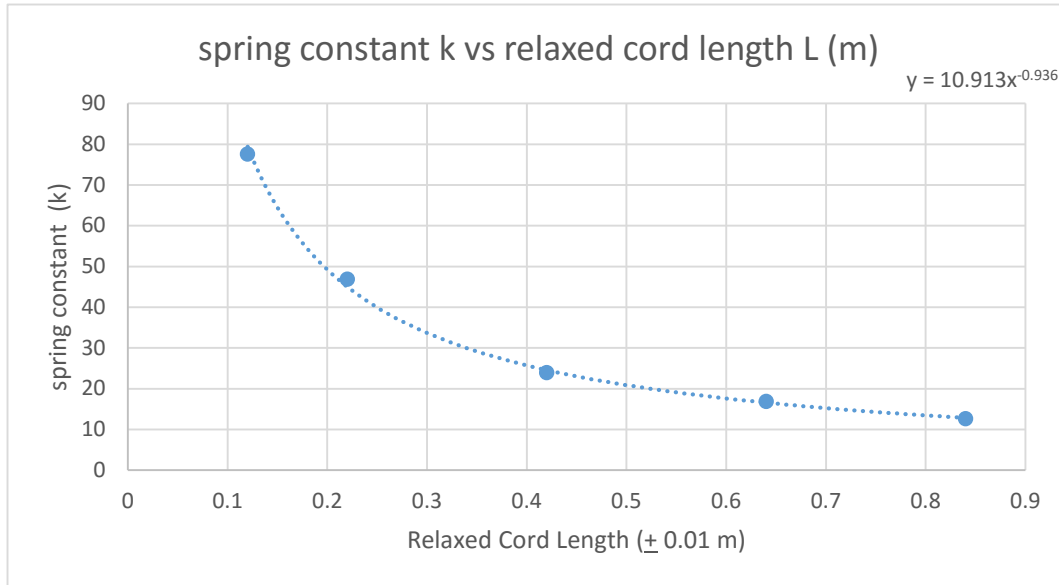
**Fig. 1c.** The force ( $m \cdot g$ ) vs its corresponding stretch distances at initial unstretched cord length of 0.42 m. k value is determined from the slope and equals  $23.9 (\pm 0.8 \text{ N/m})$ .



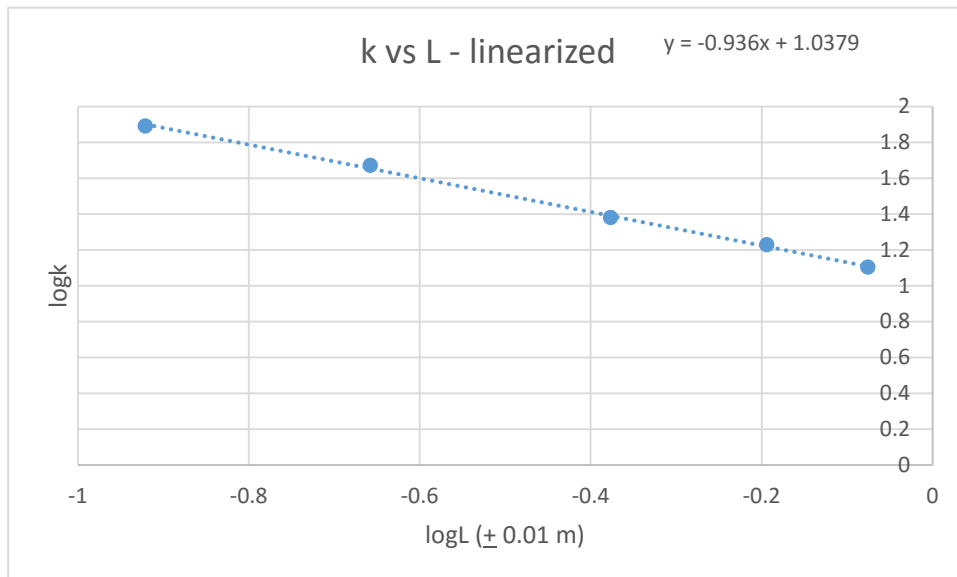
**Fig. 1d.** The force ( $m \cdot g$ ) vs its corresponding stretch distances at initial unstretched cord length of 0.64 m.  $k$  value is determined from the slope and equals  $16.9 (\pm 1.0 \text{ N/m})$ .



**Fig. 1e.** The force ( $m \cdot g$ ) vs its corresponding stretch distances at initial unstretched cord length of 0.84 m.  $k$  value is determined from the slope and equals  $12.7 (\pm 0.4 \text{ N/m})$ .



**Fig 2.** The spring constants versus the relaxed lengths plotted to derive the equation  $y = 10.913x^{-0.936}$



**Fig. 3.** Linearized spring constant versus relaxed cord length by taking the log of k and L. The slope of the linearized graph is  $-0.936 (\pm 0.02 \text{ N})$ , the intercept is  $1.0379 (\pm 0.04 \text{ N/m})$

Our derived equation used to predict k at our bungee cord length is extracted from k vs L to get  $y = 10.913x^{-0.936}$ . Alternatively, the linearized version,  $y = -0.936x + 1.0379$  with uncertainties derived from regression analysis may be used.

**DISCUSSION:**

After extracting our k values for respective L values, we were able to determine that the relationship between the two variables does in fact result in lower k values as L increases. However, in order to determine whether our equation for this relationship was acceptable, we had to first linearized the equation to get uncertainties. Since we did not have accepted k vs L values for our cord, we were unable to compare the accuracy of our results to percent error. However, conceptually the uncertainty of our linearized k vs L graph ( $\pm 0.02$  N) is about the same as our smallest known uncertainty, which is displacement,  $\pm 0.01$  m. Therefore, we can accept that our equation is accurate. Reasons for uncertainty may come from extra oscillating lateral movement and overstretching due to testing of the heaviest mass at the beginning of the trials.

**CONCLUSION:**

The point of this experiment was to help estimate the spring constant at different lengths. Since we will be using the CWE theorem in the next lab to make sure the force is not too great on the egg, we will be adjusting k and x values to make sure that they do not exceed  $3(m \cdot g \cdot h)$ . Therefore, we can refer to our equation of  $y = 10.913x^{0.936}$  to select appropriate bungee cord lengths at different k values.