Bungee II-- CWE Theorem

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Characterizing the Behavior of an Elastic Cord Using the Conservation of Work and Energy Theorem

Abstract:

The purpose of this experiment was to design a bungee model using kinetic and potential energy derived from the conservation of work and energy (CWE) theorem. In an experiment similar to that in week 6, five different masses were all dropped at a constant cord length of 0.5 m. The changes in length of the cord for each mass were then recorded and plotted against potential energy (mass*gravity*height) in order to quantify values for the work on the system. We found a that the power equation $PE = 0.9453\Delta x^{2.2897}$ was a better fit for an elastic cord with a free falling mass on it. As the elastic cord was not an oscillating spring, we found that the the k constant characterized by the formula $k=1.1351x^{-0.891}$ does not adequately describe the behavior of the cord. Ultimately, the model for the bungee jump was determined to be $h = L + (mgh(L - 0.891)/\ (1.0054))^{1/2.2897}$ where m,g and h are given. However, we further tested out this model for a desired height of 1.50 m and the mass travelled about a third of the distance than expected. Further manipulation of this equation is needed to elucidate a more fitting model for the bungee jump in 3 weeks.

Introduction:

The ultimate goal of this experiment was to design a quality bungee cord experience that allows for an egg to come as close to the ground as possible without incurring any damage. In week 6, the behavior of an elastic cord was characterized by Hooke's Law (F_{spring}= kL) where k was the spring constant and the length of the bungee was L. The model using Hooke's law did well in characterizing the force of a given mass as it was falling (m*g). However, in week 6, we did not take into account that the egg would be experiencing free-fall prior to a mass-on-spring effect. The CWE theorem describes conservative systems similar to the elastic cord system in our experiment. It states that the total energy (both the potential and kinetic energies) before and after the drop are conserved. At the top of the system, gravitational potential energy is present and at the elastic potential energy is equal to zero. This relationship is modelled below where m is the mass of the falling object (in g), g is the gravitational acceleration constant 9.81 m/s^2, h is the height of the jump (in m), k is the spring constant (unitless) and x (in m) is the amount of elongation that the elastic cord incurs.

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$$mgh = 1/2\ kx^2$$

By calculating the energy (as opposed to force) exerted on the bungee for different masses, the goal is to derive an equation (or equations) that will allow for an accurate prediction of the length that the bungee will stretch accounting for the complex nuances of the systems described above.

Methods:

This report provides an easier method in analyzing the behavior of a bungee using CWE theorem. A range of 5 different masses from 100-170 g were hung at 0.5 m (x initial) on a bungee cord and the x final was determined using slow motion videos from an iPad.

The potential energy (mgh) was graphed in relation to the change in distance, and the slope of the line characterized the energy (PE and KE) of the whole system. It was assumed that energy was conserved throughout the system. This equation was then used to improve the representation of the k constant for the bungee drop.

Experiment 1:

STEP 1: The bungee was tied securely to hanging apparatus

STEP 2: A loop was tied in the bungee at 0.5 m (50 cm) and a securing knot was also tied under the loop to strengthen the hold on the mass

STEP 3: 5 different hanging masses were placed on the bungee: 0.1 g, 0.11 g, 0.07g, 0.08 g and 0.09 g.

STEP 4: The bottom of the mass was determined to be the measuring point for the duration experiment for consistency.

STEP 5: One member dropped each mass and after a “3,2,1 GO” countdown as the other recorded the length of the fall using an iPad.

STEP 6: The drop was played back on the iPad in slow motion and we used a ruler to measure where the mass was at its lowest point. The difference between this final x value and the initial x value were plotted in excel

STEP 7: Graphed mgh v. Δx and performed a regression analysis

Figure 1: Diagram of experimental set-up where x0 is the distance of object free-fall, Δx is the total displacement of the bungee after a mass was dropped, m represents the mass in g and h is the height, which represents the total distance from the top of the system to the ground in meter. V is the velocity as the mass approached the ground m/s.

Results:

5 masses between 100 grams and 170 grams were attached to an elastic string mounted on a metal apparatus. The weight was tied onto the elastic cord at 0.5 m. Displacement of the string was measured for each trial and compared to the potential energy of the system prior to the drop. Raw data for trials, regression analysis and results are shown below:

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Table 1

Δx (m) (± 0.01 m)	mgh (J)
1.34	1.80504
1.365	2.012522
1.04	1.057518
1.201	1.334945
1.21	1.509759

Table 1: Raw data for displacement and potential energy for 5 trials.

Figure 2

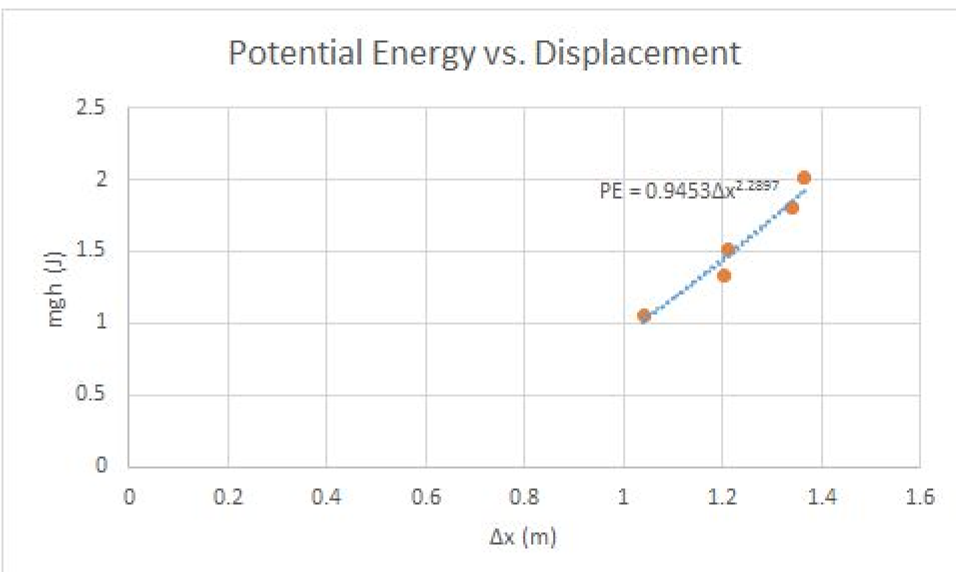

Figure 2: Relationship between potential energy and cord displacement for varying masses.

Table 2:

	Coefficients	Standard Error
Y-Intercept	-1.93433	0.468987
Slope	2.825118	0.37923

Table 2: Y-intercept and slope of the linearized graphs (Figure 2), in addition to the standard error for each of the values obtained through regression analysis. Uncertainties were estimated based on tools and standard deviation in regression analysis.

Discussion:

There was some success in deriving an equation for elastic behavior in terms of potential energy and Δx. However, our group found that the k constant found in week 6 did not properly account for the complexities of the system. Because of this, it was concluded that the constant k, which is meant to quantify the behavior of springs, cannot appropriately be applied to springs. Still, this model using Hooke's law addressed the question of force,which was just the weight of the falling object. This means that the model can be used for the sake of measuring force for the bungee jump.

The five different data points that plot displacement versus potential energy clearly illustrate a power relationship for the cord's behavior. While the model derived from the equation using CWE theorem considered the lag time between free-fall and elastic effects, it did not accurately predict the length at which a weight would fall when it was tied at 1.50 m. It was The actual distance was about a third of what we measured. This could be due to a number of reasons looking back at the experimental design. Our group depended on the recordings that we took on our iPad to measure the Δx. We could not get an accurate measurement to the nearest tenth of a meter which resulted in the eye-balling of distances to the nearest meter using a ruler on that we put on the screen. We did not account much for any stretching and hysteresis that could have affected the cord as well. In future experiments, we can do more trials for each mass and measure any changes in cord length caused by hysteresis. These sources of uncertainty could have had a drastic affect on the model and because of this more tests are and manipulations of the equation are needed to accurately model and predict the behavior of the bungee cord before the egg drop in week 12.

Conclusion:

The formula $PE = 0.9453\Delta x^{2.2897}$, derived through CWE theorem, provides a model for the behavior of our specific elastic band. In the upcoming weeks, this model will be implemented in the design of an elastic cord system that will be expected to get a raw egg as close to the ground as possible without any damage. Knowing that elastic materials like rubber cords have the tendency to recover when they are released with weight on them, and that the object free-falls before the elastic cord has any effect, it was determined that the model $h = L + (mgh(L - 0.891)/(1.0054))^{1/2.2897}$ was not sufficient in describing the behavior of the cord as the expected length was significantly more than the observed length when the mass fell a predetermined distance h (1.50 m). Further manipulation of this model is needed for better accuracy. Possibly we can start by multiplying the equation by a factor of 1/3 as the expected length was three times larger than the observed length.