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Bungee Jump 1 Lab Report

Abstract

The goal of this lab was to find the spring constant (k) of a specific cord. Given that we expected the cord to behave like a spring, we used Hooke's law ($F = kx$) to solve for k. To find k, we varied the force on the system by changing the hanging mass and measuring the resulting displacement. Then, taking the measurements and comparing weight to displacement on a graph, we were able to solve for k. However, the specific value of k varies with the length of the cord. Therefore, we repeated the experiment four times, changing (x_L), to find different values of k. Then graphing k vs. x_L, and taking the linearized form of the graph, we discovered that $k = 0.85{x_L}^{-1} + 0.14 N/m$. Thus, given the length of the cord, we can find the spring constant (k) of a specific cord.

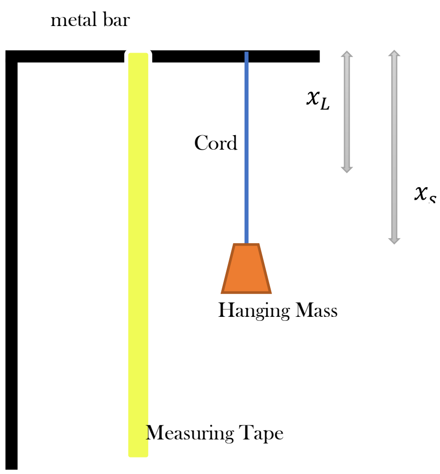
Introduction

- Primary question: What is the spring constant k of a specific cord?
- Secondary question: How does the spring constant k vary with the length of the cord?
- Set up: the system consisted of a metal bar at a set height from which we tied one end of the cord. Alongside the cord was a vertical measuring tape used to find lengths of the cord. Then, attaching a hanging mass to the other end of the cord, we measured displacement by taking the difference of the static stretched length from the original, un-stretched length.
- Background: We assumed the cord behaved like a spring in that its restorative force (F) equaled the displacement (x) times a spring constant (k). Because the system included a vertical spring-like cord connected to a mass, the only force was that of gravity (mg). To measure displacement x, we subtracted the static stretched length (x_s) (when connected to a mass) from the original un-stretched length (x_L) (without the mass).
- Hypothesis: The spring constant k will vary with the length of the spring, so in order to find k, we need to first find how k varies with x_L.

Methods

By changing the hanging mass and measuring the displacement of the cord, we solved for k for a specific x_L. Secondly, by altering x_L, we were able to compare how values of k vary with different lengths of the cord.

Sketch 1. System of Experiment. The sketch shows the set-up of the system. The static stretched length (x_s) minus the un-stretched length (x_L, the original length of the cord) gives the displacement x used in the equation $F = kx$.



Collecting Data

1. First, we tied two loops in the cord, attaching one to the bar and letting the other hang.
2. Then, we measured the original length of the cord (x_L).
3. To vary the force (mg), we added weights to the second loop in increments of 0.020 or 0.025kgs from 0.05kg to 0.25kg to the cord and measured the new length (x_s).
4. Finally, to find the displacement, we calculated $x_s - x_L$.

Steps 1-4 were repeated four times altering the between the two loops of the cord to later compare k to x_L.

Results

These results include the tables and graphs comparing displacement (x) to weight (mg), spring constant (k) to original length (x_L), and finally, the linearized graph comparing k to $x_L{}^{-1}$.

Table set 1. Trials 1-5. These tables include the mass, weight, and displacement measured in each trial. Weights were added in increments of 0.025 kg or 0.020 kg on a range from 0.050 kg to 0.250 kg. The later trials have less data points because a longer string reached the floor with fewer weights.

Trial 1. $x_L = 0.27\ (\pm 0.01)\ m$

Mass (kg)	Displacement (±.01m)	Weight (N)
0.050	0.08	0.49
0.075	0.12	0.74
0.100	0.18	0.98
0.125	0.25	1.23
0.150	0.33	1.47
0.175	0.42	1.72
0.200	0.50	1.96
0.225	0.57	2.21
0.250	0.65	2.45

Trial 2. $x_L = 0.36\ (\pm 0.01)\ m$

Mass (kg)	Displacement (±.01m)	Weight (N)
0.050	0.07	0.49
0.070	0.12	0.69
0.090	0.17	0.88
0.110	0.23	1.08
0.130	0.31	1.28
0.150	0.40	1.47
0.200	0.61	1.96
0.250	0.82	2.45

Trial 3. $x_L = 0.49\ (\pm 0.01)\ m$

Mass (kg)	Displacement (±.01m)	Weight (N)
0.050	0.10	0.49
0.075	0.18	0.74
0.100	0.28	0.98
0.125	0.42	1.23
0.150	0.55	1.47
0.175	0.71	1.72
0.200	0.86	1.96
0.225	1.00	2.21
0.250	1.14	2.45

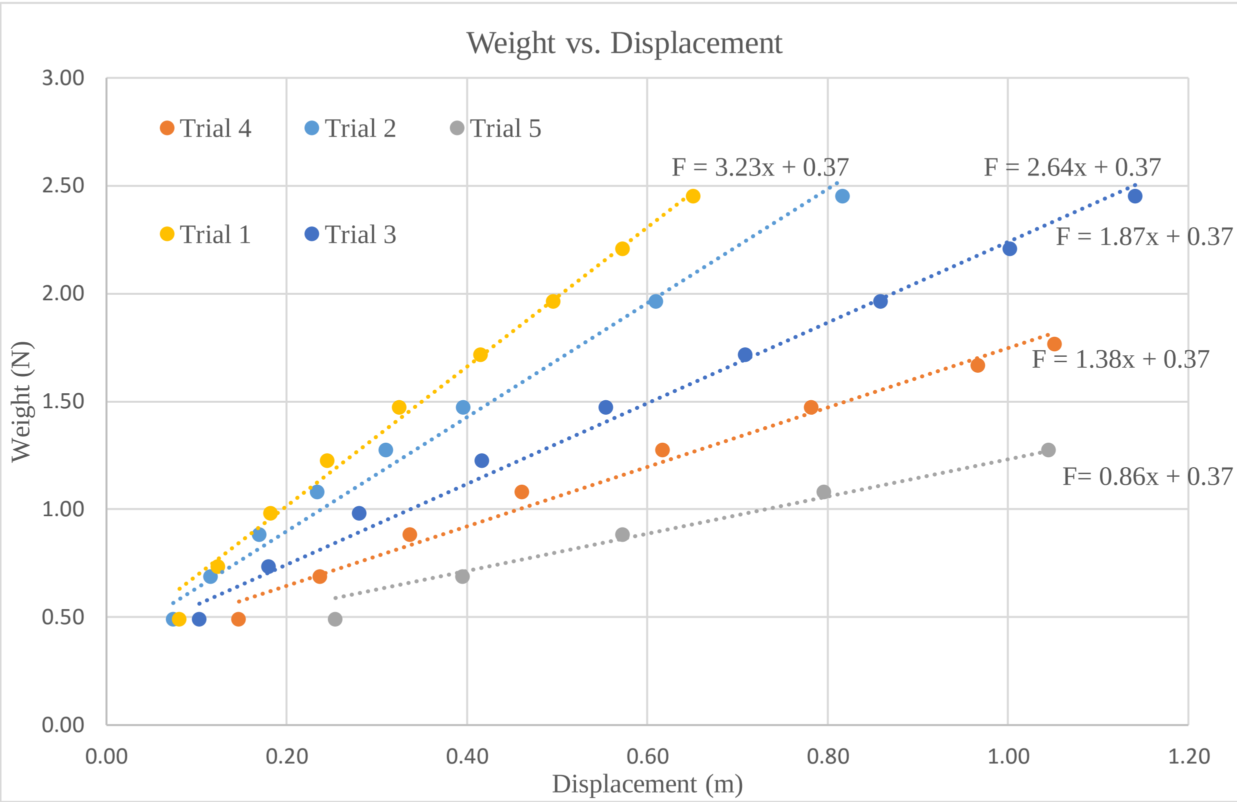
Trial 4. $x_L = 0.68\ (\pm 0.01)\ m$

Mass (kg)	Displacement (±.01m)	Weight (N)
0.050	0.15	0.49
0.070	0.24	0.69
0.090	0.34	0.88
0.110	0.46	1.08
0.130	0.62	1.28
0.150	0.78	1.47
0.170	0.97	1.67
0.180	1.05	1.77

Trial 5. $x_L = 1.16\ (\pm 0.01)\ m$

Mass (kg)	Displacement (±.01m)	Weight (N)
0.050	0.25	0.49
0.070	0.40	0.69
0.090	0.57	0.88
0.110	0.80	1.08
0.130	1.05	1.28

Chart 1. Weight vs. Displacement. This graph compares the weight of the hanging mass to the displacement given a consistent k. The slopes of the graphs give us the k value of that specific trial. Because x_L differs, there are differences in the slopes of the graphs. The common y-intercept was found by averaging the original intercepts.



Looking at the original graphs, we discovered that the experiment resembled the equation $F = kx + b$ where b acted as the y-intercept. In order to have better consistency when comparing the different k values in Graph 2, we decided to find a common y-intercept. We took the average of the original y-intercepts and got 0.37 ± 0.06 (N) (16%).

Table 2. Data from Graph 1. This table shows the original y-intercepts along with the final equations on graph 1.

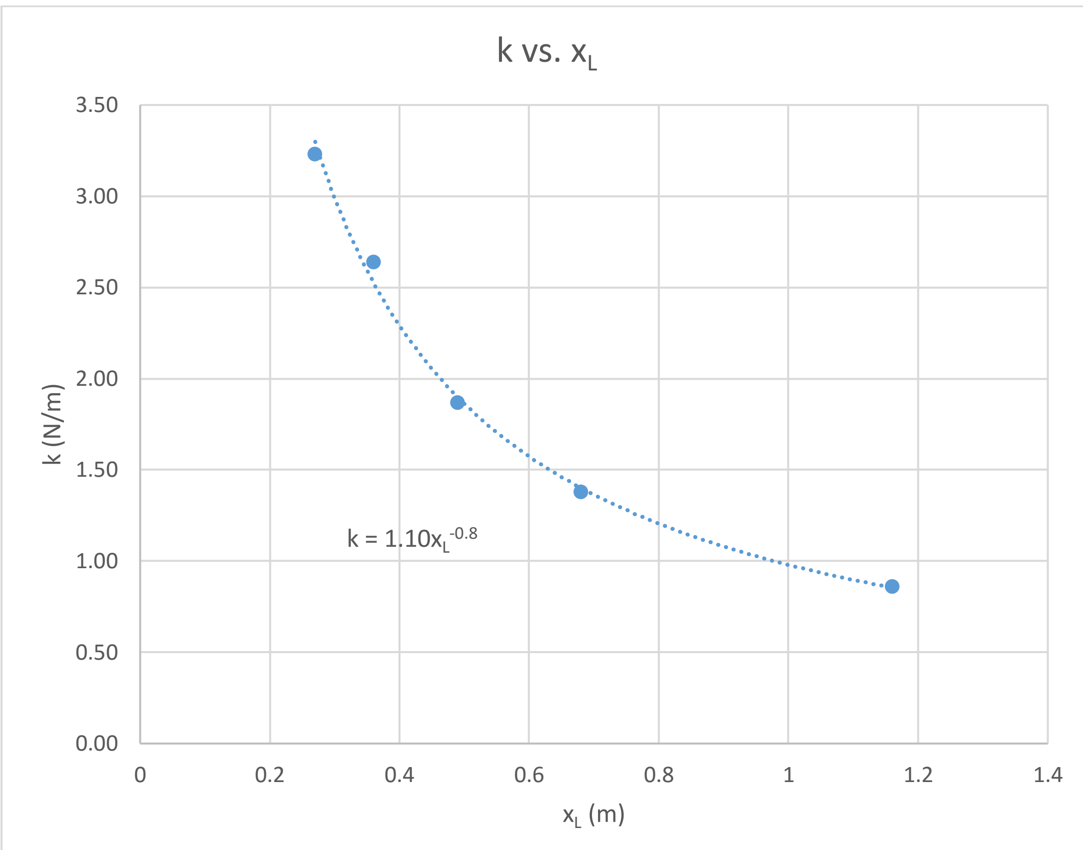
y-intercepts (N)	y_{avg} (N)	σ_t (N)	Equations	
0.28	0.37	0.06	Trial 1	F=3.23x + 0.37
0.38			Trial 2	F=2.44x + 0.37
0.42			Trial 3	F=1.87x + 0.37
0.42			Trial 4	F=1.38x + 0.37
0.34			Trial 5	F=0.86x + 0.37

Because our initial equation states $F_g = mg = kx$, then the slope of the equations gives us the k. value for that specific x_L.

Table 3. Comparing k to x_L. This table compares the k constants found in Graph 1 and Table 2 to the respective cord lengths (x_L). It also contains the inverse of x_L which is used in Graph 3.

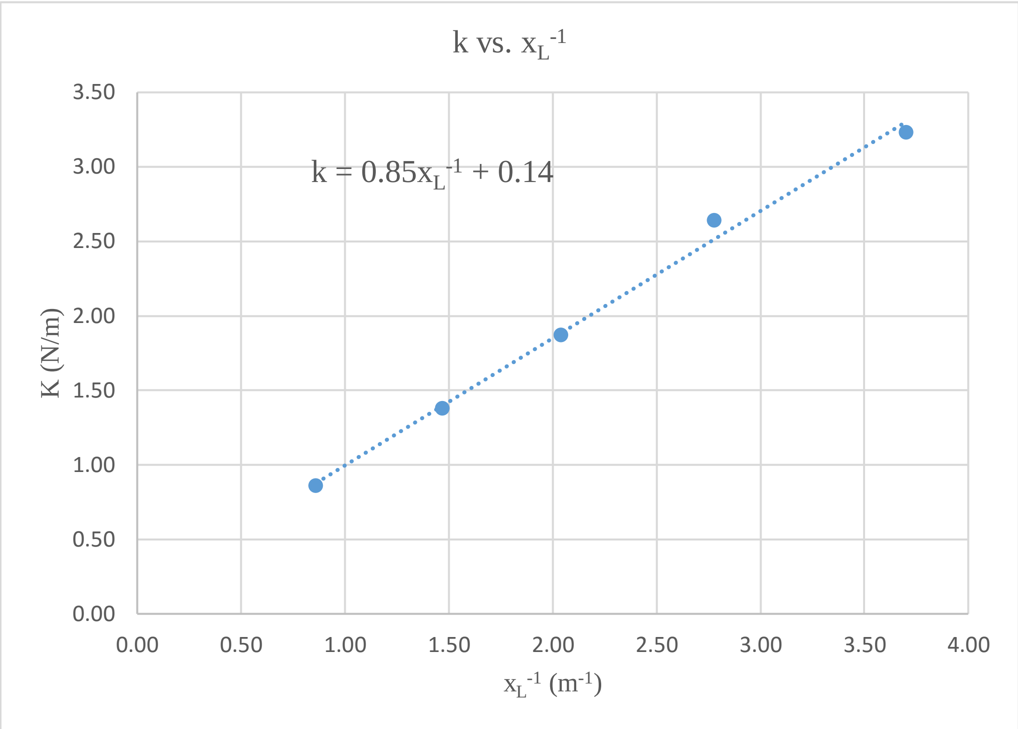
k (N/m)	x_L (m)	${x_L}^{-1}$ (m^{-1})
0.86	1.16	0.86
1.38	0.68	1.47
1.87	0.49	2.04
2.64	0.36	2.78
3.23	0.27	3.70

Graph 2. k vs x_L. This table directly compares the discovered k values to the measured x_L distances in an attempt to find the relationship between the length of the cord and its spring constant.



Equation of Graph 2. $k = 1.10{x_L}^{-0.8}$. Because the cord length appears to be roughly inversely related to the spring constant, we will compare k to the inverse of x_L for the linearized form. these values of ${x_L}^{-1}$ can be found in Table 3.

Graph 3. k vs $x_L{}^{-1}$. This graph is the linearized version of Graph 2. It demonstrates the relationship between the spring constant and the inverse of the cord length.



Equation for Graph 3. $k = 0.85x_L{}^{-1} + 0.14$. With calculated uncertainty, the equation of Graph 3 is $k = 0.85\ (\pm 0.04)x_L{}^{-1} + 0.14\ (\pm 0.09\ N/m)$ or a 5% uncertainty for the slope and 64% for the intercept.

Discussion

By comparing the force (weight) to the displacement, we were able to find direct values of k because of the form of Hook's Law $F = kx$. However, we found that our results lead to an alternate version of Hook's Law $F = kx + b$. In order to get better comparisons of k values later, we averaged the y-intercept. This way each k value is based on the same formula with the same b constant. However, the Because k varies with x_L, the cord does not have a universal spring constant. Therefore, we had to repeat the test with different x_L values. This gave us the equation $k = 0.85x_L{}^{-1} + 0.14N/m$ and so therefore we can find k if we first measure the length of the cord.

The uncertainty of Graph 3 was $\pm 0.04 m^{-1}$ which is only 5%. However, though we consolidated the y-intercepts in Graph 1 to better compare k vs x_L in Graph 2, there is still carried over error because the slopes of the lines in Graph 1 were altered when we manually set the intercepts. Secondly in Graph 2, k did not perfectly relate inversely with x_L. Finally, if the cord continued to stretch during each trial to a new x_L after adding hanging mass, then that would add uncertainty.

Because the floor limited the length of x_L, we are uncertain how accurate our equation is with longer cords. To test its accuracy, we could set the cord at a higher point, attach a mass m and measure the displacement x and then solve for k using Hooke's law $k = \frac{mg}{x}$. We could then compare the two k constants to determine the accuracy for a longer x_L.

Conclusion

By charting the comparison of weight to displacement we were able to find a k value for a specific x_L. Then, altering x_L we found multiple k values. Finally comparing k to x_L we were able to find an equation modeling the relationship between the spring constant and original cord length. From this equation we can find the value of k given any cord length. This way when using the cord, we can now solve more equations and problems with the cord as we now have a way to find the k value for a given length.