Determining the Spring Constant of Our Bungee Cord

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ABSTRACT

The goal for the overall Bungee Challenge is to create an ideal bungee experience for an egg, where it does not decelerate too quickly during the fall, and where it comes very close to the floor without hitting it. For the first step in this challenge, we used Hooke's Law ($F_s = -kx$) to determine the spring constant of our elastic bungee cord. In order to do this, we conducted ten trials, adding ten different masses to the end of the hanging bungee, measuring the bungee's unstretched length without any mass, the bungee's length at equilibrium (stretched at rest with the mass hanging from it), as well as its maximum length when dropped into oscillation (for analysis in future weeks). We calculated the displacement, or stretch, of the cord with each new mass using it's unstretched length and its stretched length at equilibrium. We then obtained the spring constant, k=5.12 ± 6%, using the slope of the weight vs. displacement graph for this setup.

Knowing this value will be particularly helpful when calculating the bungee cord's length for the final drop – the spring constant must be included in these calculations to ensure the egg does not hit the floor as the cord stretches at the end of the fall.

INTRODUCTION*:* The purpose of these tests was to determine the spring constant of our particular bungee cord. In the Bungee Challenge, this constant will be used when calculating the bungee length necessary for an ideal bungee experience.

Relevant equations and variables:

Hooke's Law: $\vec{F}_s = -k\vec{x}$

Equilibrium conditions: $\vec{F}_s = -\vec{W}$

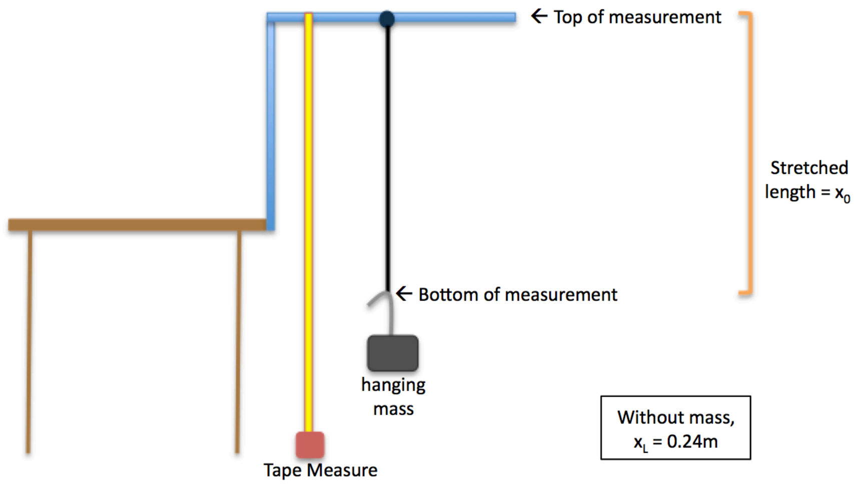
F_s: "spring force" or force of the cord (opposite of the weight of the system when in equilibrium)
x: displacement of the cord
k: spring constant of the cord
W: weight of the system

x_L: unstretched cord, without added mass
x_0: stretched cord, with a hanging mass

Background:
Newton's 2nd Law equation can be applied for this system in equilibrium – thus, the force of the cord is equal and opposite to the weight of the system. We can then use the force of the cord in Hooke's Law to find the spring constant of the cord.

Hypothesis (or expectations):
We expect linear results when plotting weight vs. displacement for the different masses, with the slope of our line being the spring constant of the cord.

METHODS: We let the cord hang above the ground at a set length (x_L) with a loop tied at the top and bottom. We then attached a hook to the bottom loop and added ten different masses, measuring the new length of the cord (x_0) when stretched different amounts and sitting at equilibrium.



Setup and Procedure:

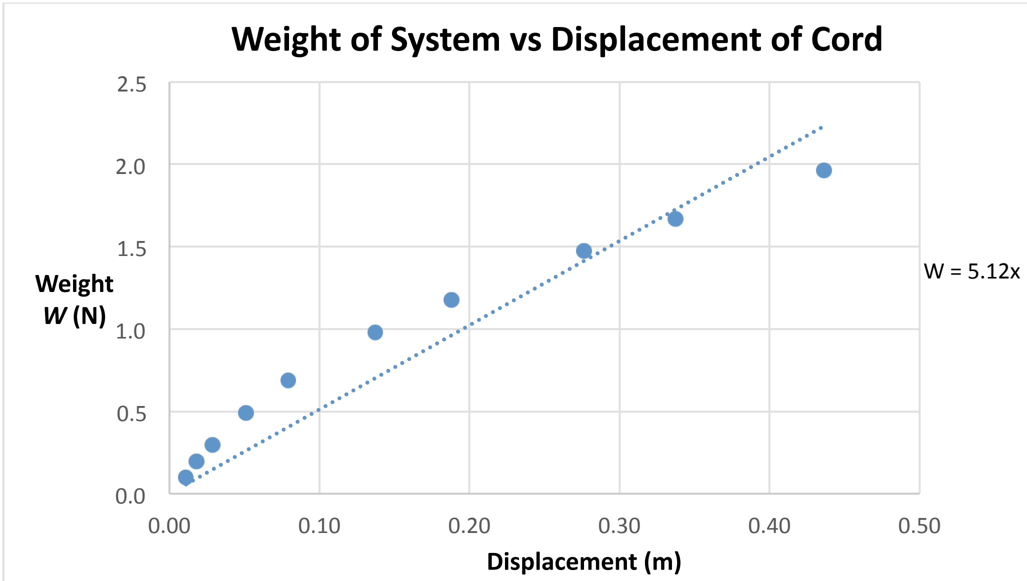
- A small loop was tied at either end of the bungee cord
- The loop at the top was used to hang the cord from a level beam above the ground. Any excess cord was wrapped off to the side, out of the way (as we did not use the entire length of the cord for these particular measurements). A tape measure was also hung from this beam next to the cord.
- The unstretched length of the cord (x_L) was measured from the top of the loop on the beam to the bottom of the loop hanging down. x_L in this setup was 0.24m for each of the ten trials.
- For the 10 separate trials, different masses were individually added to the bottom loop by a hook. Again, the new length of the stretched cord (x_0) was measured for each different mass from the top loop to the bottom loop once the system was at rest.
- Displacement of the cord was calculated by subtracting $x_0 - x_L$
- Deeming the mass of the cord negligible, we used the mass of the system to then calculate the weight of the system.

RESULTS: We plotted Weight of the System vs. Displacement of the Cord in excel to obtain the linear model W = 5.12x. Excel regression analysis was used to calculate the uncertainty of the slope.

Fig. 1: Displacement Values for Each Mass. An extra significant figure was included in some of the displacement measurements to better differentiate the values. No uncertainty was calculated for the weight because the sizes of the masses added were predetermined and labeled on the actual object – a scale was not needed for that measurement.

Weight of System (N)	Displacement ($x_0 - x_L$) (m) ± 0.001m
0.1	0.01
0.2	0.02
0.3	0.03
0.5	0.05
0.7	0.08
1.0	0.14
1.2	0.19
1.5	0.28
1.7	0.34
2.0	0.44

Fig. 2: Weight of the System vs. Displacement of the Cord
Equation: W = 5.12x
%uncert. of slope = 6% (obtained using Excel regression analysis)



Experimental value of interest:
The experimental value of interest here is the slope of the Weight vs. Displacement graph. The equation from our data fits the form of Hooke's law, $F_s = -kx$, thus making the slope the value for the spring constant of our cord (slope=k=5.12).

value obtained = 5.12
uncertainty of experimental value = 0.3
% uncert = 6%
Obtained using Excel regression analysis

Summary: After plotting our results, we obtained the model W = 5.12x, with a 6% uncertainty of the slope.

DISCUSSION

We could not quantitatively compare our slope value, as there was no way to calculate or predict any expected slope, and no "accepted value" considering the slope would vary with any cord. In the future, we could record these same measurements for displacement but change the unstretched length of the cord (x_L). If our results are precise, the slope of that graph should remain the same for any unstretched length of this particular cord.

Sources of uncertainty:
Because our graph is not completely linear, there were definitely sources of uncertainty affecting our results. For example, the cord could have permanently stretched more with each trial, not always reverting back to its initial unstretched state. This would affect our k value, as it is a measure of the stretch character of the cord. We could have mitigated this by stretching the cord more before conducting any trials. The stretch of the cord could have also been affected by the loops tied at both ends, stretching differently than unknotted cord would. This will be important to consider in the final Bungee Challenge, as we won't be able to retie the loops exactly as they were tied in this scenario.

There was also potential for error with each measurement recorded on the tape measure, especially with the initial measurement of the unstretched cord. Without any mass pulling the cord straight down, it was more difficult to accurately determine where then end of the cord fell.

Though we did not have a hypothesis for the specific value of our slope, we did expect the model to be more linear than it came out to be. According to Hooke's Law, the slope of our model should represent the spring constant of the cord; however, there is obviously a significant amount of error affecting this value as shown by the non-linear nature of the model.

CONCLUSION:

These tests allowed us to calculate a spring constant that we will be able to use in further experiments and calculations throughout the bungee challenge. Because of the obvious error and uncertainty affecting our results, we may have to conduct additional trials with different masses to broaden our range and obtain a more accurate spring constant. As noted before, we also recorded the maximum stretched length (x_{max}) for each mass when oscillating. We also ran a few tests holding the mass constant, varying the unstretched cord length, and then measuring the x_{max}. With this side data, and our now determined spring constant, we have a better chance of creating an ideal bungee experience, where the egg comes close to the ground without hitting it, and where it does not reach too great of an acceleration during the fall.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Ryan Hodgson