

**Names:** Lauren McManus, Emily Perszyk

**Section:** Tuesday      **Date:** 10/23/16

**TITLE:** Determining a relationship between spring constant (k) and bungee length

**ABSTRACT:** For a bungee cord the relationship between the amount of stretch and the force causing the stretch can be modeled by  $k$ , or the spring constant, as a bungee cord shows spring-like behavior. However, the magnitude of  $k$  depends on the length of the bungee cord. In order to later use Hooke's Law to find the ideal bungee length for our egg drop, we needed to find how  $k$  varies with different initial cord lengths. We hung the bungee cord from an apparatus and measured the change in length at each weight we put on, subtracting the initial unstretched length to the length once the weight was added. We ran tests at 8 different lengths, using the same 7 weights for each. We plotted each of these trials and got the slope of the line formed, which is the  $k$  value for each initial length. Then plotting the  $k$  value against each length resulted in equation  $K = 1.37L + 0.1016$ . Uncertainty of  $L$  is  $\pm 0.0252$ . A linear relationship ends up working well for these numbers, and  $K$  increases with length as expected. Therefore, the longer the bungee length the more it stretches. This means it will likely decelerate slower, but the egg will fall further. We must find a length in next week's experiment that strikes the right balance to keep a raw egg uncracked.

**INTRODUCTION:** Bungee cords often behave simply, like a linear spring. Therefore, the relationship between the amount of displacement and the force applied to the bungee cord can be exemplified by the value  $k$ , or the spring constant. However, the magnitude of this value is dependent on the initial length,  $L$ , of the cord. To choose the length of bungee cord to use in the later dare devil experiment, we must first find the relationship between  $K$  and  $L$ . Our chosen length will help us model our final egg drop experiment, to ensure the egg does not drop too far or decelerate too quickly. We also made sure to linearize the relationship we found to make sure the cord's behavior was linear enough to be characterized simply like a spring.

Relevant equations:

**F = -Kx** – Hooke's Law (here it stands for the force of the bungee on the mass)

- $K$  is spring constant
- $X$  is the displacement of the spring (bungee), or how far it will stretch from its initial length,  $L$

**F = mg** – to find force of mass acting on bungee

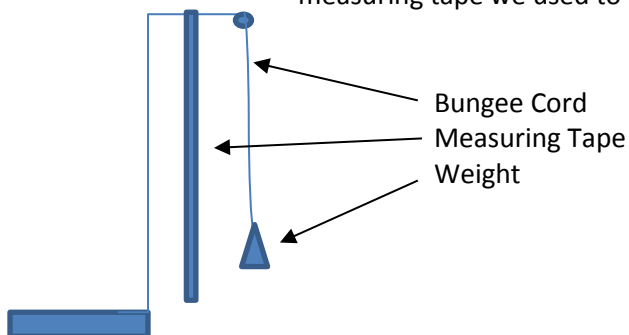
- Gravitational constant used for acceleration since weight was hanging

We expected that the bungee would behave linearly enough to model its length –  $k$  relationship like that of a spring. We also expected to see more displacement per unit of force as bungee length increased.

**METHODS:** We hung a length of bungee cord from an apparatus off the side of our desk. We tied a knot at the top of the cord and made a loop as small as possible to hang it from, to reduce and excess stretching. (Figure 1)

- Tested eight initial lengths ( $L$ ) and hung the same seven weights on each length
- Tied knot in top of cord to form a loop which hung the bungee cord from the apparatus
- Initial length of the bungee we measured from the bottom of the knot we tied, which was 1.8 cm down our cord, to the end of the cord
- Hung each weight on cord, measured new length ( $X_f$ )
- Kept weight hanging for as short a time as possible
- Recorded each initial length, force, final length and change in length in excel (Table 1)
- Computed  $dX: X_f - L$
- Plotted each of the seven length trials, Force vs.  $dX$
- Inserted linear trendline and displayed equation of the line
- Slope for each line was the  $K$  value for that length – plotted  $K$  vs  $L$
- Linearized the graph, inserted trendline and displayed equation
- Ran linear regression on  $K$  vs  $L$  data to find uncertainty in  $L$

**Figure #1 – Setup.** Diagram of our apparatus, showing the bungee cord, weight and the measuring tape we used to measure length and  $dX$ .



**RESULTS:** We recorded the mass, initial length and final length for each trial. From these we then calculated force (mg) and change in position ( $dX$ ). (Table 1). We plotted  $F$  vs  $dX$  for each trial and recorded the slope of the trendline for each graph – this was our  $K$  value for each length. We then plotted  $K$  vs  $L$ , recorded the equation of that trendline, then linearized the graph and recorded the equation of that line (Table 2). From this linearized data set we performed a linear regression to get the uncertainty in our  $L$  value.

Initial  $K$  equation:  $K = 1.4026L^{-0.945}$

Linearized  $K$  equation:  $K = 1.37L + 0.1016$ .

Uncertainty in  $L$ :  $\pm 0.0252$

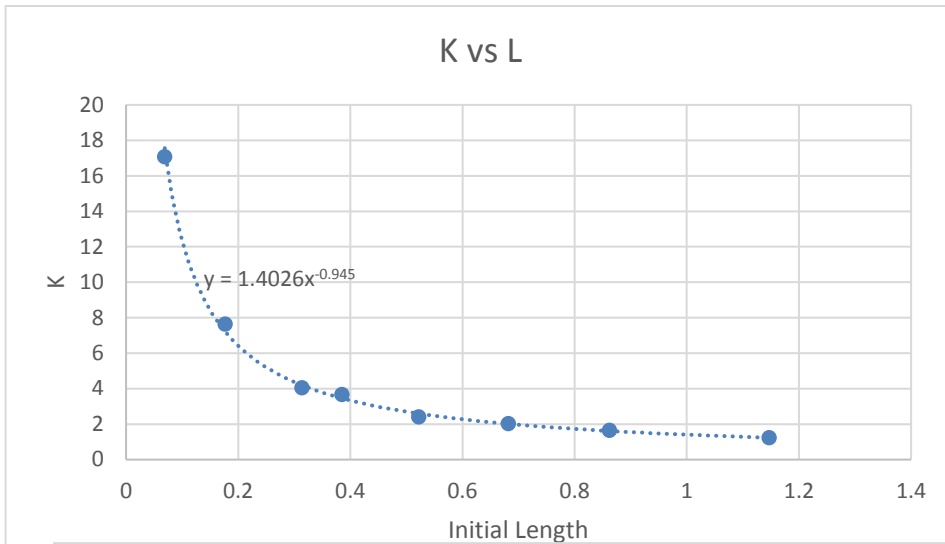
	Mass (kg)	$X_i$	$X_f$	$dX$ (m)	$F$ (mg)
<b>L1</b>	0.025	0.177	0.192	0.015	0.24525
	0.05	0.177	0.211	0.034	0.4905
	0.07	0.177	0.235	0.058	0.6867
	0.09	0.177	0.256	0.079	0.8829
	0.1	0.177	0.266	0.089	0.981
	0.12	0.177	0.301	0.124	1.1772
	0.15	0.177	0.349	0.172	1.4715
<b>L2</b>	0.025	0.682	0.73	0.048	0.24525
	0.05	0.682	0.801	0.119	0.4905
	0.07	0.682	0.862	0.18	0.6867
	0.09	0.682	0.943	0.261	0.8829
	0.1	0.682	1	0.318	0.981
	0.12	0.682	1.107	0.425	1.1772
	0.15	0.682	1.322	0.64	1.4715
<b>L3</b>	0.025	0.385	0.419	0.034	0.24525
	0.05	0.385	0.46	0.075	0.4905
	0.07	0.385	0.501	0.116	0.6867
	0.09	0.385	0.55	0.165	0.8829
	0.1	0.385	0.579	0.194	0.981
	0.12	0.385	0.643	0.258	1.1772
	0.15	0.385	0.746	0.361	1.4715
<b>L4</b>	0.025	0.862	0.931	0.069	0.24525
	0.05	0.862	1.019	0.157	0.4905
	0.07	0.862	1.107	0.245	0.6867
	0.09	0.862	1.213	0.351	0.8829
	0.1	0.862	1.285	0.423	0.981

	0.12	0.862	1.413	0.551	1.1772
	0.15	0.862	1.662	0.8	1.4715
<b>L5</b>	0.025	1.147	1.238	0.091	0.24525
	0.05	1.147	1.361	0.214	0.4905
	0.07	1.147	1.481	0.334	0.6867
	0.09	1.147	1.632	0.485	0.8829
	0.1	1.147	1.713	0.566	0.981
	0.12	1.147	1.902	0.755	1.1772
	0.15	1.147	2.217	1.07	1.4715
<b>L6</b>	0.025	0.069	0.077	0.008	0.24525
	0.05	0.069	0.085	0.016	0.4905
	0.07	0.069	0.094	0.025	0.6867
	0.09	0.069	0.105	0.036	0.8829
	0.1	0.069	0.111	0.042	0.981
	0.12	0.069	0.124	0.055	1.1772
	0.15	0.069	0.147	0.078	1.4715
<b>L7</b>	0.025	0.314	0.341	0.027	0.24525
	0.05	0.314	0.374	0.06	0.4905
	0.07	0.314	0.411	0.097	0.6867
	0.09	0.314	0.453	0.139	0.8829
	0.1	0.314	0.482	0.168	0.981
	0.12	0.314	0.537	0.223	1.1772
	0.15	0.314	0.636	0.322	1.4715
<b>L8</b>	0.025	0.522	0.566	0.044	0.24525
	0.05	0.522	0.623	0.101	0.4905
	0.07	0.522	0.678	0.156	0.6867
	0.09	0.522	0.755	0.233	0.8829
	0.1	0.522	0.799	0.277	0.981
	0.12	0.522	0.892	0.37	1.1772
	0.15	0.522	1.061	0.539	1.4715

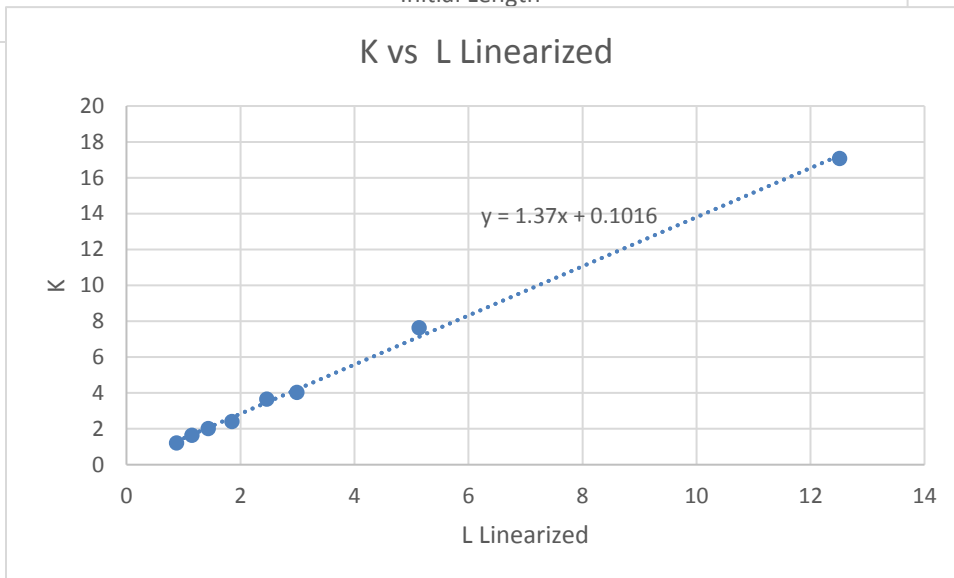
**Table 1: Stretch and force recorded for each trial.** Each of the eight lengths we tested with the initial and final position after the application of each force.

L	K	L <sup>-.945</sup>
0.177	7.645	5.136476
0.682	2.025	1.435733
0.385	3.665	2.464561
0.862	1.643	1.150656
1.147	1.227	0.878441
0.069	17.08	12.51087
0.314	4.044	2.988144
0.522	2.411	1.848423

**Table 2: Lengths and K values.** The initial lengths and recorded K values, as well as the linearized K value.



**Graph 1:** Initial lengths and recorded K values.



**Graph 2:** Linearized K vs L graph.

Use **Excel regression analysis** on any graph that has a **linear** fit only (see EG), to obtain:

uncertainty for slope= 0.0252    % uncert= 2%

**Identify experimental value(s) of interest**, why it is of interest, and how/from where obtained, briefly:

value obtained = K (in relation to length)

- K allows us to find the force of the bungee (using Hooke's Law) which we can use later to determine the best length of bungee cord for the dare devil part of this lab

uncertainty of experimental value(s) = .0252    % uncert= 2%

- Propagated this uncertainty using linear regression

We got a linear relationship between length and k, with equation  $K = 1.37Lb + 0.1016$ . So, as the length of the bungee increases so does the displacement in relation to force. This equation for k we can sub into Hooke's Law to get the force of the bungee on the egg, and therefore deceleration, for the next part of our experiment.

**DISCUSSION:** What do you make of your results? Evaluate them.

**Error analysis-** slope of K equation.

- Experimental value – 1.37. % uncert = 2%.
- Acceptable value – 1.35
- % Error – 1.5%
- %uncert > % error, so our result is accurate

**Sources of uncertainty** – We had to tie the bungee into a loop at the top to hang it from our apparatus. We dismissed this length of cord, but it could have accounted for extra stretch in the cord. We used the same piece of cord for each of our 8 trials so as the experiment went on the continuous weight being added on the cord could have caused some permanent stretch and made it less elastic.

Our observations support the idea that we can model the behavior of a bungee like that of a spring. As expected, the k value increases with length, meaning more displacement per unit of force applied occurs as length increases.

**CONCLUSION:**

We were able to determine a linear relationship between length and spring constant. The spring constant  $K = 1.37L + .1016$ . This shows the longer length of bungee cord we use the more displacement occurs in relation to force.

In our next experiment, we can use this K equation in Hooke's Law, along with known and derived forces acting in the bungee system, to find the ideal length of bungee cord to thrill an egg.

**On my honor, I have neither given nor received any unacknowledged aid on this assignment.**

***Pledged: Lauren McManus***