Lab Report Outline—Bungee Lab 1

Your name and your lab partner(s): William Schirmer, Daniel Clark **Section:** 113-04 **Date:** 10/23/16

TITLE: Determining a Modified Hook's Law Equation for the Bungee Cord

ABSTRACT: The goal of this lab was to determine an adjusted Hook's law equation of the bungee cord by changing the hanging mass on the cord and measuring equilibrium points. The bungee cord was hung from a metal rack and varying masses were tied to the end of the cord. The mass was then allowed to come to equilibrium and the difference between the un-stretched length and the stretched length of the cord were measured. The data we collected allows for a modified version of Hook's law to be applied to our cord. The modified equation is found to be

$$F_S = -(1.17(\pm.01)x + .73(\pm.01))^2$$

From this equation it can be seen that the spring is nonlinear and the force of the spring can be predicted using the modified equation. The uncertainty in the spring force is will most likely depend on the precision of the value of x, stretch, that is used. The analysis of our experiment showed that many cords do not act linearly and modified equations for hook's law must be found.

INTRODUCTION:

Purpose: The goal of this lab was to collect pertinent information regarding a bungee cord that will later be used in an egg bungee jump. The egg will be dropped from a balcony and must not hit the floor or experience to great a force during its fall. We chose to analyze the bungee cord and determine if it followed Hook's law and if not, to determine an adjusted Hook's law equation. To do this the bungee cord was suspended from a metal rack and varying masses were tied to the end of the cord. The mass was then allowed to come to equilibrium and the difference between the un-stretched length and the stretched length of the cord were measured. This allowed us to evaluate our bungee cord as a linear spring or create a modified Hook's law equation.

Theoretical Background: If the bungee follows Hook's law it is said to be linear and if it deviates form the law it is nonlinear. In a linear spring Hook's law is as follows,

$$F_S = -Kx$$

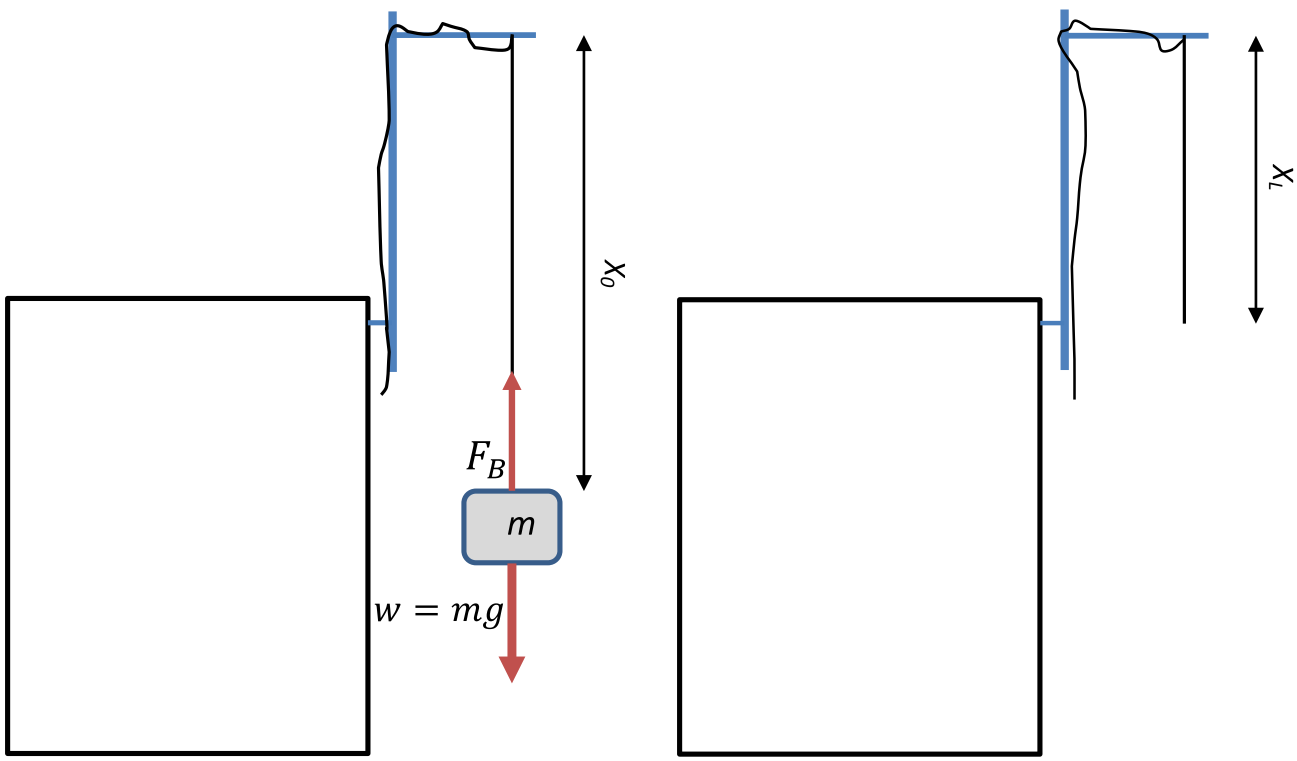
where F_S is the total force of the spring, K is the spring constant, and x is the displacement from the equilibrium position of the spring. For a linear spring the restorative force of the spring is directly proportional to the distance that the spring is stretched. For a non-linear spring this is not true, for example there could be varying degrees of resistance based on how far the spring is stretched. In this case there is no general equation and it must be determined through experimentation.

Hypothesis: The bungee cord is expected to act in a linear fashion and obey Hook's Law, meaning that the force of the spring is directly proportional to the distance the spring stretches.

METHODS:

Intro: In order to determine if the bungee cord followed Hook's law we hung the cord from a holding rack and attached masses to the end. We then compared the stretched length of the cord due to attached masses versus its un-stretched length.

Figure 1: Stretched and Un-Stretched Bungee Setup
In this diagram the bungee setup with mass, m, is shown on the left and the un-stretched bungee setup is shown on the right.



$x_L = Un - stretched\ Bungee\ Length$
$x_o = Stretched\ Bungee\ Length$
$F_B = Force\ of\ bungee$
$mg = weight\ of\ mass$
$m = mass\ attatched\ to\ spring$

Setup:
1. Secure the metal rack to the side of the table.
2. Stretch out the bungee cord at least three times using your hands. This is done to prevent stretching after experimentation has started.
3. Tie a knot in the bungee cord at the end and at about .26m from the knot at the end.
4. Hang the Cord by the second knot and tuck the rest of the bungee cord behind the metal rack.

Procedure:
1. Measure the un-stretched length of the bungee cord from the top knot to the bottom knot.
2. Attach mass of .05 kg to the end of the bungee cord and allow the mass to equilibrate.
3. After mass equilibrated, measure the length of the stretched bungee cord from the middle of the top knot to the middle of the bottom knot.
4. Repeat steps 2 and 3 nine times. Increase the mass by .05 kg each trial. Make sure to leave masses on bungee only long enough to equilibrate then remove them to minimize additional stretch of bungee cord.

RESULTS: In order to determine if the bungee cord followed Hook's Law we collected data regarding the length the bungee stretched when a variable mass was allowed to equilibrate. With this data we plotted the the weight vs. stretch first, then linearized our data by plotting the square root of weight vs. stretch. This linearized equation is the modified Hook's law for the equation. In addition, the original graph of weight vs. stretch is plotted with a linear trend line to provide a simplified Hook's law equation. This equation could be used more easily for continued experimentation with the cord.

Figure 2: Weight of Mass and Stretch of Bungee.

Each mass was allowed to equilibrate and the stretch was measured from middle of top knot to middle of bottom knot before mass. Weight is calculated by multiplying mass on end of cord by $9.81\,{}^{m}/_{s^2}$.

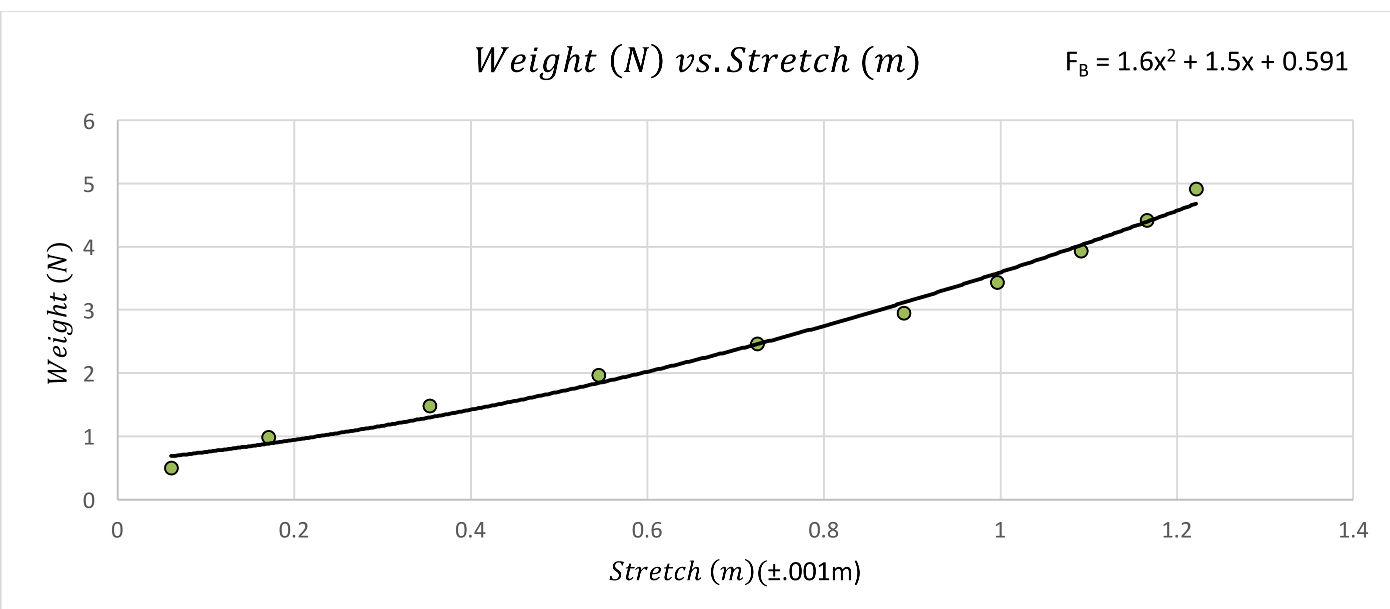
Trial	$\boldsymbol{Weight\ (N)}$	$\boldsymbol{Weight^{.5} (N^{.5})}$	$\boldsymbol{Stretch\ (x_0 - x_L)(m)}$ $(\pm\mathbf{.001\ m})$
1	0.491	0.700	0.061
2	0.981	0.990	0.171
3	1.472	1.213	0.354
4	1.962	1.401	0.545
5	2.453	1.566	0.725
6	2.943	1.716	0.891
7	3.434	1.853	0.997
8	3.924	1.981	1.092
9	4.415	2.101	1.166
10	4.905	2.215	1.222

$x_0 - x_L = Total\ length\ of\ bungee\ with\ weights - Unstretched\ length\ of\ bungee$

We then analyzed this data by first plotting the weight vs. stretch of the bungee.

Figure 3: Weight of mass vs. Stretch of Bungee Graph

The graph shows the weight of the hanging mass plotted vs. the stretch of the bungee from its equilibrium point when the weight is added. The line of best fit is a quadratic of the second degree.



Equation of Non-Linearized Data: F_B=1.6x^2+1.5x+.6

Uncertainty for x^2 = ±.001 N^2/m^2 % Uncert= .1%

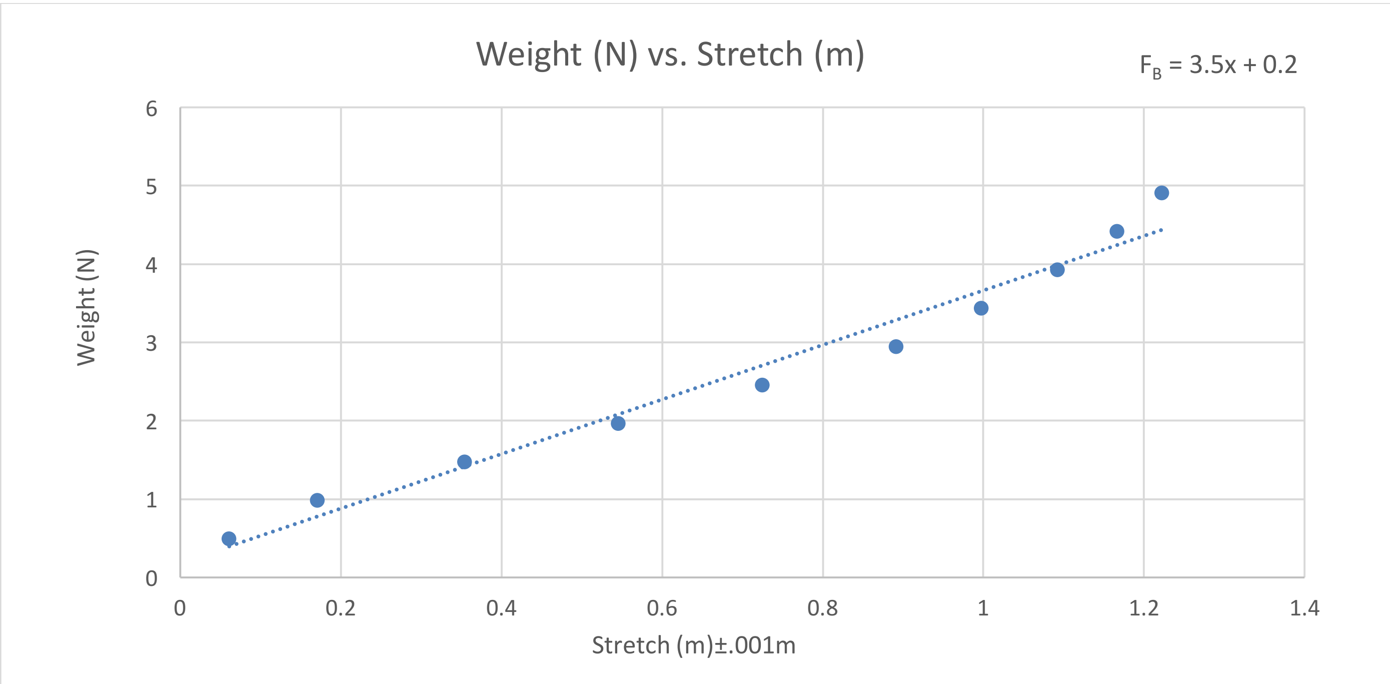
Uncertainty for x = ±.001 N/m % Uncert= .1%

Uncertainty for y-intercept= ±.001 N % Uncert= .2%

In order to linearize our data, we plotted the square root of the weight vs. the stretch as our non-linear graph was a quadratic. The results can be found in Figure 4.

Figure 4: Linearized Graph of Weight vs. Stretch

For simplicity's sake a linear trend line was applied to the initial data. The resulting trend line and equation could be used as a simplified version of Hook's law for further calculations.



Linearized Equation: $F_B = 3.5x + .2$

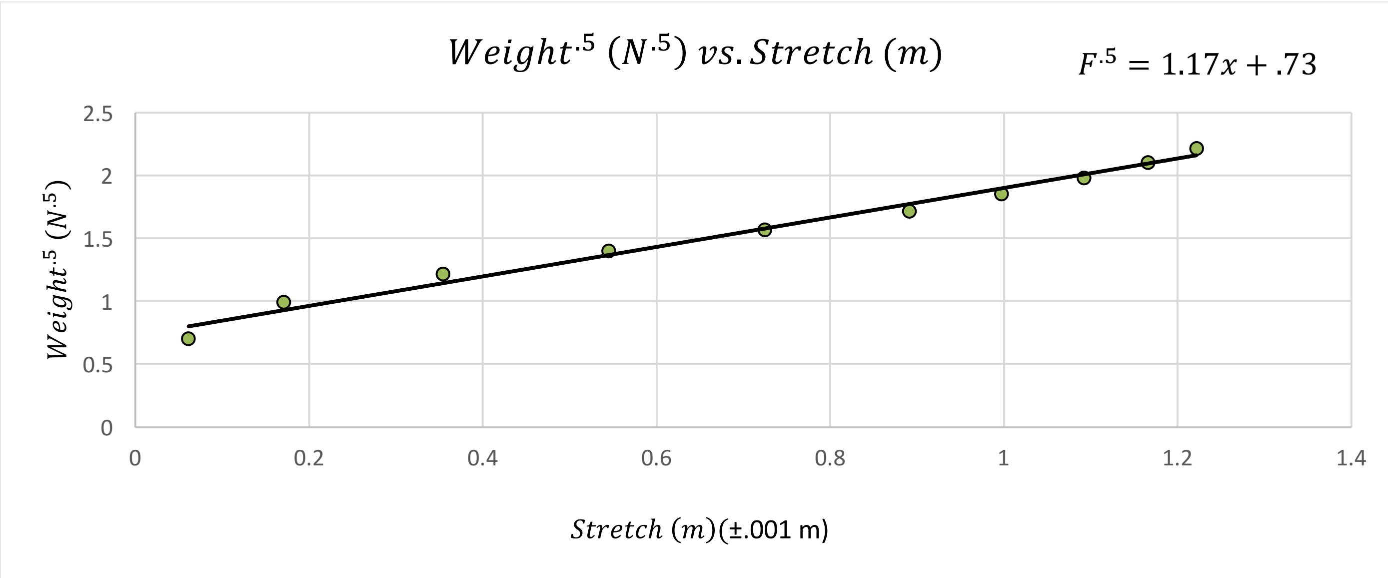
Uncertainty in x coefficient = ±.1 N/m % Uncert= 3%

Uncertainty in y-intercept = ±.1 N % Uncert= 50%

This uncertainty was derived from a linear regression of the data.

Figure 5: Linearized Graph of Weight$^{1/2}$ vs. Stretch

The graph shows the square root of the weight of the hanging mass plotted vs. the stretch of the bungee from its equilibrium point when the weight is added. The line of best fit is linear.



Linearized Equation: $\sqrt{F_B} = 1.17x + .73$

Uncertainty in x coefficient = ±.01 N/m % Uncert= 1%

Uncertainty in y-intercept = ±.01 N % Uncert= 1%

This uncertainty was derived from a linear regression of our linearized data.

Identify experimental equation of interest: The linear equation of Figure 5 is the most pertinent derived experimentally derived equation. We then squared this equation and made it negative to give the restorative force of the spring. This is possible because the system was in equilibrium when x_0 was measured so the $F_B - mg = 0$ so $F_B = mg = weight$. The equation allows one to calculate the force (N) that the bungee cord provides in resistance to stretch. This equation is the modified Hook's Law for our bungee cord. In addition, when the equation is multiplied out it is quite similar to the non linearized equation for bungee force.

Equation: $Restorative\ Force\ of\ Cord = -(1.17x + 0.73)^2 = -2.34x^2 - 1.71x - .53$

In addition to the linear equation from Figure 5, the linear equation of Figure 4 can be used as a simplified version of Hook's Law $F_B = 3.5x + 0.2.$ This equation could be especially useful if the bungee force is being used in further calculations such as a potential energy analysis of the bungee jump.

Summary: The further it is stretched the greater the restorative force the spring has. As the force of the spring is restorative the value of F_B must be made negative. The restorative force can be found using the following equation: $F_B = -(1.17x + 0.73)^2$ This information can help us to determine the acceleration of the egg during the bungee jump as well as the forces acting on the egg. This equation should help to ensure a safe and thrilling jump for the egg.

DISCUSSION: As there is no known value of the Hook's Law equation for this specific bungee we must analyze our results qualitatively. The uncertainty of our coefficients in our modified Hook's equation are relatively small when compared to the weight of the egg, so the uncertainty in our results should not be a major cause for concern. However, a test of our results would be appropriate to gain a value for comparison. A suitable test would be to attach the bungee cord to a hook on stationary block and stretch the cord in the horizontal direction using a force meter and record the force required to stretch the cord a measured distance. This would give us another value to evaluate the accuracy of our results. We then could compare the two modified Hook's law equations to determine the percent difference.

Sources of Uncertainty: I believe the greatest source of uncertainty comes from the hysteresis of the bungee cord. As the band was loaded each time it seems like the amount of force, weight, needed to produce each stretch was inconsistent. However, I do not believe the permanent deformation of the cord to be a significant cause for error as the length of the cord, with no mass attached, was measured before and after the experiment and found to be the same within uncertainty. Another source of error that should be considered is parallax when reading the measurement of stretch on the measuring tape. The individual reading the measure has to ensure that the end of the tape measure is even with the knot while simultaneously reading the measurement. If the lab were repeated the estimated uncertainty for the measurement of stretch should be larger and another person should hold the top of the tape measure while the first reads the value. A possible source of random error is in the inconsistent stretch of the bungee due to oils from experimenter's hands, distortion in the sun, and changes in the temperature of the band that produce different amounts of stretch for each force applied.

Hypothesis Revisited: The hypothesis that the bungee would act like a spring and return a restorative force directly proportional to distance stretched was wrong. The bungee was found to act according to a modified and non-linear version of Hook's law. While the general idea that, when a bungee is stretched there is a restorative force opposing the stretch is consistent with Hook's law the ratios are not the same and a modified version must be created for each bungee.

CONCLUSION: The goal of the experiment was achieved by determining that the bungee did not act like a linear spring and a modified version of Hook's law for our bungee is $F_S = -\left(1.17(\pm.01)x + .73(\pm.01)\right)^2$. These results suggest that many elastic cords do not behave ideally, contrary to my belief, and that each must be tested to ensure accurate predictions in its behavior. In reference to the greater egg bungee jump problem, this means that a sophisticated analysis of gravitational potential energy and elastic potential energy will have to be conducted to predict the appropriate amount of inelastic leader and bungee to be used in the jump. If the computations involved in predicting this using the linearized equation are especially difficult one could use the following equation to provide a simpler analysis of potential energy $F_B = 3.5x + 0.2$. A second test that I propose would be to double the bungee back upon itself and repeat the experiment to see if the result agreed with the theoretical implication that when there are two parallel springs the restorative force is simply doubled.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: *William Schirmer*