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TITLE: Determining k-value of the elastic cord by varying number of strands to which the mass is attached.

ABSTRACT: The goal of this experiment was to determine the k-value of the elastic cord by varying the number of strands and to examine the relationship between k and the number of strands. To test this, we assumed our cord could be modeled as a spring, utilizing Hooke's Law (F = kx). We took measurements of displacement for 5 different masses at 3 different numbers of strands while keeping the equilibrium position constant. We predicted that the k-value would increase as the number of strands increased. Different masses were attached to the elastic cord for 1 strand, 2 strands, and 4 strands and displacement distance from the equilibrium position were measured. We converted the masses to weights by multiplying by the acceleration due to gravity and displacement was graphed against force (weight) so that the slope would be equal to the k-value. The k-values were then plotted against the number of strands to determine their relationship. The k-value of the cord increased as the number of strands increased, supporting our prediction. However, the experimental determination of k-values for 2 and 4 strands did not test within experimental uncertainty when comparing these values to the theoretical values. The relationship between k and the number of strands may be helpful in determining the number of strands that will lead to best success during the egg drop portion of the bungee challenge, but further experimentation needs to take place.

INTRODUCTION:

Determining the k-value of the elastic cord in response to changing the number of strands from which the mass hangs while keeping equilibrium position constant for all three different numbers of strands.

Relevant equations:

Equation 1: Hooke's Law

$F = kx$

- F = force (N)
- k = spring constant (elasticity constant for our model)
- x = displacement from equilibrium position (m)

Equation 2: Force due to weight

$w = mg$

- w = weight (N)
- m = mass (kg)
- g = acceleration due to gravity (m/s^2)

In this experiment, the elastic cord was modeled as a spring, and therefore Hooke's Law could be used. The spring constant in Hooke's equation would represent the elasticity constant of our cord. The only force acting on the system is the weight and therefore total force can be calculated by multiplying each of our different masses times the acceleration due to gravity. These values of force divided by the displacement from equilibrium position for each weight would give us our k-value. Therefore, when graphed, the change in force over the change in displacement is equal to the slope, which represents our k-value, the variable of interest for the experiment. For 2 and 4 strands, we were able to predict the theoretical k-values because according the Hooke's Law, springs in parallel have a k-value equal to the sum of the individual k-values.

We hypothesize that as the number of strands increases the k-value will also increase.

***METHODS*:**

To obtain the k-value of the elastic cord for 1 strand, 2 strands, and 4 strands, we took measurements of displacement from equilibrium for 5 different masses at each numbers of strands. The equilibrium position was held constant for all trials to avoid introducing a second independent variable. The different masses were attached to the elastic cord and displacements from equilibrium were measured. Masses were then converted to weights by multiplying gravity and displacement was graphed against force (weight) so that the slope would be equal to the k-value. Finally, the k-values were graphed against number of strands to obtain a linear relationship between our two variables.

Pre-Measurement

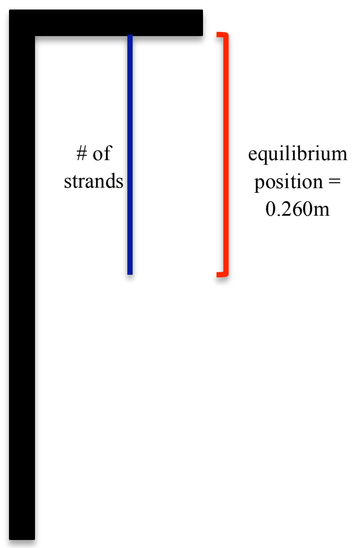


Figure 1: Prior to measurement, the number of strands, one, two, or four, were attached to a horizontal bar at a constant equilibrium position. The equilibrium set in this experiment was 0.260m.

Measurement

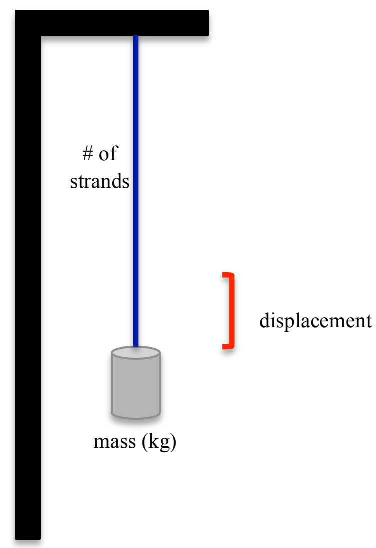


Figure 2: Once the mass was hung from the end of the strand(s), the displacement from the equilibrium position was measured and recorded. Five different masses were tested for each number of strands.

Setup:

The cord was hung from a horizontal bar and a mass was then hung from the bottom of the cord. For 1 strand, the mass was hung from a knot that was tied in the end of the cord as close to the end as possible to prevent the k-value of that small portion of the cord below the knot to interfere with the recorded displacement and calculations of k-value. For 2 and 4 strands, the mass was simply hung from the loop of the folded cord so that no knot had to be tied. Equilibrium position was set at 0.260m for each test.

Procedure:

- Establish an equilibrium position (0.260m) and keep it constant throughout.
- Hang 5 different masses (chosen to prevent cord from overstretching to more than 20% of its length) from the cord and record the displacement from the equilibrium position using a measuring tape.
- Calculate the force, mass times the acceleration due to gravity, for each mass used.
- Graph the displacement (x) against force (y) such that the slope will give the k-value according the Hooke's Law for that given number of strands.
- Determine the linear relationship between k-value and number of strands by graphing the slope values from the previous graphs against the number of strands.

RESULTS:

Once the force was calculated for each mass for the different number of strands, displacement was graphed against force. Force was graphed on the y-axis and displacement on the x-axis. This setup was selected so that the slope (force over displacement) would be equal to the k-value according the Hooke's Law. The k-value increased as the number of strands increased. Then, the k-values were plotted against the number of strands, obtaining a linear relationship. Additionally, the uncertainties in the models of k-values were calculated.

Tables:

Table 1: Displacements of masses hung from one strand of the elastic cord. Light masses were selected when testing one strand to prevent the cord from overstretching. Forces were calculated for each mass.

ONE STRAND	**Equilibrium: 0.260m**	
mass (kg) (± 0.001)	x (distance from equilibrium) (m) (± 0.005)	$F_{grav} = mg$ (N)
0.010	0.003	0.098
0.020	0.015	0.196
0.030	0.026	0.294
0.040	0.037	0.392
0.050	0.050	0.490

Table 2: Displacements of masses hung from two strands of the elastic cord. Slightly heavier masses were selected because 2 strands were able to hold more mass without overstretching. Forces were calculated for each mass.

TWO STRANDS	**Equilibrium: 0.260m**	
mass (kg) (± 0.001)	x (distance from equilibrium) (m) (± 0.005)	$F_{grav} = mg$ (N)
0.025	0.012	0.245
0.050	0.027	0.490
0.075	0.042	0.735
0.100	0.056	0.980
0.125	0.081	1.225

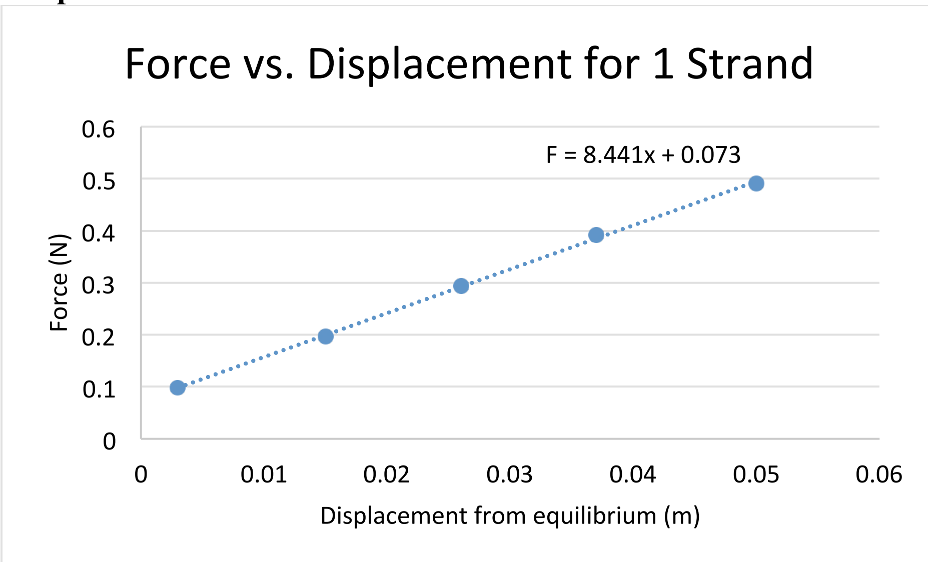
Table 3: Displacements of masses hung from four strands of the elastic cord. Even heavier masses were selected because 4 strands were able to hold more mass without overstretching. Forces were calculated for each mass.

FOUR STRANDS	**Equilibrium: 0.260m**	
mass (kg) (± 0.001)	x (distance from equilibrium) (m) (± 0.005)	$F_{grav} = mg$ (N)
0.050	0.018	0.490
0.100	0.034	0.980
0.150	0.048	1.470
0.200	0.066	1.960
0.250	0.085	2.450

Table 4: The number of strands was compared to the k-values calculated from the above measurements.

# of strands	k-value (N/m)
1	8.441
2	14.462
4	29.420

Graphs:

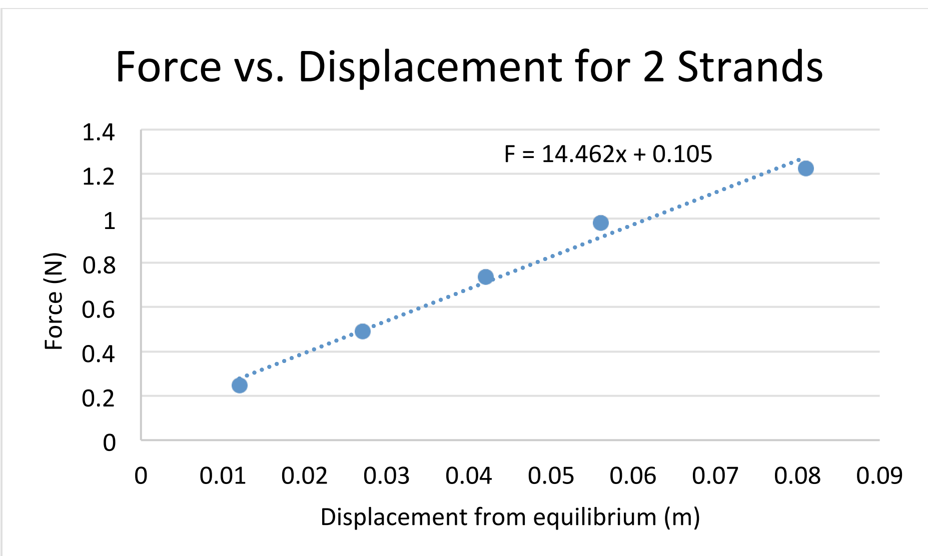


Equation:
F = 8.441x + 0.073

Regression analysis on linear fit graph:

uncertainty for slope = 0.1455 — % uncertainty = 2%

uncertainty for y-intercept = 0.0045 — % uncertainty = 6%

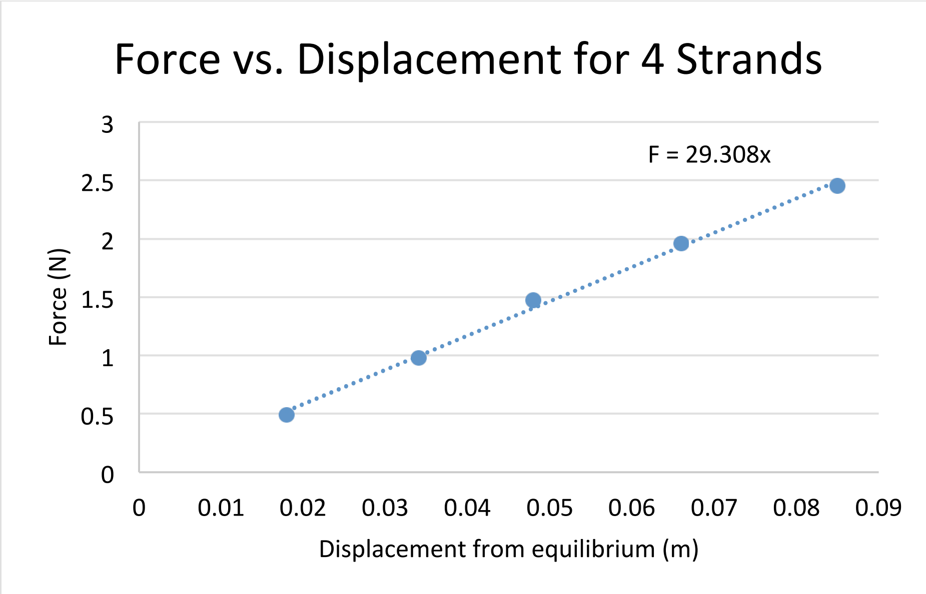


Equation:
F = 14.462x + 0.105

Regression analysis on linear fit graph:

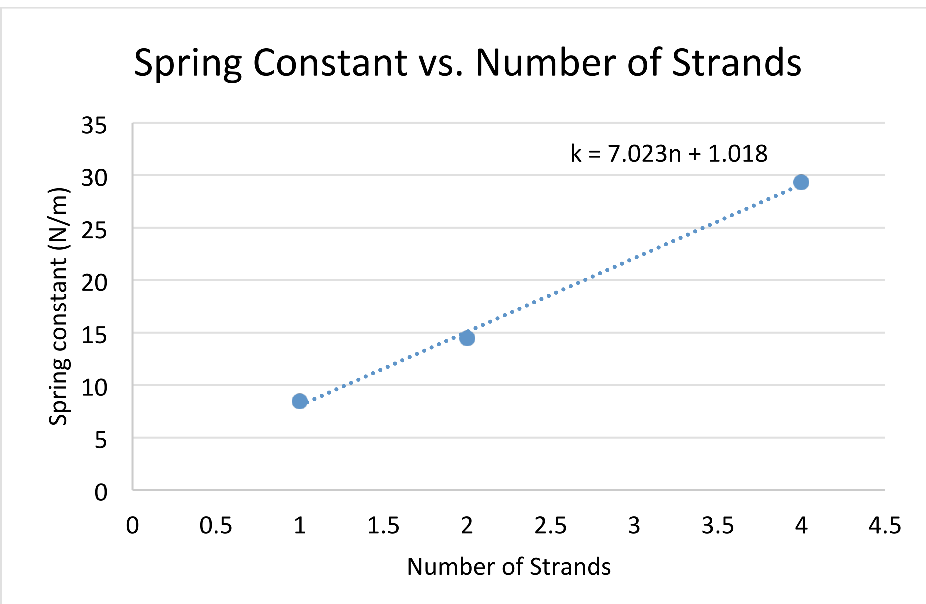
uncertainty for slope = 1.0037 — % uncertainty = 7%

uncertainty for y-intercept = 0.0498 — % uncertainty = 50%



Equation:
F = 29.308x

Regression analysis on linear fit graph:
uncertainty for slope = 0.9814 % uncertainty = 3%



Equation:
k = 7.023n + 1.108

Regression analysis on linear fit graph:
uncertainty for slope = 0.3607 % uncertainty = 5%
uncertainty for y-intercept = 0.9543 % uncertainty = 86%

Experimental values of interest:
Our values of interest are the experimentally determined k-values and the relationship between these k-values and the number of strands of the elastic cord. How the k-value changes in response to the number

of strands may be helpful information for the bungee challenge. The slope of the graph of 1 strand is 8.441, which is the specific k-value to the equilibrium position used in this experiment, 0.260m. The slope of the graph of 2 strands is 14.462, which represents that k-value at this same equilibrium constant. The slope of the graph for 4 strands is 29.308, which is the experimental k-value for 4 strands also at this equilibrium distance. There exists a relationship between k and number of strands such that the k-value of 2 strands is about double that of 1 strand and the k-value of 4 strands is about double that of 2 strands.

k-value of 1 strand
value obtained = 8.441
uncertainty of experimental value = 0.1455 % uncertainty = 2%

k-value 2 strands
value obtained = 14.462
uncertainty of experimental value = 1.0037 % uncertainty = 7%

k-value 4 strands
value obtained = 29.308
uncertainty of experimental value = 0.9814 % uncertainty = 3%

Uncertainty was determined through linear regression analysis on the data. The percent uncertainty was determined by dividing the uncertainty by the value of interest and multiplying that by 100.

The force, or weight, for each mass was calculated by multiplying the mass times the acceleration due to gravity. The displacements from the equilibrium position were graphed against the force for each of the different number of strands tested so that the slope would be equal to the k-value; this is according to Hooke's Law. In addition to determining the k-values for the varying number of strands, the k-values were graphed against the number of strands to determine if the plot would show a linear relationship, The k-value of the cord increased as the number of strands increased and the relationship between k-value and number of strands was linear.

DISCUSSION:

According to Hooke's Law, springs in parallel should have a k-value that is equal to the sum of the k-values of the individual springs. Since there is not true theoretical k-value of the cord, we use our experimental k-value of 1 strand (8.441 N/m) to evaluate the theoretical values for 2 and 4 strands. Since our elastic cord is modeled as a spring, the k-value of 2 strands should be double that of 1 strand and the k-value of 4 strands should be double that of 2 strands.

Error Analysis:

experimental k-value of 2 strands = 14.462 N/m
theoretical k-value of 2 strands = 8.441+ 8.441= 16.882 N/m
% error = 14%

experimental k-value of 4 strands = 29.308 N/m
theoretical k-value of 4 strands = 8.441+ 8.441+ 8.441+ 8.441= 33.764 N/m
% error = 13%

% uncertainty vs. % error:

For 2 strands, the % error in our k-value is 14% while the % uncertainty is only 7%. Because % uncertainty is smaller than % error, the result for k-value of 2 strands is not accurate within experimental uncertainty. For 4 strands, the % error in our k-value is 13% and the % uncertainty is only 3%. This means that this result for k-value of 4 strands is also not accurate within experimental uncertainty.

Sources of uncertainty:

The most important source of uncertainty for this experiment was the possible error in setting the equilibrium position to exactly 0.260m for each number of strands tested. When 2 and 4 strands were tested, the cord did not hang straight downwards. In order to get the equilibrium position as precise as possible, the cord had to be straightened by hand, but it was difficult to get the cord as straight as possible without stretching it at all. If not completely straight or stretched to some degree, the equilibrium distance would not be precise and this would interfere with the calculations of k since k is dependent on equilibrium distance. Another source of uncertainty included that when 1 strand was tested, a knot had to be tied at the end of the cord in order to hold the hanging weight. The portion of the cord below the knot has a different k-value and was not accounted for. This could explain part of the uncertainty in the k-value calculated. In this experiment, we modeled the elastic cord as a spring and used Hooke's Law. The elasticity of the cord may not exactly match the spring constant of a spring, possibly explaining a portion of the uncertainty.

There are a few limitations of this experiment. One main limitation includes that the k-values calculated are only relevant for one equilibrium position, 0.260m. This equilibrium distance is not useful for the actual bungee challenge experiment since the drop will be much higher. Although this experiment was helpful to determine the relationship between k and the number of strands, the specific values of k will not transfer to a much longer equilibrium position. Further experimentation should be done. For example, a possible useful experiment would be keeping the mass and number of strands constant while varying the length of the equilibrium positions. A mass similar to the mass of the egg could be used and a relationship between this mass and increasing equilibrium positions would allow us to develop a model that would let us determine the k-value at the height of the bungee drop to be calculated.

The main results of this experiment show that k-values increase as the number of strands from which the mass is hung increases. Additionally, this experiment shows the relationship between the k-value and the number of strands. The k-value of 2 strands is approximately double that of 1 strand and the k-value of 4 strands is about double that of 2 strands despite not leading to results accurate within experimental uncertainty. Overall, these results support the hypothesis that was made at the outset of this experiment.

CONCLUSION:

In conclusion, our prediction that as the number of strands increases, the k-value of the elastic cord also increases was correct. The calculated k-value of 1 strand was 8.441 N/m, the k-value of 2 strands was 14.462 N/m, and the k-value of 4 strands was 29.308 N/m. Additionally, the relationship between k and the number of strands was shown. The k-value for 2 strands was approximately equal to the k-value for 1 strand doubled and the k-value for 4 strands was approximately equal to the k-value for 2 strands doubled. Implications of these conclusions tell us that for a given equilibrium position, the k-value will be greater for a larger number of strands, which tells us that for a certain mass, we can alter displacement from equilibrium by altering the number of strands. If we want the cord to stretch more and the mass to have a greater displacement (egg move closer to the ground in the bungee challenge), use fewer strands. However, if we want the cord to stress less and the mass to have a smaller displacement (egg stay farther from the ground), use more strands. Also, based on the relationship between number of strands and k-value, if we know the k-value of 1 strand for a certain equilibrium position, we can calculate the k-value of 2 strands, 4 strands, etc. for that same equilibrium.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: Andrea Ferrero