Lab Report Outline—Bungee 1

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Section: 6

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TITLE: Does the Spring Constant Vary with Length of Bungee Cord

ABSTRACT:

We have been given a bungee cord in lab and we need to determine as many of its properties as possible. We need to know about the properties of this cord because will be using it in a future lab for a bungee jumping competition. We hypothesize that the spring constant will vary with the length of the cord, and we do not yet know what length of the cord we will use for the challenge. The first aspect of the cord that we decided to measure is the spring constant, k, for varying lengths of the cord. This was done by measuring the displacement of different lengths of the cord when various masses were hung from the cord. The weight was then plotted against the displacement and we were able to obtain the k value for seven different lengths of cords. Once the k values were found, they were plotted against the unstretched length of the bungee cords. After linearizing the data, a model was found that shows how k varies with length for our bungee cord. In conclusion, we were able to experimentally obtain a model to use in the Bungee Challenge of the k value that corresponds to any given length of cord.

INTRODUCTION:

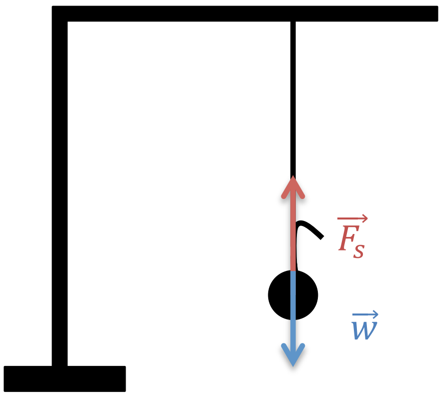
We are going to simulate a bungee jumping scenario where an egg is our dare devil, or "devilled egg," and a thin bungee cord is the tether. The egg will be dropped from a certain distance and the goal is to have the egg in free fall as long as possible with the acceleration not exceeding three times the force of gravity. In order to figure out how we are going to accomplish this, we must first find out different characteristics of our system, such as the "spring" constant of the bungee cord. The bungee cord acts as a simple harmonic oscillator, therefore we can use Hooke's Law in order to figure out the k constant of the cord. We predict that the k constant will be different for varying lengths of the cord, and since we do not know the length we will be ultimately using, we will test several different lengths of the cord to find the k constant(s). Ultimately, we will end up with an equation that can be used to find the k value of the bungee cord, given a specific length.

In order to determine the k value, we must use Hooke's Law. Hooke's Law is a "restoring force" law, which means that the further something is displaced, the more force it exerts back towards equilibrium. This is why we expect k to vary with the length of the bungee cord. The equation that we are going to use for this experiment is based off of Newton's Second Law as well as Hooke's Law. The equation is $F_S = kx$ where F_S is the force of the spring (our bungee cord), *k* is the spring constant of our bungee cord, and *x* is the displacement of our bungee cord. Because the mass is hanging off the bungee cord, there are only two forces acting on it: weight (mass times gravity) and the force of the spring (our bungee cord). This means that the vectors $\vec{F_S}$ and $\vec{w}$ are opposite of each other (as diagramed in the Methods section below). The magnitudes of these vectors will therefore be equal and we can use the weight of the masses hanging off the bungee cord as the value for the force.

METHODS:

Hooke's Law is a "restoring force" law, where the further something is displaced, the more force it exerts back towards equilibrium. A spring exhibits this behavior and can be modeled as a simple harmonic oscillator. In this experiment, we treat our bungee cord as a spring and therefore can model its behavior after a simple harmonic oscillator. Hooke's Law states that $F_S = kx$ where F_S is the force of the spring (our bungee cord), *k* is the spring constant of our bungee cord, and *x* is the displacement of our bungee cord.

Fig. 1: Diagram of Experimental Setup. *This setup was used to measure the displacement of the bungee cord with various weights attached to it.*



Procedure:

1. We first decided that we wanted to measure the k constant of our bungee cord
2. Since we do not yet know the length of bungee we are going to use for the challenge, we decided to test a wide range of bungee lengths (7 total), since the k value is expected to be different for each length
3. The stand we needed to use was already at our lab station so we made sure that the clasps were secure to the table
4. We then tied a slip knot to the top of our bungee cord and hung it on one of the pegs at the top of the stand
5. Next, we got out our measuring tape and hooked it on to the top of our stand and unwound it to rest on the ground, so that we would be able to easily measure our bungee cord
6. We then tied a slip knot into the bungee cord
7. Then we measured the length of the unstretched bungee cord (x_L) and recorded the length in meters
8. Next, we placed varying masses (from 0.05 to 0.20 kg) on the small loop at the end of our bungee
9. For each mass we added, we measured the length of the bungee cord at equilibrium (x_0) and subtracted the unstretched length (x_L) in order to get the displacement of the bungee cord (x)
10. We repeated steps 6 through 9 until we had measured 7 different lengths of the bungee cord
11. We recorded all of the data in Excel, making sure that everything was in proper SI units
12. In Excel, we multiplied all of the mass values (kg) by gravity ($9.81\frac{m}{s^2}$) to obtain the weight (force, N) acting on the bungee cord
13. Once all of the data was in the proper format to analyze it, we plotted weight (y-axis) versus displacement (x-axis) and obtained a linear line for each of the seven trial's data
14. We added a trendline to each graph to get the equation of the line through our data
15. The slope of the equation we obtained for each graph is the k value for the bungee cord at that length since $F_s = kx$
    a. F_s has the same magnitude as the weight acting on the bungee cord, these forces are acting in opposite directions as shown in the diagram of the setup
16. So now we have obtained k values for seven different lengths of our bungee cord
17. We want to be able to, for any length of bungee cord we use in the Bungee Challenge, find the k value associated with that length
18. To do so, we plot k (y-axis) versus the unstretched length of the bungee cord (x-axis), which gives us an exponential graph, and add a trendline to the graph
19. To linearize the data, we will plot k (y-axis) versus $\frac{1}{\text{length}}$ (x-axis) because in the initial equation had the length to the power of -1
20. Now, we have an equation that models the k value for various lengths of our bungee cord which will be present in the results section below

RESULTS:

In order to obtain our final model of the value of k with varying bungee lengths, we first had to determine the different k values of the bungee at various lengths. Figures 2 through 8 contain the data collected for the various lengths of the bungee cords. The unstretched length of the bungee cord was recorded as well as the displacement of the cord. The k value for each length was determined from the slope of the graph of weight versus displacement.

Fig. 2: Displacement of Bungee Cord at x_L = 0.113 m. *The slope of the graph of weight versus displacement, k=9.79 N/m, is the k constant of the bungee cord at this length.*

Displacement **x** (m) (±0.001)	Weight (N) (±0.001)
0.021	0.491
0.035	0.736
0.058	0.981
0.081	1.226
0.108	1.472
0.138	1.717
0.167	1.962

Fig. 3: ***Displacement of Bungee Cord at x_L = 0.205 m.*** *The slope of the graph of weight versus displacement, k=4.95 N/m, is the k constant of the bungee cord at this length.*

Displacement **x** (m) (±0.001)	Weight (N) (±0.001)
0.044	0.491
0.074	0.736
0.112	0.981
0.162	1.226
0.227	1.472
0.277	1.717
0.328	1.962

Fig. 4: ***Displacement of Bungee Cord at x_L = 0.344 m.*** *The slope of the graph of weight versus displacement, k=2.91 N/m, is the k constant of the bungee cord at this length.*

Displacement **x** (m) (±0.001)	Weight (N) (±0.001)
0.070	0.491
0.124	0.736
0.193	0.981
0.280	1.226
0.372	1.472
0.463	1.717
0.565	1.962

Fig. 5: ***Displacement of Bungee Cord at x_L = 0.505 m.*** *The slope of the graph of weight versus displacement, k=1.98 N/m, is the k constant of the bungee cord at this length.*

Displacement **x** (m) (±0.001)	Weight (N) (±0.001)
0.105	0.491
0.141	0.589
0.188	0.736
0.230	0.834
0.277	0.981
0.358	1.128
0.416	1.226
0.546	1.472
0.694	1.717
0.832	1.962

Fig. 6: ***Displacement of Bungee Cord at x_L = 0.690 m.*** *The slope of the graph of weight versus displacement, k=1.42 N/m, is the k constant of the bungee cord at this length.*

Displacement **x** (m) (±0.001)	Weight (N) (±0.001)
0.156	0.491
0.264	0.736
0.408	0.981
0.603	1.226
0.766	1.472
0.974	1.717
1.162	1.962

Fig. 7: ***Displacement of Bungee Cord at x_L = 0.863 m.*** *The slope of the graph of weight versus displacement, k=1.16 N/m, is the k constant of the bungee cord at this length.*

Displacement **x** (m) (±0.001)	Weight (N) (±0.001)
0.186	0.491
0.323	0.736
0.504	0.981
0.660	1.128
0.719	1.226
0.970	1.472
1.216	1.717

Fig. 8: ***Displacement of Bungee Cord at x_L = 0.942 m.*** *The slope of the graph of weight versus displacement, k=1.15 N/m, is the k constant of the bungee cord at this length.*

Displacement **x** (m) (±0.001)	Weight (N) (±0.001)
0.203	0.491
0.322	0.638
0.353	0.736
0.472	0.981
0.706	1.128
0.778	1.226
1.044	1.472

Once all of the k values are determined, we need to compare these values to the various lengths of the bungee cord. The data used in the graphs is shown in Figure 9.

Fig. 9: ***Length of Bungee Cord Compared to the Bungee Constant, k.*** *The raw data as well as the data used when the graph was linearized are both shown in this table.*

Length Trial Number	Unstretched Length x_L (m) (±0.001)	Bungee Constant **k** (N/m) (±0.01)	1/Length (m) (±0.001)
1	0.113	9.79	8.450
2	0.205	4.95	4.878
3	0.344	2.91	2.907
4	0.505	1.98	1.980
5	0.690	1.42	1.450
6	0.863	1.16	1.159
7	0.942	1.15	1.062

Fig. 10: ***k Value vs. Unstretched Bungee Cord Length.*** *This graph is not linear. The exponent tells us that this is an exponential graph and we can use 1/length in order to linearize our data.*

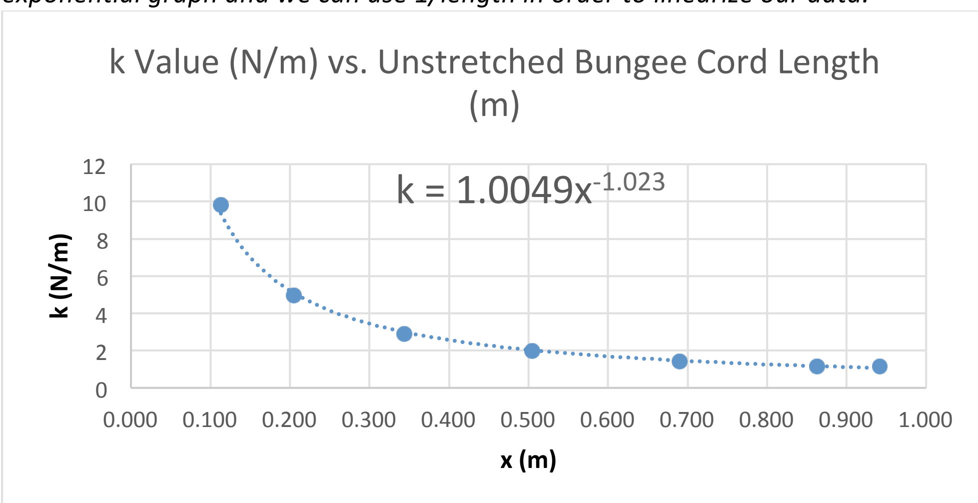
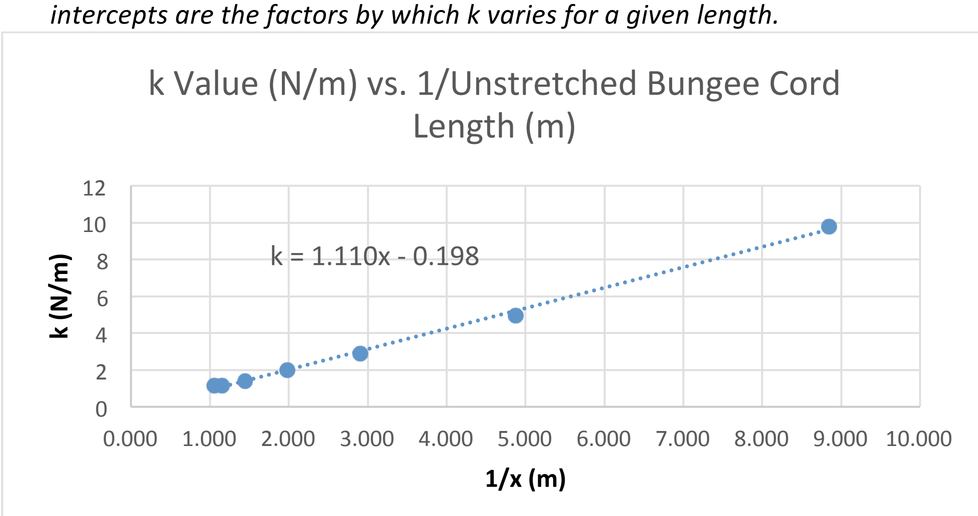


Fig. 11: ***k Value vs. 1/Unstretched Bungee Cord Length.*** *Linearized graph of the data where our slope and y-intercepts are the factors by which k varies for a given length.*



Equation 1: *Equation of the linearized graph of the data.*
k = 1.110x – 0.198

Uncertainty for slope= $\pm$ 0.02 N % uncert= $(\frac{uncertainty}{slope})$ * 100 = 1.8%

Uncertainty for y-intercept= $\pm$ 0.1 N/m % uncert= $(\frac{uncertainty}{slope})$ * 100 = 50.5%

Experimental Value of Interest: *The equation obtained from the k value versus 1/unstretched bungee cord length. This will give us a model we can use during the Bungee Challenge to find the k value of the length of cord we decide to use.*

Value Obtained = k = 1.110x – 0.198

Uncertainty for slope= $\pm$ 0.02 N % uncert= $(\frac{uncertainty}{slope})$ * 100 = 1.8%

Uncertainty for y-intercept= $\pm$ 0.1 N/m % uncert= $(\frac{uncertainty}{y-intercept})$ * 100 = 50.5%

Technique Used for Propagation of Uncertainty: *Regression Analysis on Excel.*

DISCUSSION:

There are no k values of our bungee cord available for comparison; therefore we must determine the "acceptability" of the uncertainty in our values. We will use the uncertainty in our k equation (k = 1.110x – 0.198) obtained from regression analysis in Excel to determine this acceptability. The calculations are shown above, in the Results section of this outline. The percent uncertainty of the slope is 1.8%, which is a very low uncertainty and can be accepted. In order to determine the significance of the y-intercept, we must compare the value of the y-intercept to the uncertainty of the y-intercept. The magnitude of the y-intercept is 0.1977 N/m and the magnitude of the uncertainty of the y-intercept is 0.1019 N/m. Since the y-intercept is basically the same as the uncertainty in the y-intercept, we can deem that the y-intercept in our equation is not significant. Based on our results, and the uncertainty in our results, we can deem that our k equation that we obtained is acceptable.

Sources of Uncertainty:
There are numerous places in our experiment where error could have been introduced. The masses we were using have been used for many physics labs and are therefore no longer exactly the mass that they claim to be. Another possibly source of error is that the reading for the length of the stretched bungee cord was supposed to be done at rest and sometimes it was difficult to determine if the cord was completely still. Even though we were tying slipknots into the cord, one of them got stuck and was tied into our cord for the entire experiment. We are assuming in our calculations that this knot has no effect on the cord when, in reality, the cord most likely is weakened by the knot in it. Also, the experiment called for us to stretch the cord by various weights. While it was not our intention to alter the integrity of the cord, the stretching may have weakened the cord as well.

Do the Main Results Support Our Hypothesis?
The main results do support our hypothesis in the sense that the k value does vary with different lengths of bungee cords. We were also able to obtain a model of the k value for different lengths of the bungee cord. These results were deemed "acceptable" based upon the uncertainties in the values.

CONCLUSION:

From our experiment, we were able to determine that the spring constant of the cord does vary with length of the bungee cord, as we expected. We were also able to obtain a model to determine the k value of our bungee cord depending on the length of the cord that we are using. Therefore, we will be able to use our spring constant equation in the Bungee Challenge to determine the k value of the length of the bungee cord we decide to use. Which will allow us to make more accurate decisions when it comes to our cord in the Bungee Challenge.

On my honor, I have neither given nor received any unacknowledged aid on this assignment.

Pledged: *Hayden Johnson*