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TITLE: The Affect of Length on the Force – Displacement Relationship of a Bungee Cord

ABSTRACT:

This experiment explores the relationship between the spring force and displacement of a bungee cord and to what extent the length of the cord affected that association. By assuming the bungee cord acts like a spring, we were able to use Hooke's law to find an equation that allows us to characterize the cord. We found the spring constant values for different cord lengths by measuring the displacement of a hanging mass. We then were able to graph these values against the corresponding un-stretched cord lengths to find an equation that would reveal the relationship between force and displacement. Our results supported our hypothesis so we can use these experimental equations later to determine the desired characteristics of our cord that would give us the best bungee jumping experience.

INTRODUCTION:

Bungee jumping is a thrilling experience that involves somebody free falling while attached to a cord. In order to design a proper experiment that would create the most thrill to a jumper without causing harm to him or her, we must first find out the characteristics of the bungee cord. The cord itself is made up of elastic material that will stretch out when pulled and return to the equilibrium position when released. The relationship between the force causing the displacement and the stretch of the cord depends on the type of material used. Some material will stretch farther than others without causing damage to the cord.

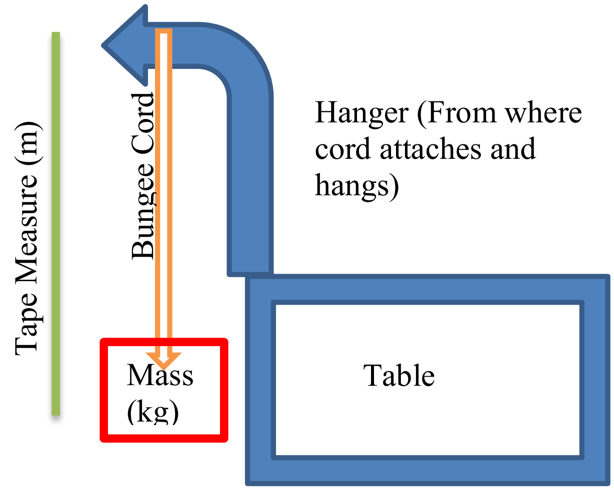
The force of a spring, also referred to as the spring force, can be defined as the amount of force required to stress or compress the spring. Hooke's law states that this spring force is equal to a "spring constant" multiplied by the displacement of the spring. The equation states that F_{spring} = -spring constant X displacement. We believe that a bungee cord behaves similarly to a spring. Thus we used the equation from Hooke's law to characterize our bungee cord. The net forces acting on our system included weight (the mass times gravity) and the spring force. Since our experiment was designed so that the system was not accelerating, we could conclude that the weight must be equal to the spring force when at rest. This would allow us to find the spring constant, k, for varying lengths of the cord.

We hypothesized that there is an inverse relationship between the spring constant and the length of the bungee cord. If we are correct, the length of the bungee cord will affect the relationship between the force and displacement. This information is important in creating the perfect bungee jump because it will allow us to figure the exact length for our cord so that there is as much force as possible without causing harm to the jumper.

METHODS:

This set up allows us to find the spring constant, k, of the cord given various lengths by allowing us to measure the displacement depending on the weight of the hanging mass. Since the system is at rest, the weight is equal to the spring constant X displacement.

Diagram 1. Set up of the hanging mass, attached to the bungee cord, at a given length.



A hanging mass was attached to the bottom of the bungee cord of length l. The cord was taped to a hanger so that the mass would not hit the ground when the cord stretched. A tape measure was used to measure the length of the cord, both with the hanging masses, and without them.

For our experiment, we first taped the bungee cord of a certain length to a device on a table. We measured the length of the cord by itself. We then attached a hanging mass of .005 grams and measured the corresponding displacement. We then attached more mass to the cord, and again recorded the displacement. We continued this process of adding mass and measuring displacement a total of 5 times (except for our first trial as explained later) before adjusting the length and repeating the same steps. We graphed the weight against the displacement to find the spring constants for each length. We then graphed these constants against the un-stretched length to find the affect of cord length on the relationship between force and displacement.

RESULTS:

This data was collected by measuring the displacement of a mass hanging on our bungee cord using eight different lengths. The first trial contains many different masses but due to time constraints, we only measured the displacement of five different masses for the following cord lengths. This data gave us a spring constant for each length which we then graphed against the un-stretched length of the bungee cord to find how length affected the relationship between force and displacement.

Fig. 1: The hanging mass, weight and corresponding displacement for a cord of length .625 (+/- .001 m) using raw uncertainties.

Hanging Mass (+/- .001 kg)	Weight of Hanging Mass (N)	Length of Cord (+/- .001m)
0	0	0.625
0.005	0.04905	0.662
0.025	0.24525	0.691
0.05	0.4905	0.761
0.07	0.687	0.833
0.1	0.981	0.989
0.12	1.177	1.151
0.14	1.373	1.232
0.16	1.57	1.39
0.18	1.766	1.54
0.2	1.962	1.672

Fig. 1: The hanging mass, weight, and corresponding displacement for a cord of length .877 (+/- .001 m) using raw uncertainties

Hanging Mass (+/- .001 kg)	Weight of Hanging Mass (N)	Length of Cord (+/- .001m)
0	0	0.877
0.05	0.049	0.894
0.5	0.4905	1.043
0.1	0.981	1.344
0.15	1.472	1.771
0.08	0.785	1.241

Fig. 2: The hanging mass, weight and corresponding displacement for a cord of length .710 (+/- .001 m) using raw uncertainties.

Hanging Mass (+/- .001 kg)	Weight of Hanging Mass (N)	Length of Cord (+/- .001m)
0	0	0.710
0.005	0.049	0.712
0.05	0.491	0.853
0.1	0.981	1.098
0.15	1.472	1.456
0.17	1.962	1.635

Fig. 3: The hanging mass, weight and corresponding displacement for a cord of length .625 (+/- .001 m) using raw uncertainties.

Hanging Mass (+/- .001 kg)	Weight of Hanging Mass (N)	Length of Cord (+/- .001m)
0	0	0.625
0.005	0.04905	0.662
0.025	0.24525	0.691
0.05	0.4905	0.761
0.07	0.687	0.833
0.1	0.981	0.989
0.12	1.177	1.151
0.14	1.373	1.232
0.16	1.57	1.39
0.18	1.766	1.54
0.2	1.962	1.672

Fig. 4: The hanging mass, weight and corresponding displacement for a cord of length .530 (+/- .001 m) using raw uncertainties.

Hanging Mass (+/- .001 kg)	Weight of Hanging Mass (N)	Length of Cord (+/- .001m)
0	0	0.53
0.005	0.049	0.56
0.05	0.491	0.663
0.1	0.981	0.862
0.15	1.472	1.401
0.2	1.962	1.465

Fig. 5: The hanging mass, weight and corresponding displacement for a cord of length .415 (+/- .001 m) using raw uncertainties.

Hanging Mass (+/- .001 kg)	Weight of Hanging Mass (N)	Length of Cord (+/- .001m)
0	0	0.415
0.005	0.049	0.426
0.05	0.491	0.501
0.1	0.981	0.642
0.15	1.472	0.861
0.2	1.962	1.091

Fig. 6: The hanging mass, weight and corresponding displacement for a cord of length .315 (+/- .001 m) using raw uncertainties.

Hanging Mass (+/- .001 kg)	Weight of Hanging Mass (N)	Length of Cord (+/- .001m)
0	0	0.315
0.005	0.049	0.332
0.05	0.491	0.393
0.1	0.981	0.502
0.15	1.472	0.662
0.2	1.962	0.832

Fig. 7: The hanging mass, weight and corresponding displacement for a cord of length .222 (+/- .001 m) using raw uncertainties.

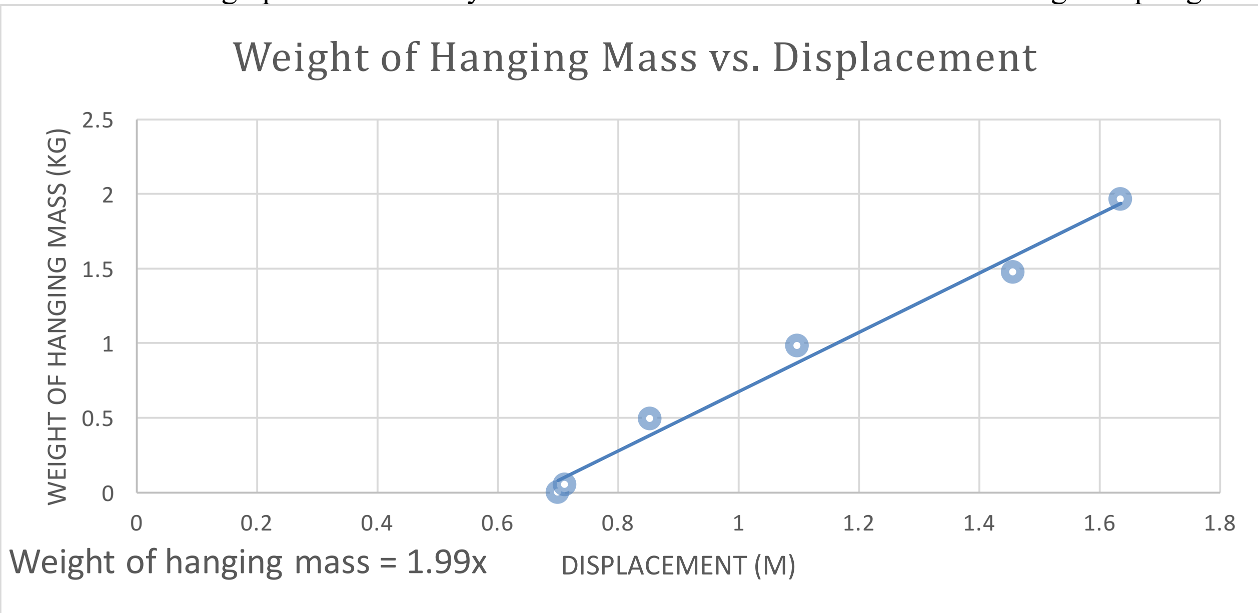
Hanging Mass (+/- .001 kg)	Weight of Hanging Mass (N)	Length of Cord (+/- .001m)
0	0	0.222
0.005	0.049	0.234
0.05	0.491	0.281
0.1	0.981	0.349

0.15	1.472	0.472
0.2	1.962	0.594

Fig. 8: The hanging mass, weight and corresponding displacement for a cord of length .175 (+/- .001 m) using raw uncertainties.

Hanging Mass (+/- .001 kg)	Weight of Hanging Mass (N)	Length of Cord (+/- .001m)
0	0	0.175
0.005	0.049	0.184
0.05	0.491	0.215
0.1	0.981	0.269
0.15	1.472	0.353
0.2	1.962	0.439

Fig. 9: Weight vs Displacement. The weight of the hanging mass and the corresponding stretch for a cord with length .710 (+/- .001 m). For this given length, the spring constant is 1.99. For each length of the cord, the displacement and hanging mass were graphed in the same method as shown below to give us differing values of k. This graph was randomly chosen to demonstrate our method of finding the spring constant.



Equation: Weight of hanging mass = 1.99x

Fig. 10: The spring force constants (using regression uncertainties) of each cord of un-stretched length (using raw uncertainties). The spring force for each length was found by graphing the weight of the hanging mass vs. the displacement for the eight differing cord sizes.

Spring Constant (K Value)	Length of Cord (+/- .001 m)
1.65 (+/- .16)	0.877
1.99 (+/- .12)	0.710
1.84 (+/- .11)	0.625
1.83 (+/- .23)	0.530
2.89 (+/- .26)	0.415
3.81 (+/- .30)	0.315
5.29 (+/- .45)	0.222
7.46 (+/- .59)	0.175

Fig. 11: Spring Constant vs. Length of the Cord. The equation of the best fit line suggests that the spring constant is equal to the force times the inverse of the length.

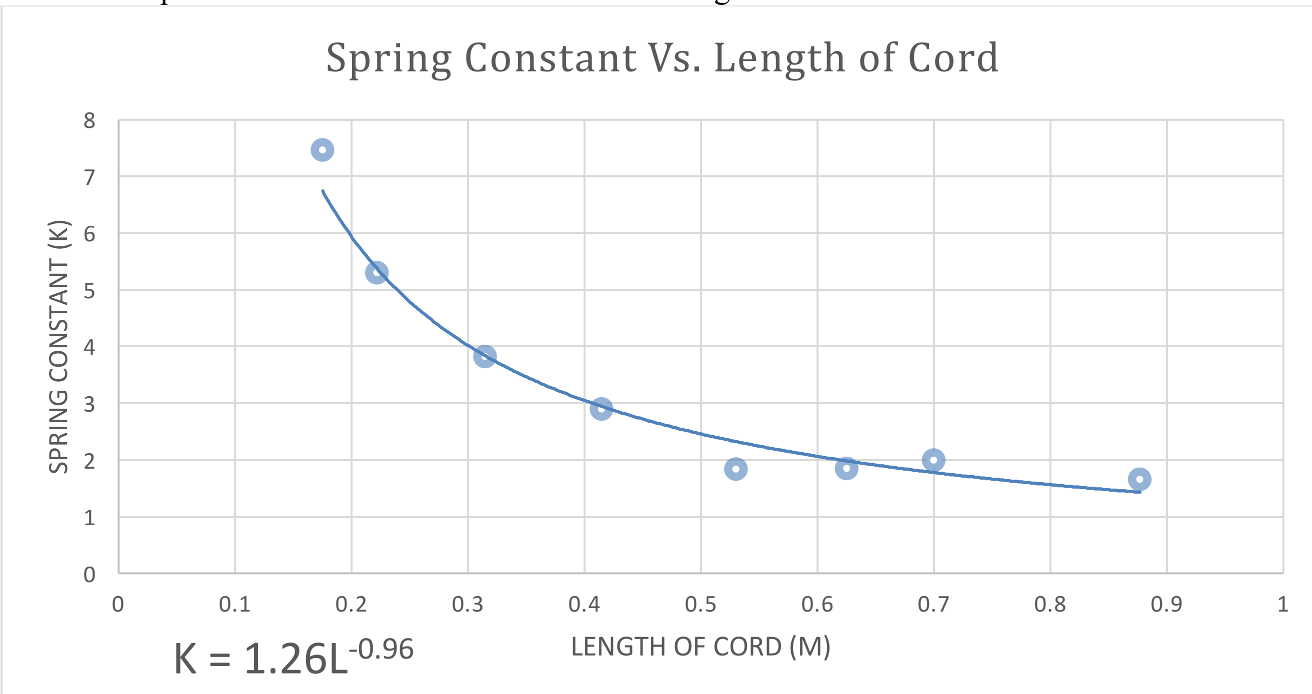
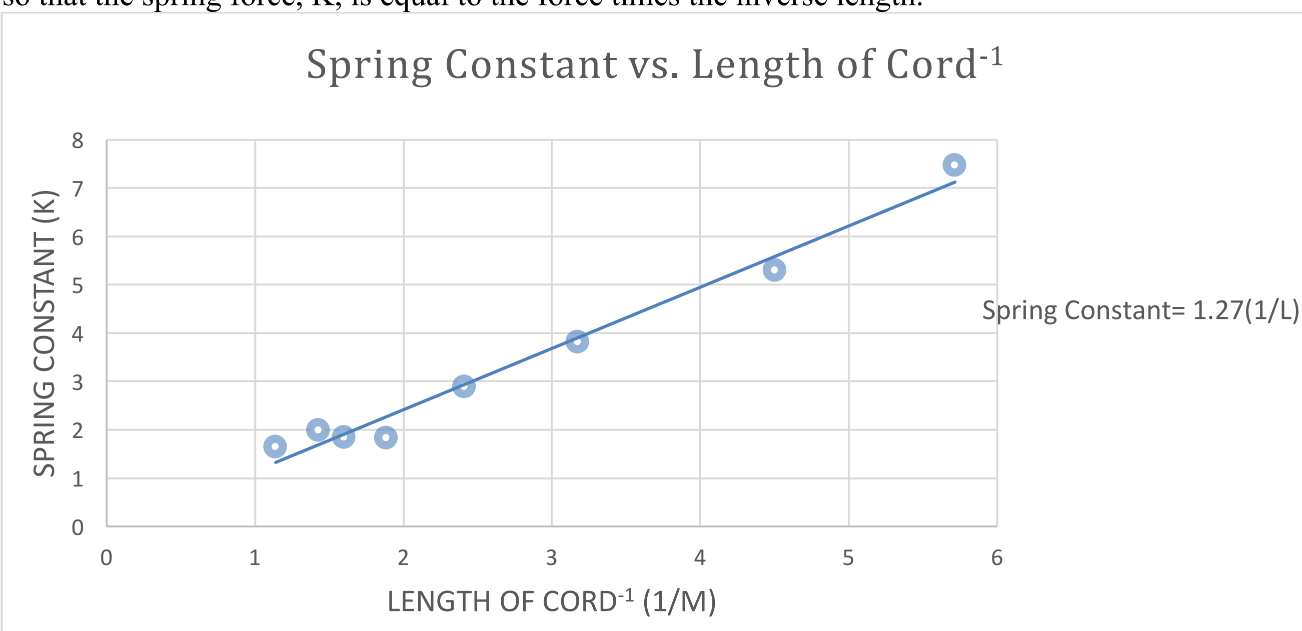


Fig. 12: Spring Constant vs. Inverse Length of Cord. The graph was linearized in order to fit the equation so that the spring force, K, is equal to the force times the inverse length.



Linearized equation: Spring Constant (K) = 1.267(1/L) with a .074 (or 5.83%) uncertainty in the slope coefficient.

Our first values of interest were the coefficients of x for our equations: weight = kx. This equation comes from Hooke's Law, which states that the spring force is equal to a spring constant times the displacement. The only forces acting on our system were weight and the spring force. Since the system was not accelerating, the net force was 0 so we can assume that the spring force is equal to the weight. The coefficient, k, was the spring constant for that particular length of the bungee cord. The values of each spring constant are listed in figure 10, with corresponding uncertainties found through regression analysis. Using this data, we could find the length's affect on the relationship between force and displacement.

Our next value of interest was the inverse length, L, in our linearized equation. The coefficient value was 1.27 with a .074 uncertainty (5.83%). Since in our original graph the spring constant equation took the shape of an inverse function, we were able to take 1 over each un-stretched length to linearize the graph and find a linearized equation. Since this coefficient is almost 1, the spring constant is equal to the inverse length of the cord. This indicates the affect of length on the relationship between force and displacement.

To summarize, we were able to find the relationship between the force and displacement of our bungee cord by measuring the displacement of a hanging mass for 8 different string lengths. This gave us 8 different K values that we used to determine how the length of the cord would affect the force and displacement as well. The spring constant will change according to the ratio of 1.27 (which is about 1) to the length of the cord.

DISCUSSION:

Based on our results, we have found an experimental relationship between the force and the displacement of our bungee cord, which varies according to the length of the cord. The spring constant is thus equal to the coefficient, 1.27 (+/- .074), divided by the length of the cord. By finding this relationship, we can figure out the necessary length we want for our cord, in order to compress it by a certain amount, K. The uncertainty of the inverse length coefficient is .074, or 5.83%, which was found using regression analysis. This means that our coefficient of the inverse length captures the true relationship between force and displacement about 94% of the time. Although this is not ideal, we have considered this an acceptable value of uncertainty, especially given our time constraints which would make it hard to repeat the experiment to find a more accurate coefficient. There is no accepted value for us to compare to our experimental value. However, we can test our equation by taking a new cord length and using the equation to predict the spring constant. We could then take this spring constant, K, and use it to predict the displacement, X, for a known weight of a hanging mass. If our prediction for the displacement is accurate, then we can conclude that there is in fact an inverse relationship between the spring constant and the length of the cord.

The data shows a trend between the value of the spring constant and the length of the cord. For a shorter length of bungee cord, we will have a much higher spring constant. For a longer cord, there will be a smaller spring constant. This trend is consistent with the fact that the equation demonstrates an inverse relationship. As one value increases, the other decreases. More importantly, this trend makes sense because a longer cord does not have to stretch as far to counteract the weight hanging from it.

Possible sources of uncertainty would result from the measuring of the displacement. It was difficult to measure the hanging mass when it was completely still. This would skew our data and give us inaccurate spring constant values, which would then result in an inaccurate coefficient of the inverse length. Another source of uncertainty would result from the cord being stretched out over time. Since we had to stretch the cord repeatedly, it's possible that towards the end of the experiment, the cord wasn't as elastic as it was to start. Additionally, the cord stretched the floor as the hanging mass became heavier, making it difficult to calculate a large range of values. With more space, and time, we could obtain more accurate measurements that had more variation.

Our experiment supported our hypothesis in that the bungee cord had a relationship that could be described by Hooke's Law. The weight was related to the spring force constant and the displacement through the linear equation, weight = kx. The law suggests that the relationship between the spring constant for any given length of cord was inversely related to the displacement of the cord. It thus makes sense that we found the relationship to be the spring constant = 1.27(1/L).

CONCLUSION:

The result of our experiment gave us an equation that determined the relationship between the force and displacement and how that relationship was affected by the individual length of the cord. These relationships allow us to characterize our bungee cord. In doing so, we can determine the necessary length of our cord to obtain a desired spring constant for a harmless but thrilling bungee jump. In later experiments, we will determine our desired values for the spring constant and length of the bungee cord.