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PHY113-02
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Defining the Relationship between Spring Constant and Spring Length of a Bungee Cord

Abstract

This report presents the relationship between an experimentally derived value for the spring constant (k) of a bungee cord and the initial length of the bungee. The former was derived via comparing changes in the bungee's length under the stress of a hanging mass. The hanging mass was varied over a range of up to 130 grams, including the 50-gram mass of the hanger itself. This allowed for the comparison between length of the cord under the stress of the mass hanger alone ($x_{initial}$) and the change in x over the range of masses, x_f correlating to the equilibrium point, where F_{weight} (Equation 3) equals F_{spring} (Equation 1). The slope of this graph is $k_{experimental}$. The length of the cord was also varied, in order to determine the initially mentioned relationship. This relationship is defined by the non-linear expression $y = 1.2908x^{-1.024}$. However, when linearized by raising the x-variable length to the power of x in the previous equation, the relationship is defined as $y = 1.3697x - 0.1188$, where the intercept is directly related to error in the experiment. Error may have been propagated by inaccurate measurements in length with raw uncertainty of ± 0.002 meters and an average statistical uncertainty of 0.146. It was more likely augmented, however, by expansion of the cord due to stress caused by the weight of the masses. It was observed that after placing the five masses on the hanger and recording the Δx for each mass, the cord expanded on average 0.001 meters. After five tests it expanded 0.5 cm, which is significant and is thus represented in the intercept value for the determined expression (Equation 4). After experimentation, it was concluded that the bungee cord acts with non-uniform force, such that the longer the cord the lesser the value of $k_{experimental}$. This result will be used to predict the actions of the cord under the stresses of a falling mass. In particular, this result will aid in predicting the stretch of the cord under the strain of the mass and inversely the magnitude of the cord's upward tension force on the mass.

Introduction

The experiment discussed in this report was designed to determine the relationship between the $k_{experimental}$ of a spring-like oscillator, a bungee cord, and the length of the cord used.

Equation 1: This is the equation for the force of a spring, also known as Hooke's Law, with variables substituted for their equivalent in the terms of this experiment. It was used here as it defines the relationship between $k_{experimental}$ and Δx.

$$F_{spring} = k_{experimental} \cdot \Delta x$$

Equation 2: This is Newton's First Law of Motion. It defines the relationship between two forces that are in mechanical equilibrium to be zero.

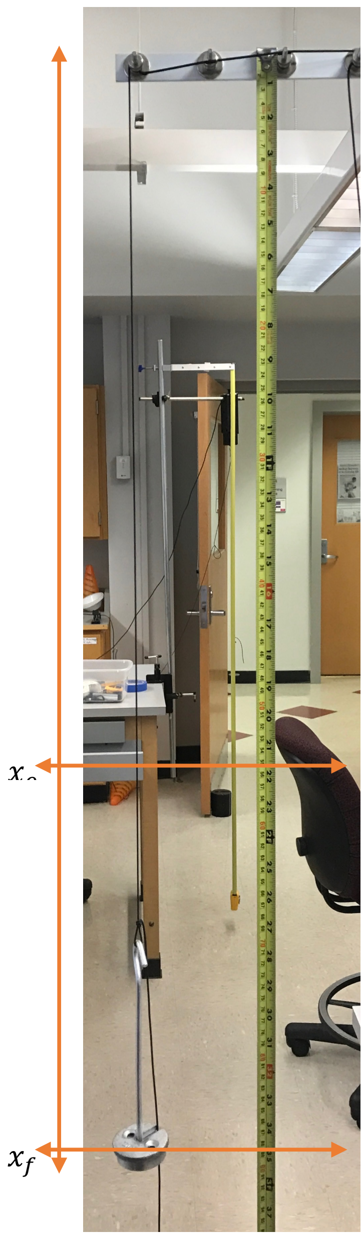
$F_{total} = 0$ in Mechanical Equilibrium

Equation 3: This is the equation for the force caused by weight. Which can be defined as the direct relationship between an object's mass and the force of gravity at its location.

$$F_{weight} = m_{total} \cdot g_{local}$$

In order to collect raw data for this experiment the system was placed into mechanical equilibrium with a known mass hanging from the cord. This means that the weight of the mass on the cord was equal to the spring force of the cord. In order to derive a value for $k_{experimental}$, the hanging mass was increased and the change in the position of the hanger, the Δx, was recorded. It was anticipated that as $Length_{cord}$ increased the value for $k_{experimental}$ would decrease, since the bungee cord does not act with uniform spring force.

Methods



The most important value quantified in this report, in terms of deriving a value for $k_{experimental}$, is the change in the location of the equilibrium point as mass was added to the hanger, depicted in Figure 1. The $Length_{cord}$ was measured initially with no added mass, then measured with only the mass of the hanger, and finally with the added mass. The masses added were kept constant for each test, in order to eliminate any variants in tests. The $Length_{cord}$ with no added mass was also measured after the final Δx for the last length was recorded in order to approximate the amount the cord stretched in the duration of the tests. This value was minimal, yet not negligible.

Figure 1: The set up pictured to the left was used for this experiment. The cord and the measuring tape are held by nuts against the white plastic backing seen at the top of the image. The cord is supporting both the mass of the hanger, seen in the bottom left of the image, and a mass of 50 grams, for a total of 100 grams. This system eliminates large amounts of error caused by measurement by holding both the measuring tape and the cord at even heights.

Experiment 1: Relating Weight and Δx to obtain $k_{experimental}$

- STEP 1: Choose a set of 5 lengths and a set of 5 masses to use for static testing.
- STEP 2: Using the backing piece picture at the top of Figure 1, attach the cord at the first length and the measuring tape.
- STEP 3: Record the length of the cord
- STEP 4: Place the mass hanger from the cord, and record the length value as $x_{initial}$.
- STEP 5: Add the first mass to the hanger, and record the x_{final} length of the cord.

- STEP 6: Determine and record the change from $x_{initial}$ to x_{final} as Δx.
- STEP 7: Quantify the weight force using Equation 3.
- STEP 8: Repeat STEPS 4-7 for the four remaining masses selected to be used for testing.
- STEP 9: Repeat STEPS 3-8 for the four remaining lengths selected.
- STEP 10: With all the raw data now collected, the values found for F_{weight} and Δx, they can be plotted against the other for each length, in order to obtain a value for $k_{experimental}$ in the form of slope.
- STEP 11: With all values of $k_{experimental}$ derived from the raw data (the slope of Figures 2-6), now relate the $k_{experimental}$ values to the lengths at which they apply by graphing.
- STEP 12: The previous step will compute a power relationship. Linearize the graph by raising the $Length_{cord}$ to the power of the previous graph's x-variable (example can be seen in Figure 7).

Results

The quantities found in this experiment rely basically on the values Δx and F_{weight}. The former was measured using a measuring tape, taking the initial and final measurements. The latter was calculated using the m_{total} of the system and the known value of gravity near Earth. These two measures were then used to derive the desired value $k_{experimental}$, using linear graphing.

Table 1: This table contains the raw data collected for the first $Length_{cord}$ tested. It shows the range of masses used, which will remain consistent for each length, the changes in x, and the relationship of m_{total} and gravity, or F_{weight}. The latter two values were used to determine the $k_{experimental}$ for the $Length_{cord}$ examined.

$Length_{cord}$ (m)	m_{hanger} (kg)	$x_{initial}$ (m)	x_{final}(m)	m_{added}(kg)	Δx (m)	m_{total}(kg)	F_{weight}(mg) (N)
0.452	0.05	0.548	0.578	0.01	0.03	0.06	0.5886
			0.604	0.02	0.056	0.07	0.6867
			0.633	0.03	0.085	0.08	0.7848
			0.687	0.05	0.139	0.1	0.981
			0.799	0.8	0.251	0.13	1.2753

Figure 2: This graph plots the F_{weight} and Δx, or F_{spring}, founded in the first test of $Length_{cord}$, the data for which is found in Table 1. The linear trendline gives the slope, which is equivalent to the $k_{experimental}$, that in this case was 3.1 ± 28.5%. The slope was not adjusted for error in the intercept value.

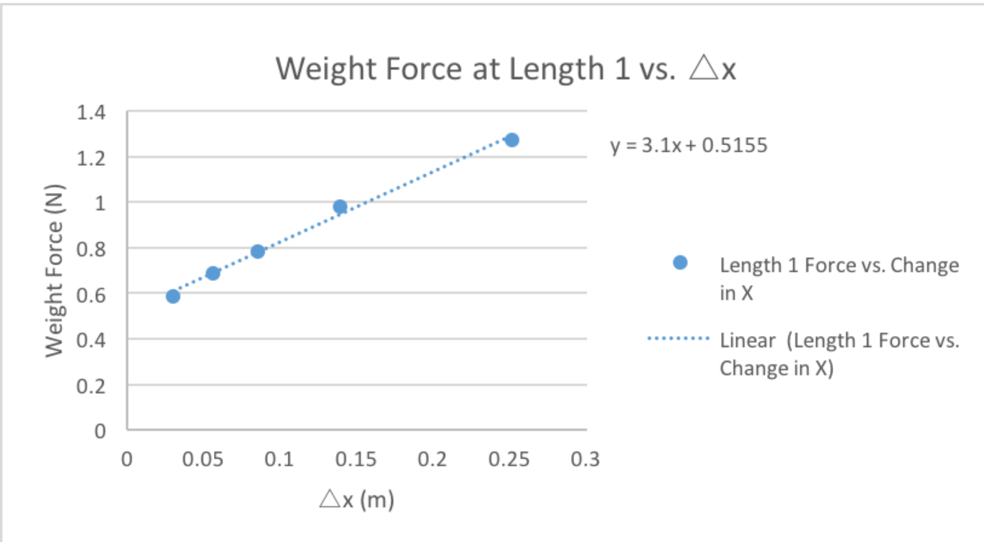


Table 2: The variability of the X-variable, $k_{experimental}$ for Length 1, for which the information is contained in Table 1, is shown here in the form of standard error in the set of data, obtained via regression analysis.

	Coefficients	*Standard Error*
Intercept	0.515459487	0.02001562
$k_{experimental}$	3.100004569	0.146303788

Table 3: This table contains the raw experimental data for the second $Length_{cord}$ tested, which was 0.539 $\pm$ 0.001m. This data was used in the same way as the data in Table 1, to determine a value for $k_{experimental}$ at this length of cord.

$Length_{cord}$ (m)	m_{hanger} (kg)	$x_{initial}$ (m)	x_{final}(m)	m_{added}(kg)	Δx (m)	m_{total}(kg)	F_{weight}(mg) (N)
0.539	0.05	0.652	0.689	0.01	0.037	0.06	0.5886
			0.71	0.02	0.058	0.07	0.6867
			0.741	0.03	0.089	0.08	0.7848
			0.831	0.05	0.179	0.1	0.981
			0.958	0.08	0.306	0.13	1.2753

Figure 3: The relationship between the force of F_{weight} and the Δx for the second length tested is shown here in Figure 2. The slope of the graph is equivalent to the value for $k_{experimental}$ for the length of cord tested.

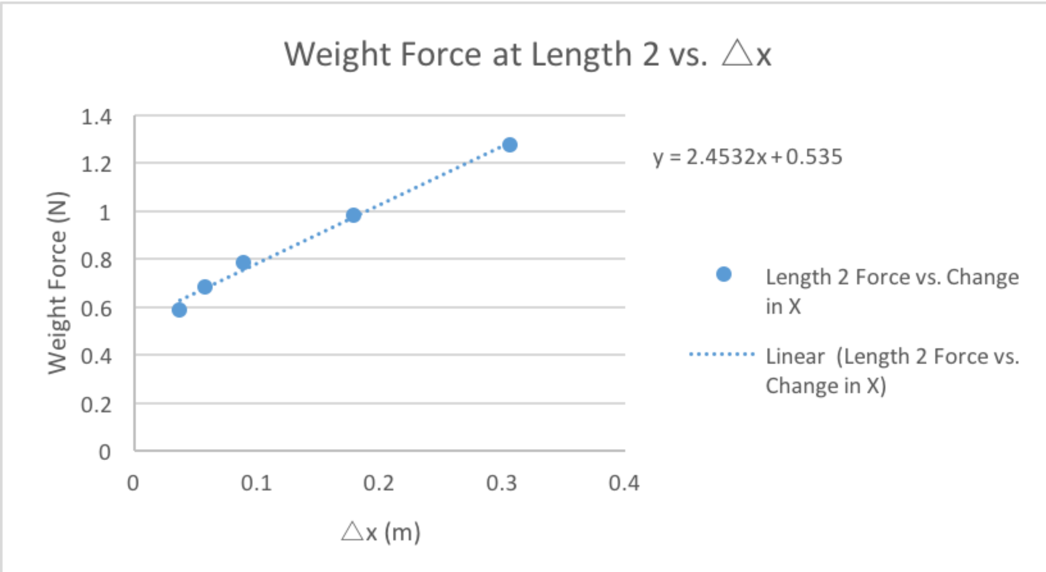


Table 4: This table displays the standard error for $k_{experimental}$ determined for the second $Length_{cord}$, 0.539 $\pm$ 0.001m. The relationship between the two k values determined thus far shows that as Length increases, $k_{experimental}$ decreases. This relationship will appear in later analysis.

	Coefficients	*Standard Error*
Intercept	0.535046774	0.022226577
$k_{experimental}$	2.453163126	0.133628109

Table 5: The raw data for the third $Length_{cord}$, 0.152 ± 0.001 m, can be found in the table below. Like the first and third tables, this data was used to derive a value for $k_{experimental}$.

$Length_{cord}$ (m)	m_{hanger} (kg)	$x_{initial}$ (m)	x_{final}(m)	m_{added}(kg)	Δx (m)	m_{total}(kg)	F_{weight}(mg) (N)
0.152	0.05	0.198	0.208	0.01	0.01	0.06	0.5886
			0.216	0.02	0.018	0.07	0.6867
			0.228	0.03	0.03	0.08	0.7848
			0.248	0.05	0.05	0.1	0.981
			0.284	0.08	0.086	0.13	1.2753

Figure 4: The relationship between F_{weight} and Δx can be found here in this graph, in the same way that this relationship is conveyed in the previous Figures one and two. The slope of the graph is equal to the $k_{experimental}$ value for the third length used for experimentation.

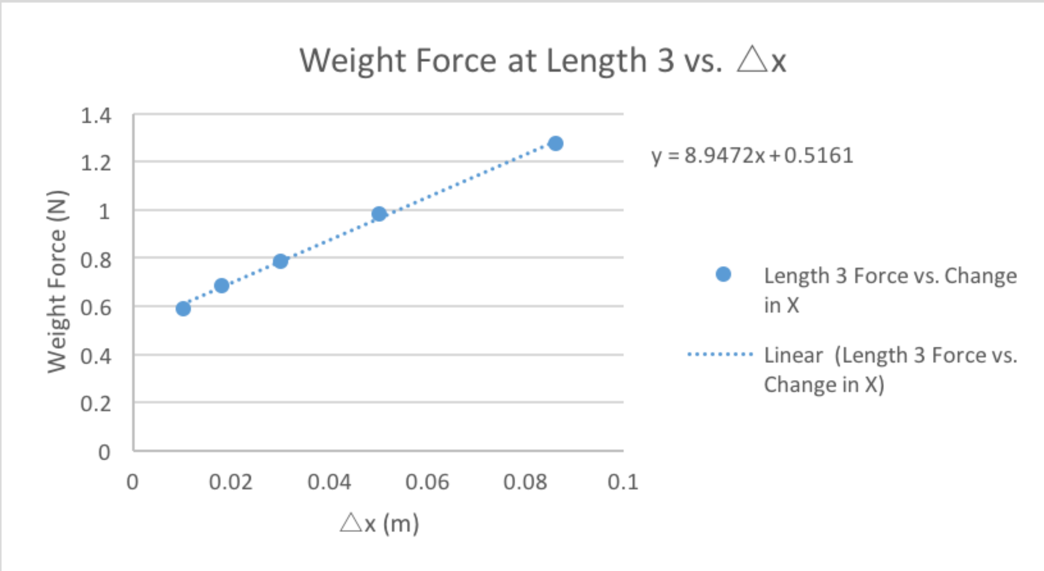


Table 6: The regression analysis for the data shown in Figure 3 is shown here. The standard error in this particular table is higher than the other error values sited in this report. This is in part due to this length of cord being much shorter than the other lengths tested, leading to higher error in measurement.

	Coefficients	*Standard Error*
Intercept	0.516130113	0.012667789
$k_{experimental}$	8.947162045	0.267417229

Table 7: This table contains the raw data for the fourth $Length_{cord}$ used in this experiment. The data will be analyzed in the same way as that found in Tables one, three and five.

$Length_{cord}$ (m)	m_{hanger} (kg)	$x_{initial}$ (m)	x_{final}(m)	m_{added}(kg)	Δx (m)	m_{total}(kg)	F_{weight}(mg) (N)
0.609	0.05	0.732	0.75	0.01	0.018	0.06	0.5886
			0.8	0.02	0.068	0.07	0.6867
			0.842	0.03	0.11	0.08	0.7848
			0.931	0.05	0.199	0.1	0.981
			1.086	0.08	0.354	0.13	1.2753

Figure 5: This figure plots the values for the F_{weight} and the Δx found in Table 7. The slope of the linear trendline given to this graph, as in each Figure proceeding this one, is equivalent to the $k_{experimental}$ for the $Length_{cord}$ used, 0.609 ± 0.001m.

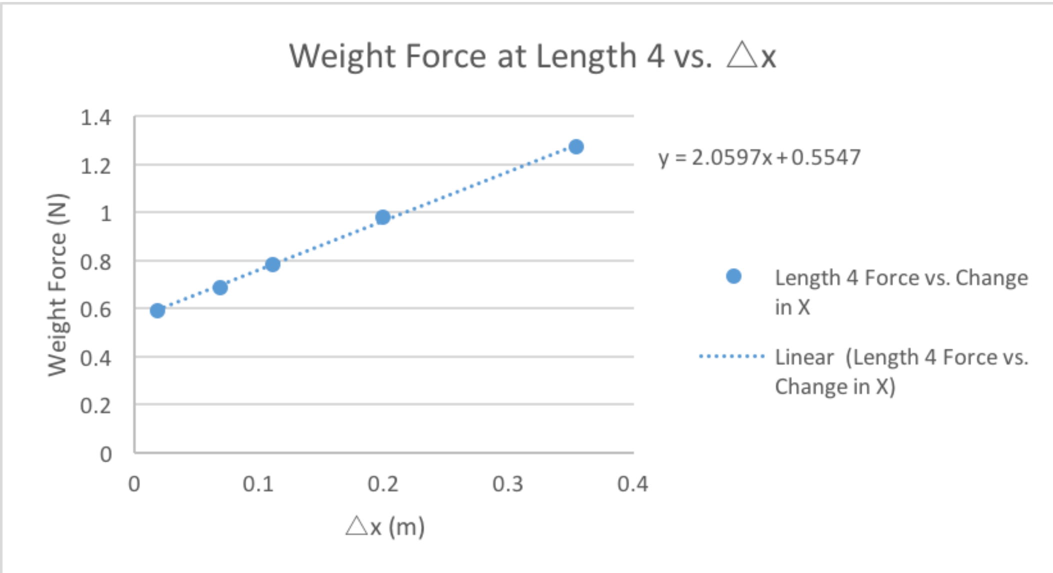


Table 8: This table contains the regression data for the fourth $Length_{cord}$ examined, expressing the standard error in the slope of the graph.

	Coefficients	*Standard Error*
Intercept	0.554731955	0.008646402
$k_{experimental}$	2.059733275	0.045323819

Table 9: The data for the fifth, and final, $Length_{cord}$, 0.283 ± 0.001m, is shown below. This data will be used to derive a value of $k_{experimental}$ for the length.

$Length_{cord}$ (m)	m_{hanger} (kg)	$x_{initial}$ (m)	x_{final}(m)	m_{added}(kg)	Δx (m)	m_{total}(kg)	F_{weight}(mg) (N)
0.283	0.05	0.349	0.363	0.01	0.014	0.06	0.5886
			0.377	0.02	0.028	0.07	0.6867
			0.4	0.03	0.051	0.08	0.7848
			0.443	0.05	0.094	0.1	0.981
			0.512	0.08	0.163	0.13	1.2753

Figure 6: The values listed in Table 9 are graphed below. The slope of the graph, as before, is equivalent to $k_{experimental}$.

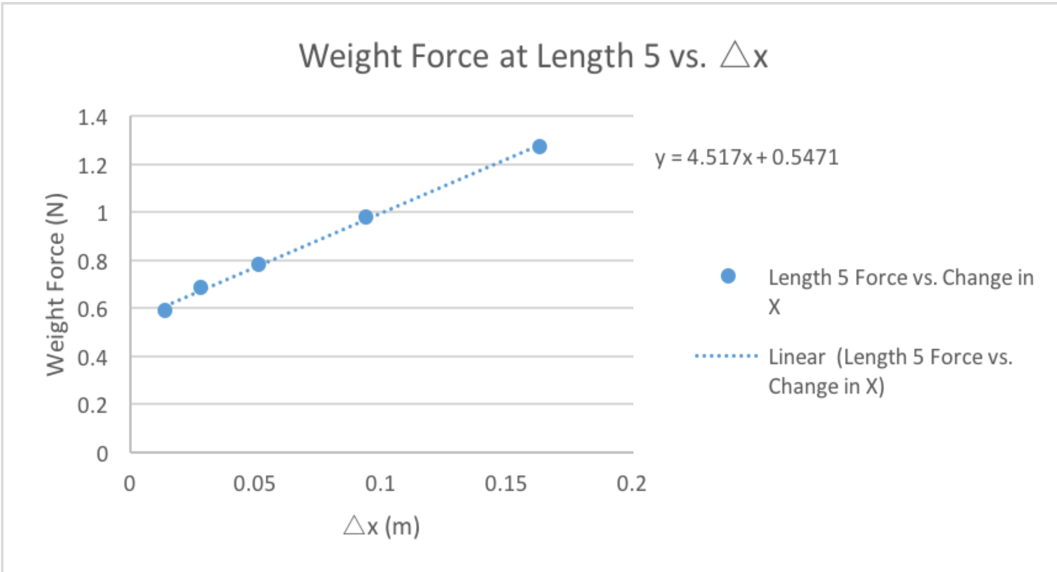


Table 10: This table displays the regression data for Figure 5.

	Coefficients	*Standard Error*
Intercept	0.547092716	0.012350948
$k_{experimental}$	4.516961204	0.139872054

Table 11: The values for $k_{experimental}$ are collected here and placed alongside the $Length_{cord}$ for which they were recorded. The relationship between these two values is important as it will allow for approximation of the behavior of the cord with respect to its length. Hence this data will be used to derive an experimental force equation for the cord used in this report.

$k_{experimental}$	$Length_{cord}$ (m)
8.9472	0.152
4.517	0.283
3.1	0.452
2.4532	0.539
2.0597	0.609

Figure 7: The decaying power function nature of the relationship between $k_{experimental}$ and the $Length_{cord}$ can be seen in the graph below. This means that as length increases along the x-axis the spring constant (k) of the cord decreases in its efficiency. The equation $y = 1.2908x^{-1.024}$ defines the relationship between the two quantities as a power function.

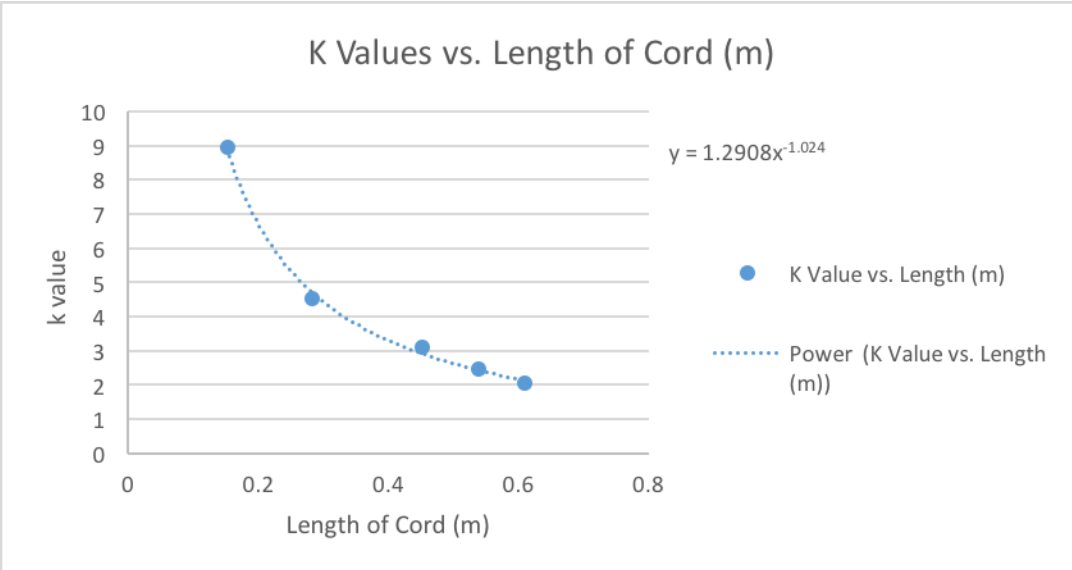


Figure 8: The linearized function shown below is created by raising the $Length_{cord}$ to the negative first power. The intercept of the trendline was not adjusted for error, hence the error is represented in the equation for the relationship between the spring constant (k) and the length of cord used.

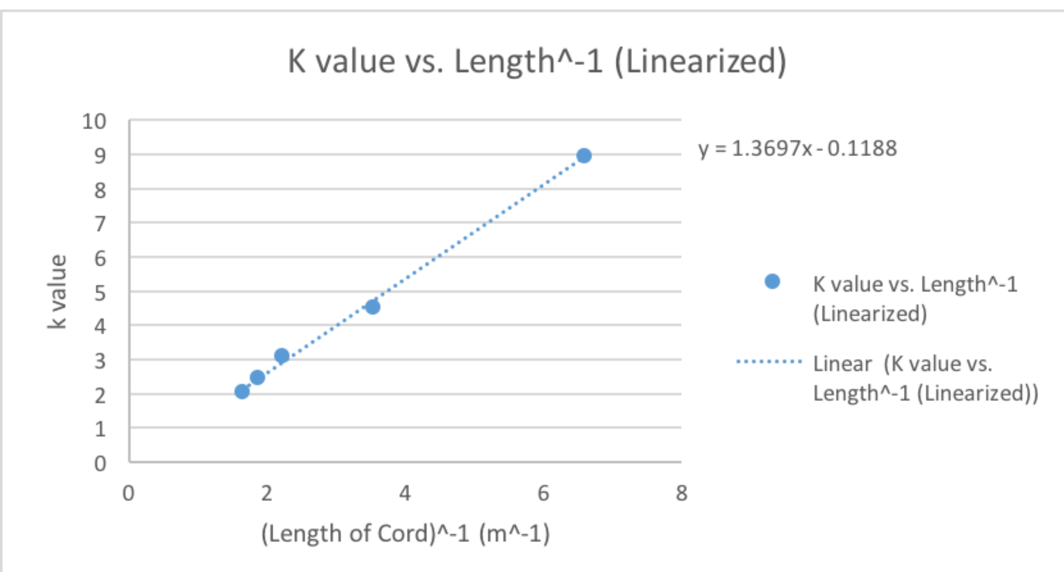


Table 12: The regression data for the linearized function found for the relationship between the spring constant (k) and the length of cord used, shown in Figure 7, is recorded in this table. The standard error is low which is ideal as this data set is derived from the whole set of raw data collected for this report.

	Coefficients	*Standard Error*
Intercept	-0.118827928	0.151408611
$k_{experimental}$	1.369670428	0.041423026

The objective of this experiment was to derive an expression that estimates the relationship between the $k_{experimental}$ of the bungee cord and the length of the cord used. Equation 4, below, is the determined expression corresponding to that idea.

Equation 4: This is the equation of the linearized function relating the spring constant of the bungee cord and the length corresponding to that $k_{experimental}$ value.

$$y = 1.3697({Length_{cord}}^{-1}) - 0.1188$$

Discussion

Error was propagated in the expression derived the relationship between the spring constant (k) and the length of cord used, shown in Equation 4, from multiple sources. Dissemination of error occurred through inaccurate measurements of length, previously quoted to have a raw uncertainty of ± 0.001 meters, and through the expansion of the bungee cord due to strain from weight. The latter is a fundamental procedural flaw. Since the function defined in Equation 4 draws directly upon the length of the cord being known, any changes in length hence principally inhibit the expression from predicting a $k_{experimental}$ value at any length with accuracy. It was found that after the stresses of one test of a full range of five masses at a medium length the cord expanded 0.001 meters. Over the course of the five tests of a full range of masses at differing lengths conducted in this report, therefore, the cord expanded a measured 0.005 meters, which, while not a sizeable percentage of the total $Length_{cord}$ (~0.25%), is not negligible. Another source of error may have been the masses added to the cord, which were labeled only to the ones place, and hence have an uncertainty in mass of $\pm$ 0.01 kg.

Equation 4, despite sources of experimental error, does fit to the expectations that were set forth at the beginning of the experiment. It was known that the relationship between $k_{experimental}$ and $Length_{cord}$ would be non-linear, since the bungee cord is a non-uniform spring. Equation 4, thus, matches theory.

Conclusion

This experiment was principally designed to better understand the nature of the bungee cord used. In relating different values for F_{spring} at different cord lengths, the graphs for which are found in Figures 2-6, it was discerned that the bungee acts with a non-uniform force, and this is crucial to understand moving forward. This data will be used to predict the force on an object as it falls and is caught by the bungee cord, and also predict the stretch that will occur in the cord by doing so, based on the weight added to the cord.