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Measuring Displacement of Bungee Cord at Varying Mass

Abstract: We developed a model to find the relationship between the mass of an object dropped and the displacement when attached to an elastic cord at constant length to find the spring constant k of our cord. We hypothesized that our value of k would accurately depict our cord's actual value of k. We dropped a hanger containing various masses from the top of our bungee setup, and measured the total displacement undergone at each mass. By using Hooke's Law, $mgh=1/2kx^2$, we found k to be equal to $(2mgh)/x^2$. In context of our model, we remodeled this equation to $k=(2mgX_{max})/(X_{max}-X_L)^2$, where X_{max} was the maximum elongation of the cord and X_L was the un-stretched length of the cord. We analyzed the relationship between $(X_{max}-X_L)^2$ and $2mgX_{max}$, and found that our experimental value of interest, k was equal to 2.63 N/m when our cord length was 0.36 m. With this value of k, we will be able to predict the maximum distance a given mass will fall at a cord length of 0.36 m.

Introduction: We dropped hangers with various masses attached to an elastic cord, and measured the total displacement at each mass to solve for the spring constant k. We used Hooke's Law, $mgh=1/2kx^2$, and then remodeled it to represent our setup, $k=(2mgX_{max})/(X_{max}-X_L)^2$, where m is the mass of the hanger being dropped in kilograms, g is gravity (9.81 m/s^2), X_{max} is the length of the cord when the hanger was at its lowest point in meters, and X_L is the un-stretched length of the cord in meters. We hypothesized that our model would be relatively accurate in determining the value of k at a set length of elastic cord (X_L).

Methods: We attached a 0.05 kg hanger to the end of an elastic cord of length 0.36 m and dropped it from the top of our model (the equilibrium point). We varied mass by adding different masses to the hanger and kept the length of the cord constant. We recorded the displacement of the hanger at each mass and then used our results to find k.

Figure 1: Identifying Variables of remodeled Hooke's Law Equation

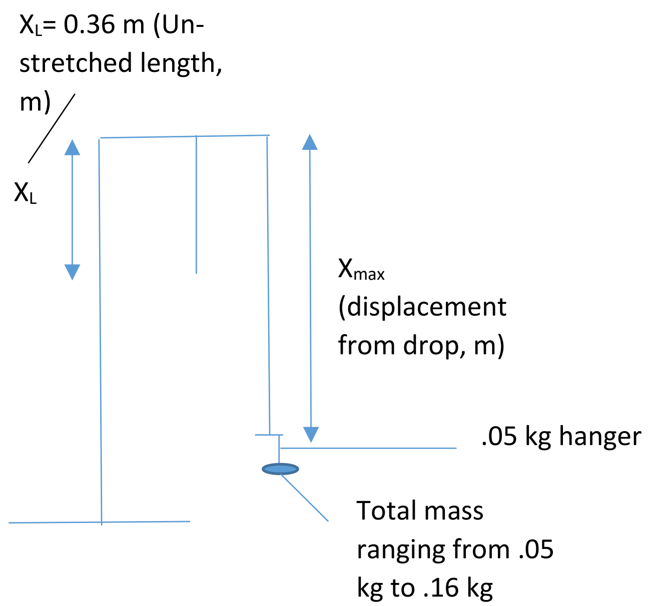
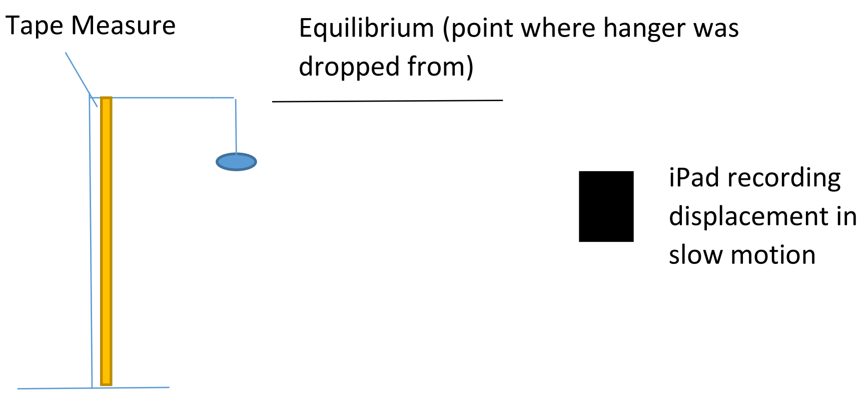


Figure 2: Experimental Setup



We attached the hanger to the elastic cord by tying two knots, one attaching the cord to the top of the stand, and the other attaching the cord to the hanger. We dropped the hanger from the equilibrium point with 10 different masses, where the total mass ranged from .05 kg to .16 kg, and recorded the displacement for each mass through use of slow motion video on iPad.

Results: We recorded the maximum distance each mass fell when attached to our elastic cord, and analyzed the relationship between $(X_{max}-X_L)^2$ and $2mgX_{max}$. By comparing those two components of Hooke's Law graphically, the slope of the graph gave us our experimental value of interest, k.

Figure 3: Raw Data and components of modified Hooke's Law equation at cord length 0.36 m

Mass of hanger plus additional mass (±0.0001 kg)	X_{max} (total height of fall), (±0.05 m)	Weight of hanger plus additional mass (N)	X max- XL (total height of fall minus un-stretched cord length, m)	2mg*(Xmax) (N*m)	(X max-XL)^2 (m^2)
0.05	0.788	0.491	0.428	0.773	0.183
0.07	0.928	0.687	0.568	1.275	0.323
0.09	1.118	0.883	0.758	1.974	0.575
0.1	1.238	0.981	0.878	2.429	0.771
0.11	1.308	1.079	0.948	2.823	0.899
0.12	1.368	1.177	1.008	3.221	1.016
0.13	1.448	1.275	1.088	3.693	1.184
0.14	1.598	1.373	1.238	4.389	1.533
0.15	1.648	1.472	1.288	4.850	1.659
0.16	1.778	1.570	1.418	5.581	2.011

Figure 4: Graph of $(X_{max}-X_L)^2$ vs. $2mg(X_{max})$ for Cord Length 0.36 m

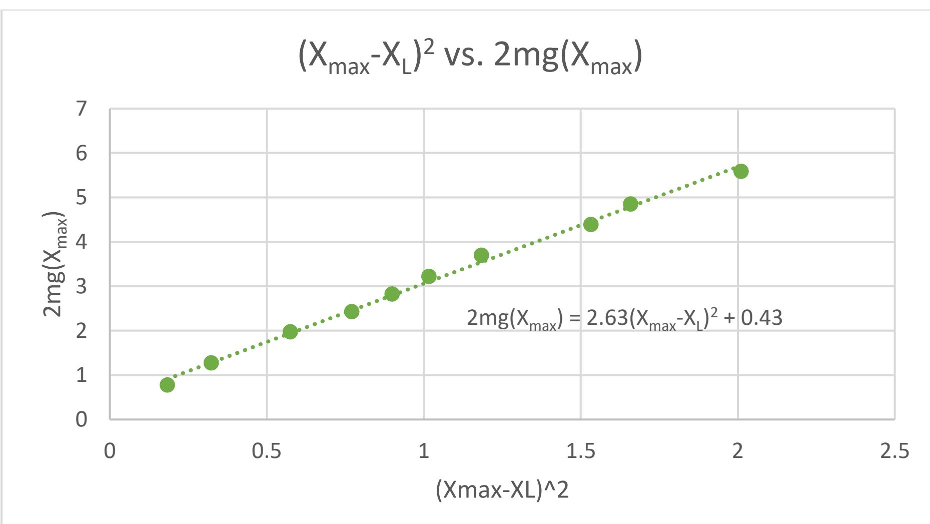
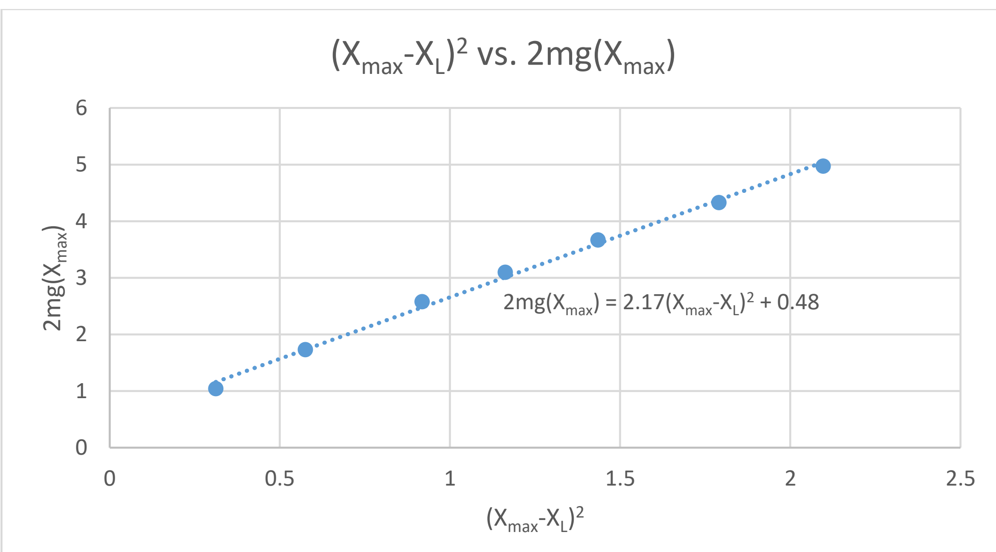


Figure 5: Raw data and components of modified Hooke's Law equation at cord length 0.50 m

Mass of hanger plus additional mass (±0.0001 kg)	X_{max} (total height of fall), (±0.05 m)	Weight of hanger plus additional mass (N)	X_{max}-X_L (total height of fall minus un-stretched cord length, m)	2mg*(X_{max}) (N*m)	$(X_{max}-X_L)^2$ (m^2)
0.05	1.058	0.491	0.558	1.038	0.311
0.07	1.258	0.687	0.758	1.728	0.575
0.09	1.458	0.883	0.958	2.575	0.918
0.1	1.578	0.981	1.078	3.096	1.162
0.11	1.698	1.079	1.198	3.665	1.435
0.12	1.838	1.177	1.338	4.327	1.790
0.13	1.948	1.275	1.448	4.969	2.097

This was not part of our main experiment, rather it was a test to see how k would differ with increased cord length.

Figure 6: Graph of $(X_{max}-X_L)^2$ vs. $2mg(X_{max})$ for Cord Length 0.50 m



Our test showed us that as cord length increases, the value of k decreases.

- Equations:
    - When X_L=0.36 m, $2mg(X_{max}) = 2.63(X_{max}-X_L)^2 + 0.43$
        - Experimental Value of Interest (k)= coefficient of $(X_{max}-X_L)^2$= 2.63 N/m
    - When X_L=0.50 m, $2mg(X_{max}) = 2.17(X_{max}-X_L)^2 + 0.48$
        - Experimental Value of Interest (k)= coefficient of $(X_{max}-X_L)^2$= 2.17 N/m
- Uncertainties:
    - When X_L=0.36 m, slope= 2.63 N/m and y-intercept= 0.43
        - Slope Uncertainty= ±0.06 N/m= 2.3% error
        - Y-intercept Uncertainty= ±0.07= 16.3% error
    - When X_L=0.50 m, slope= 2.17 N/m and y-intercept= 0.48
        - Slope Uncertainty= ±0.06 N/m= 2.8 % error
        - Y- intercept Uncertainty= ±0.08= 16.7 % error

Our experimental value of k, or slope of the graph, calculated to be 2.63 N/m when the un-stretched cord length, or X_L, was 0.36 m.

Discussion:

- By using Excel regression analysis, we found our uncertainty of the slope to be ±0.06 N/m when cord length is 0.36m, which equates to a 2.3 percent error. Ideally, we would want there to be no error in our value of k, but a 2.3 percent error is not of major significance for the sake of our model, and is therefore an acceptable value. We did have a relatively high percent error in our y-intercept of 16.3 percent. Ideally, the line of our linear equation would travel directly through the origin, with no y-intercept. This high error is most likely from a variety of sources of uncertainty that pertained to the measurements made in our experiment.

- Some sources of uncertainty may have arisen in our experiment to cause error in our experimental values. For one, the knots attaching the cord to the stand and the cord to the hanger may have tightened and loosened when we dropped the hanger from the top of our model. This would cause displacement measurements to be slightly off. Also, there may have been error in our interpretations of the displacement when using slow motion video on the iPad. Occasionally, we did not have the iPad positioned in the perfect spot, which caused there to be a slight parallax present when trying to measure X_{max}, therefore skewing measurements. We may have also dropped the hanger at slightly different heights which may have caused measurements of X_{max} to be slightly off.
- Our results supported our hypothesis in that our experimental value of k accurately depicted our cord's actual value of k. Ultimately, we will not know if this is true until we test our experimental values in the Bungee jump, but our low percent error makes us believe that our value of k is a relatively accurate representation of our cord's actual value of k.

Conclusion: We concluded that our experimental value of k is an acceptable value, and that it can be used to represent our cord's actual value of k. With our value of k, we will be able to find the maximum distance a given mass will fall as long as we keep our cord length constant. Our test showed us that as cord length increases, the value of k decreases, so we will need to take that into account when calculating how much cord we should use for the Bungee jump.

On my honor, I have neither given nor received any unacknowledged aid on this report.

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