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Title: Hooke's Law and the Bungee Cord

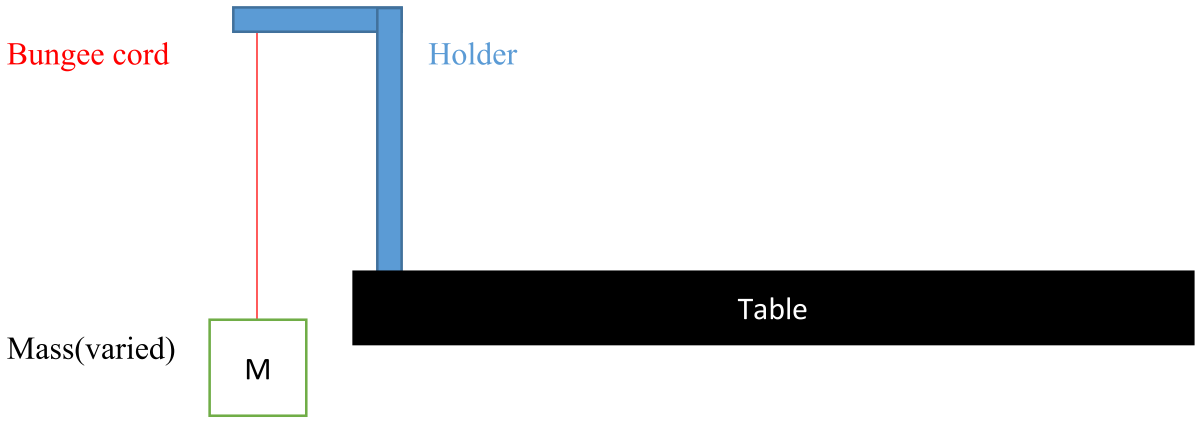
Abstract: The purpose of this experiment was to look at Hooke's law and see how it could help to create a quality bungee experience in the bungee drop. Hooke's law is useful to this experiment because it relates the force, spring constant, and the displacement using the equation $F = -kx$. Hooke's law was used by connecting a cord to a holder and a mass, and then changing the mass attached, while measuring the distance that the cord stretched. The changing force caused the displacement of the mass to change, which gave us a table of values that represented a linear equation, where the slope of the line is equal to the value of k. The slope constant had a small uncertainty and percent uncertainty, showing that it was reliable. There was still some error in the value of the spring constant, and the value of the spring constant was not always completely accurate when comparing the experimental value of force to the theoretical value for the earlier data points, but it very accurate for the later data points. The experiment showed how the spring constant is necessary when trying to design a quality bungee jump, and that further experiments using a force sensor should be conducted in order to create the ideal bungee drop.

Introduction: The purpose surrounding this experiment was to determine if and how Hooke's Law can be used in our bungee experiment to create the "best bungee experience" in our bungee drop, and then design an experiment to properly measure Hooke's law. The question for this experiment asked how Hooke's Law related to and object dropped from rest that was experiencing freefall. Relevant equations for this experiment are $F = -kx$ and $F = ma$, where the first equation is the equation for the spring force, and the second equation is for a force on an object. The coefficient k is the spring constant of the bungee cord, x is the distance that the spring is compressed, m is the mass of the system, and a is the acceleration of the system.

Substituting F for either equation, the equation $-kx = ma$ is derived, and changing the acceleration to the acceleration due to gravity, the equation becomes $kx = mg$, where the negatives cancel because the acceleration due to gravity is pointing in the negative direction. This can be simplified even further so $k = mg/x$ in order to solve for k. The basis of this experiment was to find the k value of the bungee cord and determine if the spring constant will be helpful in creating a successful bungee drop. Our hypothesis for this experiment was that Hooke's law would be a valuable equation in determining a good bungee drop because it would help us determine the spring force, which is necessary to know in order to calculate how far the object dropped will fall.

Methods: The method for this experiment was to hang the bungee cord from a hook and add varying masses to the other end of the bungee cord. The distance that the bungee cord stretched with each weight was then measured and recorded in order to determine the spring constant. The setup consisted of a varying mass, a cord, a hanger, and the table. The hanger, which sat on the table, was used to hold the cord at one end, while the other end of the cord had the mass hanging from it as shown in Diagram 1.

Diagram 1: Setup of experiment



The procedure for this experiment was to first stretch the cord out before adding any mass in order to avoid hysteresis which would cause an error in the experiment. Two loops were then tied on either end of the cord, and one loop was attached to the holder. The length of the un-stretched spring was measured from the knot of each loop in order to be able to determine the stretch of the spring with a mass attached from its un-stretched position without any mass attached to it. Ten different masses were picked, with each mass increasing by 0.020 kg, and each mass was attached to the bottom loop of the cord, and then the stretch in the cord was measured for each mass. In order to determine the stretch of the spring from its original position, we took the difference between the stretch of the cord with the mass attached and the cord without any mass attached. The values of the mass were then used to find the spring constant in the spring using the equation $mg = kx$.

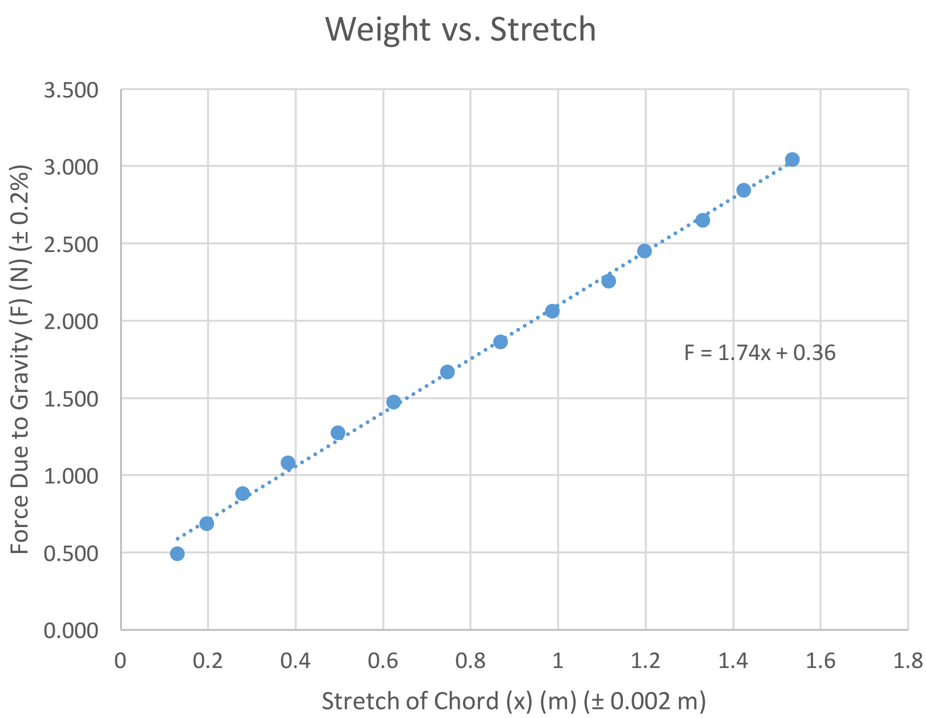
Results: The data collected shows how the force applied to a spring, or in this case a cord, has a linear relationship to the product of the spring constant and the distance the spring is stretched or compressed. In this case, the force acting on the system is equal to the weight of the mass attached to the spring, so the force is calculated by taking the product of the mass and the acceleration due to gravity, as shown in Table 1.

Table 1: Values of masses, un-stretched and stretched lengths of cord

Mass (m) (kg) (± 0.2%)	Force Due to Gravity (F) (N) (± 0.2%)	Unstretched Length (x_L) (m) (± 0.002 m)	Equilibrium Position (x_0) (m) (± 0.002 m)	Stretch (x) (m) (± 0.002 m)
0.050	0.491	0.51	0.64	0.13
0.070	0.687	0.51	0.706	0.196
0.090	0.883	0.51	0.789	0.279
0.110	1.079	0.51	0.893	0.383
0.130	1.275	0.51	1.007	0.497
0.150	1.472	0.51	1.134	0.624
0.170	1.668	0.51	1.256	0.746
0.190	1.864	0.51	1.378	0.868
0.210	2.060	0.51	1.497	0.987
0.230	2.256	0.51	1.626	1.116
0.250	2.453	0.51	1.707	1.197
0.270	2.649	0.51	1.841	1.331
0.290	2.845	0.51	1.934	1.424
0.310	3.041	0.51	2.045	1.535

The force due to gravity in Table 1 is the force that was caused by the weight, and is the force that acts on the system, causing the cord to stretch. The stretch in the cord was found by subtracting the un-stretched length of the cord (x_L) from the equilibrium positon of the cord with the mass (x_0) giving the stretch of the cord (x) as seen in Table 1. In Graph 1, the weight, also known as the force caused by gravity, is plotted against the stretch of the spring. The slope in this graph gives the value of *k*, or the spring constant, of the cord.

Graph 1: Graph of Weight vs. Stretch of the Cord



The equation for the graph of the weight against the stretch of the cord has an equation of $F = 1.74x + 0.36$, where the slope for the x-coefficient is the value of the spring constant, measured in N/m, and the constant attached is the y-intercept of the line of best fit. After taking the linear regression for the line of best fit, which can be seen in Table 2, we found uncertainties of ±0.02 N/m for the value of *k* and ±0.02 for the y-intercept. The percent uncertainties are 1.14% for the value of *k* and 5.55% for the value of the y-intercept.

Table 2: Regression Analysis for *k* and the y-intercept

	Coefficients	*Standard Error*
y-intercept	0.36	0.02
k	1.74	0.02

The experimental value of interest for this experiment was the value of *k*, which would then allow us to measure the maximum distance that the cord could stretch without exerting too much force on the egg during the bungee jump. The obtained value of *k* was 1.74 N/m, with an uncertainty of ±0.02 N/m, and a percent uncertainty of 1.14%, which was obtained using regression analysis. The value of *k* is important for the final bungee drop because it is necessary to know the spring constant of the cord in order to know how much force is exerted on the egg during the jump. If the spring constant was not known, then it would be impossible to drop the egg with any certainty that it would not break.

Discussion: The experimental values for the spring constant varies taken from the line of best fit varies from the theoretical values using the equation of the line of best fit in Table 1. The line of best fit does not contain all the data points, showing that there is a discrepancy between our experimental values and the theoretical values that we should have obtained. In order to test this discrepancy, the equation in Graph 1 can be used to calculate the experimental force and then be compared to the theoretical force of *mg*. Using the equation with the experimental value of *k* in Graph 1, the force on the system is .586 N, when the force in realty is .491 N, which is a significant difference between the experimental and theoretical values and outside the given uncertainty of force of 2%. However, using a separate data point that is closer to being on the line of best fit, the experimental value of force is 2.077 N, compared to the theoretical value of 2.060, which is within the uncertainty of force of 2%.

The variance in the differences between the experimental and theoretical values of *F* show that the value of *k* has an error, but that the error is more severe for certain data points than for others. The data points recorded at the beginning of the experiment have a much greater variation from the line of best fit in Graph 1 than the later data points do, showing that this could have been caused by a source of uncertainty rather than a flaw in the recording of the data. Hysteresis could have had some effect on the recorded data, as the cord could have not been fully stretched, thus skewing the earlier data points, while not affecting the later data points after the cord had been fully stretched. Other sources of uncertainty could have been a flaw in measurement, as the loop that was attached to the holder began to become stretched with the increase in the mass. This could have affected the measure of the stretch of the spring, and would throw off the data points. The variance in the data points could also show that Hooke's law is more accurate with a greater displacement, which would explain why that data was skewed for the masses that had displacements. The smaller the displacement of the mass, the further the data point was from the line of best fit, but as the displacement of the masses increased, the data points gradually got closer to the line of best fit. The data points for the greater masses with greater displacements should be the data points that are used when actually designing the bungee experiment, as they are the closest to the line of best fit and show a linear relationship.

The experimental value of *k* should be acceptable, as it is accurately able to calculate the force that the cord undergoes for the later data points. While the earlier data points are not very accurate when using *k*, there are enough data points that can accurately calculate the force using *k* to deem it acceptable. Our hypothesis was accurate because using Hooke's law with this experimental design allowed for us to find a value for the spring constant of the cord, which will allow us to determine the distance that the object will fall in the bungee experiment.

Conclusion: This experiment showed that the value of the spring constant is necessary to know when doing any experiment that involves spring force. In this specific experiment, using Hooke's law proved to be the most effective way to solve for the spring constant of the cord, and the spring constant was shown to be a critical part in determining the displacement that the mass would undergo. This is important for the bungee drop because the spring constant has to be known in order to determine how far a mass can travel before in hits the ground or undergoes too much force and is damaged. The next step in this experiment would be to use a force sensor and drop masses from rest, simulating a bungee drop, and then measuring the force that the mass underwent. This would help for the final bungee drop to ensure that the egg does not undergo an excessive amount of force, and would help to make sure that the actual bungee drop is a good bungee experience.